PROBABILISTIC VALIDATION OF ADVANCED DRIVER ASSISTANCE SYSTEMS

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Abstract: We present a methodological approach for validation of advanced driver assistance systems, based on randomized algorithms. The new methodology is more efficient than conventional validation by simulations and field tests, especially with increasing system complexity. The methodology consists of first specifying the perturbation set and performance criteria. Then a minimum required number of samples and a relevant sampling space is selected. Next an iterative randomized simulation is executed, followed by validation with hardware tests. The concept is illustrated with a simple adaptive cruise control problem. *Copyright© 2005 IFAC.*

Keywords: adaptive cruise control, control system analysis, randomized algorithms

1. INTRODUCTION

1.1 State-of-the-art

The increasing demand for safer passenger vehicles has stimulated the research and development of advanced driver assistance systems (ADASs). An ADAS typically consists of environment sensors (e.g. radar, laser, and vision sensors) and control systems to improve driving comfort and traffic safety by warning the driver, or even autonomous control of actuators. A state-of-the-art example of an ADAS is adaptive cruise control (ACC).

Figure 1 illustrates an ACC-equipped vehicle following a leading vehicle, with their position x and velocity v, where the subscripts 'l' and 'f' denote leader and follower respectively. Further defined are the *headway* $x_{\mathrm{r}} = x_{\mathrm{l}} - x_{\mathrm{f}}$, the relative velocity

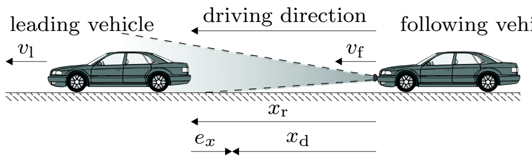

Fig. 1. Two vehicles in ACC mode.

$v_{\mathrm{r}} = v_{\mathrm{l}} - v_{\mathrm{f}}$, and the desired distance x_{d}. ACC tries to maintain a pre-defined velocity set-point, unless a slower vehicle is detected ahead. In that case vehicle 'f' is controlled to follow vehicle 'l' with equal velocity $v_{\mathrm{l}} = v_{\mathrm{f}}$ at a desired distance x_{d}. Since the ACC objective is to control the motion of a vehicle *relative* to a preceding vehicle, the vehicle state is chosen as $\mathbf{x} = \begin{bmatrix} x_{\mathrm{r}} & v_{\mathrm{r}} \end{bmatrix}^T$ with initial condition $\mathbf{x}(0) = \begin{bmatrix} x_{\mathrm{r},0} & v_{\mathrm{r},0} \end{bmatrix}^T$. We can then write the state space representation of the system as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} a_{\mathrm{f}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_{\mathrm{l}}, \qquad (1)$$

where the acceleration of the following vehicle a_{f} is the input, and the acceleration of the leading vehicle a_{l} forms the disturbance to the system.

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For distance control of the following vehicle, a control law can be used to produce a desired acceleration a_{d} that has to be achieved by lower-level actuator control, resulting in a_{f}. Assuming a given control law, the performance and stability of this controller should always be validated for a wide set of operating conditions (both from the viewpoint of system design and certification).

1.2 *Validation of ADAS control systems*

An iterative process of *simulations* and prototype *test drives* on a test track is often used for validation purposes. Test drives give realistic results, but can never cover the entire set of operating conditions, due to time and cost constraints. Test results can also be difficult to analyze, because traffic scenarios cannot be exactly reproduced. On the other hand, simulations have their limitations as well. For a low-order system as in Eq. (1), controller validation is still possible in a deterministic way or by using iterative algorithms. However, with a realistic nonlinear model and multiple traffic disturbances, the problem will become more difficult to solve, and eventually become *intractable* (Vidyasagar, 1998).

In order to make the simulation phase more efficient, a controller can be validated with a *grid search* over the operating range of all parameters (Fielding *et al.*, 2002). This is however inefficient, since an exhaustive grid search requires a very large number of experiments, perhaps even too large to be feasible. Alternatively, a worst-case analysis can be performed, but this may result in a conservative controller. Another possibility is a Monte Carlo strategy, where the system is simulated for a representative, but still very large, set of operating conditions (Stengel and Ray, 1991).

In practice the validation of an ADAS controller therefore requires much effort. In order to improve the transition from simulations to test drives, TNO has developed the VEhicle-Hardware-In-the-Loop (VEHIL) facility, a laboratory for testing ADASs. In VEHIL a real ADAS-equipped vehicle is tested in a hardware-in-the-loop simulation, providing accurate and repeatable tests, as is explained in detail in (Gietelink *et al.*, 2004). With VEHIL the validation of ADASs can be carried out safer, cheaper, more manageable, and more reliable. However, before VEHIL testing takes place, the simulation phase should be able to provide a reliable estimate of the performance.

1.3 *Objectives of this paper*

The objective of this paper is to present a methodological approach based on *randomized algorithms* (RAs) to provide an *efficient* test program in order to cover the entire set of operating conditions with a minimum number of samples. A simplified case study will be used as an illustration of this methodology for reasons of transparency.

2. RANDOMIZED ALGORITHMS

2.1 *Motivation for a probabilistic approach*

An alternative approach for solving a complex problem exactly, is to solve it *approximately* by using an RA. An RA is an algorithm that makes random choices during its execution (Motwani, 1995). The use of an RA can turn an intractable problem into a *tractable* one, but at the cost that the algorithm may fail to give a correct solution. The probability δ that the RA fails can be made arbitrarily close to zero, but never exactly equal to zero. This probability δ mainly depends on the *sample complexity*, i.e. the number of simulations performed, but also on the specification of the problem to be solved.

2.2 *Problem specification*

In this paper the controller validation is restricted to the measure of *safety*, expressed as the probability p that no collision will occur for a whole range of traffic situations. The safety measure for a single experiment is $\rho_{\mathrm{s}} \in \{0, 1\}$, where $\rho_{\mathrm{s}} = 1$ means that the ACC manages to follow the preceding vehicle at a safe distance, and $\rho_{\mathrm{s}} = 0$ means that the traffic scenario would require a brake intervention by the driver to prevent a collision.

The value of ρ_{s} for a particular traffic scenario depends on the perturbations imposed by that scenario. The disturbance to the ACC system is formed by the motion of other vehicles that are detected by the environment sensors. Apart from the acceleration of the preceding vehicle a_{l}, also the initial conditions $\mathbf{x}(0)$ determine the collision probability. These scenario parameters, together with measurement noise, unmodelled dynamics and various types of faults construct an n-dimensional *perturbation set* Δ. It is then of interest to evaluate the function $\rho_{\Delta} : \Delta \rightarrow \mathbb{R}$, as shown by the example in Figure 2.

2.3 *Formulation of an RA*

Based on the problem specification the use of a randomized approach for controller validation can be illustrated as follows (for more details see (Tempo *et al.*, 2004)). Consider an arbitrary process with only two possible outcomes: 'failure' ($\rho = 0$) and 'success' ($\rho = 1$). Suppose that we wish to determine the probability p for a successful outcome of this process. If N denotes the number of experiments with this process and N_{S} the number of experiments with successful results, then the ratio N_{S}/N is called the *empirical probability* $\hat{p}_N$ for a successful result of the process.

Suppose that the closed-loop system (in our case the ACC system) must be verified for a certain performance level ρ. It is then the goal to estimate the probability p that this performance ρ lies above a pre-specified threshold value γ. In order to compute $\hat{p}_N(\gamma)$ we can then use Algorithm 1.

Algorithm 1. Probabilistic performance verification (Tempo *et al.*, 2004).

Given a desired accuracy $\epsilon > 0$, $\delta \in (0,1)$ and a threshold $\gamma \geq 0$, this RA returns with a probability of at least $1-\delta$ an estimate $\hat{p}_N(\gamma)$ for $p(\gamma)$, such that $|p(\gamma) - \hat{p}_N(\gamma)| \leq \epsilon$.

(1) Determine the necessary sample size N with the *additive Chernoff bound* (Chernoff, 1952):

$$N = N_{\mathrm{ch}} \geq 1/2\epsilon^2 \ln 2/\delta \qquad (2)$$

(2) Draw N independent identically distributed (iid) samples $\Delta_1, \Delta_2, \ldots, \Delta_N$ in the perturbation set Δ according to its probability density function (pdf) f_Δ.

(3) Return the empirical probability

$$\hat{p}_N(\gamma) = \frac{1}{N}\sum\nolimits_{i=1}^{N} J(\Delta_i), \qquad (3)$$

where $J(\cdot)$ is the indicator function of the set $\mathcal{B} = \{\Delta : \rho(\Delta) \geq \gamma\}$, defined as

$$J(\Delta_i) = \begin{cases} 0, & \text{if } \rho < \gamma \\ 1, & \text{if } \rho \geq \gamma \end{cases} \qquad (4)$$

3. REDUCTION OF SAMPLE COMPLEXITY

3.1 Chernoff bound conservatism

Eq. (2) in Algorithm 1 gives the sufficient sample complexity N to estimate p for some values of ϵ and δ. Since $\hat{p}_N$ has a confidence interval, N is called a *soft* bound, as opposed to *hard* bounds given by a deterministic algorithm. As we will illustrate in Section 5, this soft bound is very conservative. Reduction of N for ADAS validation is therefore an important challenge.

The necessary sample size N can be reduced by reformulating the problem and using another test objective. Indeed, $\hat{p}_N$ does not say anything about the *minimum* or *maximum* level of performance that can be expected; e.g. a control system can have a good *average* performance, but also a poor *worst-case* performance. Fortunately, the necessary sample complexity for estimating worst-case performance is lower than for Eq. (2), as shown in (Tempo *et al.*, 2004). Still, N need to be reduced.

In addition, the sampling space Δ can be reduced by neglecting certain subsets that are impossible to occur. Obviously, a collision is more likely with lower values for $x_{\mathrm{r},0}$, $v_{\mathrm{r},0}$, and a_{l}, such that the collision occurrences are clustered in a specific subset Δ_{F}. This means that there is structure in the perturbation set Δ and in the function ρ_Δ that can be used to reduce the sampling space by disregarding specific subsets of Δ, of which the outcome is a priori known. An example is the subset Δ_{S} with combinations of positive acceleration and positive relative velocity that will never result in a potential collision.

3.2 Importance sampling

As stated above, it makes sense to give more attention to operating conditions that are more likely to cause a collision than others. Another possibility for using a priori knowledge on interesting samples is *importance sampling*, which is a technique to increase the number of occurrences of the event of which the probability p should be estimated (Madras, 2002).

Suppose that we want to estimate a probability p, given a perturbation Δ. If f_Δ is a uniform pdf on the interval $S = [0,1]$, denoted as $f_\Delta \in \mathcal{U}[0,1]$, our goal is then to estimate

$$p = \int_S J(\Delta) f_\Delta(\Delta) d\Delta = E[J(\Delta)], \qquad (5)$$

where we sample Δ from f_Δ, denoted as $\Delta \sim f_\Delta$. In order to highlight the interesting subset Δ_{F} it thus makes sense not to sample from the original pdf f_Δ, but instead use an artificial pdf, reflecting the 'importance' of the events, and then reweighing the observations to get an unbiased estimate.

We can now define an importance sampling pdf φ that is strictly positive on S. We can then write

$$p = \int_S \left(\frac{J(\Delta) f_\Delta(\Delta)}{\varphi(\Delta)}\right) \varphi(\Delta) d\Delta = E\left[\frac{J(\Phi) f_\Delta(\Phi)}{\varphi(\Phi)}\right] \qquad (6)$$

where $\Phi \sim \varphi$. The importance sampling estimator based on φ is

$$\hat{p}[\varphi]_N = \frac{1}{N}\sum\nolimits_{i=1}^{N} \frac{J(\Phi_i) f_\Delta(\Phi_i)}{\varphi(\Phi_i)} \qquad (7)$$

where $\Phi_1 \ldots \Phi_N$ are iid with pdf φ. Its variance is

$$\mathrm{var}\left(\hat{p}[\varphi]_N\right) = \frac{1}{N}\left[\int_S \frac{J(\Delta)^2 f_\Delta(\Delta)^2}{\varphi(\Delta)} d\Delta - p^2\right] \qquad (8)$$

An efficient estimator $\hat{p}[\varphi]_N$ is obtained by choosing φ proportional to the importance of the individual samples, with importance defined as

$|J(\Delta)f_\Delta(\Delta)|$. A rare but dangerous event can thus be equally important as a frequent but less critical event. An RA can then be formulated as follows.

Algorithm 2. Importance sampling.

Given ϵ, $\delta \in (0,1)$, γ, and the true distribution function f_Δ, this RA returns with probability at least $1-\delta$ an estimate $\hat{p}_N$ for the probability p, such that $|p - \hat{p}_N| < \epsilon$.

(1) Determine a strictly positive importance sampling pdf φ that emphasizes the interesting events;
(2) Select an initial number of samples N_{is};
(3) Draw N_{is} samples Φ_i according to φ;
(4) Return the empirical probability $\hat{p}[\varphi]_N = \frac{1}{N_{\text{is}}} \sum_{i=1}^{N_{\text{is}}} \frac{J(\Phi_i)f(\Phi_i)}{\varphi(\Phi_i)}$
(5) Determine the importance sampling variance $\sigma^2_{\text{is}} = \frac{1}{M-1} \sum_{j=1}^{M} (\hat{p}[\varphi]_{N_{\text{is},j}} - \hat{p}_M)^2$
(6) Determine the importance sampling reduction factor $\lambda_{\text{is}} = \sigma^2_{\text{is}}/\sigma^2_{\text{ss}}$, where σ^2_{ss} is the simple sampling variance;
(7) Check if $N_{\text{is}} \geq \lambda_{\text{is}} N_{\text{ch}}$; IF yes, THEN end algorithm; IF no, THEN increase N_{is} and return to step 3;

4. METHODOLOGICAL APPROACH

In this section we propose a generic methodological approach for validation of ADASs, consisting of the following steps: (1) specification; (2) simulation; (3) model validation; and (4) optimization of the performance estimate.

4.1 Specification

Firstly, define performance measures ρ, the corresponding evaluation criterion γ, and the desired δ and ϵ. Correspondingly, select the test objective in order to determine the type of bound for N:

- *Probability of performance satisfaction*: for the desired δ and ϵ, check whether ρ is below threshold γ with a certain probability level p for the whole perturbation set Δ.
- *Worst-case performance*: check if the worst-case performance $\rho_{\max}$ is within ϵ of $\hat{p}_N$ with a certain probability $1-\delta$.

Then, identify Δ and its pdf f_Δ by using preliminary field test results. Using knowledge on the structure of Δ or ρ_Δ, determine subsets of which the outcome (failure or success) is a priori known.

4.2 Simulation

Execute Algorithm 2 to cover the important part of Δ to estimate the performance $\hat{p}_N$. In general,

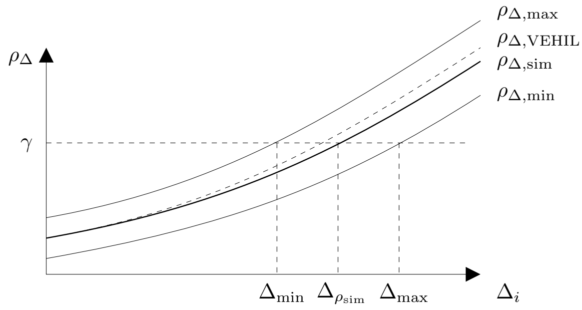

Fig. 2. Illustration of the dependency between the performance characteristic ρ_Δ and Δ.

ρ is a continuous value, although we will use a discrete value in our example.

The performance of Algorithm 2 depends heavily on the reliability of the models and the pdf's used in the simulation phase. The robustness of $\hat{p}_N$ to model uncertainty should therefore be considered when validating an ADAS in a randomized approach. The experimental relation between ρ and Δ from the simulations is then bounded between $\rho_{\max}$ and $\rho_{\min}$, as illustrated in Figure 2. This means that the estimated boundary value lies within the interval $[\Delta_{\min}, \Delta_{\max}]$, provided that ρ_Δ is a non-decreasing relation. For the sake of simplicity of the explanation we assume from now on that Δ is a 1-dimensional set (note however that the approach can easily be extended to n-dimensional perturbation sets).

4.3 Model validation

The most interesting samples Δ_i are chosen to be reproduced in the VEHIL facility, also in a randomized approach to efficiently cover Δ. These particular Δ_i are selected in the interval $[\Delta_{\min}, \Delta_{\max}]$. In VEHIL disturbances are introduced very accurately, thereby achieving a more reliable estimate than $\rho_{\Delta,\text{sim}}$. In this way model uncertainty is reduced, because of the replacement of a vehicle and sensor model by real hardware.

In addition, the test results can also be used for model validation. The estimate $\hat{p}_N$ may indicate necessary improvements in the system design regarding fine-tuning of the controller parameters. The simulation model can then be improved using the VEHIL test results until the simulation model proves to provide adequate performance, convergence in $\hat{p}_N$ and sufficient samples N.

4.4 Performance measure

In an iterative process the simulation results in step 2 and thus the estimate $\hat{p}_N$ can be improved. Subsequently, the VEHIL test program in step 3 can be better optimized by choosing a smaller interval $[\Delta_{\min},\ \Delta_{\max}]$. From the combination of simulation and VEHIL results the performance $\hat{p}_N$ of the ADAS can then be estimated with a high level of reliability, and the controller design can be improved. Finally, determine ρ_{VEHIL} with Eq. (7), correcting it for the higher occurrence rate in the interval $[\Delta_{\min},\ \Delta_{\max}]$.

5. EXAMPLES OF APPLICATION OF AN RA

In this section we will apply step 1 and 2 of the methodology to an illustrative case study, of which the 'true' outcome is exactly known. We will therefore use the system of Eq. (1) in combination with a simple linear controller.

5.1 Distance control law

Referring to Figure 1, the desired acceleration a_{d} is given by proportional feedback control of the distance separation error $e_x = x_{\mathrm{d}} - x_{\mathrm{r}}$ and its derivative $e_v = \dot{e}_x = v_{\mathrm{d}} - v_{\mathrm{r}}$

$$a_{\mathrm{d}} = -k_2 e_v - k_1 e_x = k_2 v_{\mathrm{r}} + k_1 (x_{\mathrm{r}} - s_0). \quad (9)$$

The distance x_{d} and the feedback gains k_i can be tuned in order to achieve a natural following behavior. We will use x_{d} equal to a constant value $s_0 = 40\,\mathrm{m}$, $v_{\mathrm{d}} = 0$, $k_1 = 1.2\,\mathrm{s}^{-2}$, and $k_2 = 1.7\,\mathrm{s}^{-1}$. This control law regulates both e_x en e_v to zero, provided that $k_1,\ k_2 > 0$.

In this paper we also neglect sensor processing delay and vehicle dynamics by assuming that the desired acceleration is realized at the input of the controlled system without any time lag, such that $u = a_{\mathrm{d}}$. However, we do introduce an actuator saturation, since ACC systems usually restrict the minimum and maximum control input u for safety reasons. In this case study we use the restriction that a_{f} is bounded between -2.5 and $2.5\,\mathrm{m/s}^2$.

5.2 Example 1: Gaussian distributed disturbance

Scenario definition We choose a highway scenario where the leading vehicle approaches a traffic jam and brakes to a full stop. The initial conditions are $x_{\mathrm{r},0} = x_{\mathrm{d}} = 40\,\mathrm{m}$, and $v_{\mathrm{l}}(0) = v_{\mathrm{f}}(0) = 30\,\mathrm{m/s}$. We assume that the deceleration of the preceding vehicle is the only disturbance. Typical measurements during ACC field testing (Fancher *et al.*, 1998) suggest that the acceleration profile can be roughly described as a random signal with a Gaussian pdf f^{N} with mean $\mu = 0$ and standard deviation $\sigma = 1.5$, denoted as $\mathcal{N}(0, 1.5)$, truncated on the interval $[-10, 10]\ \mathrm{m/s}^2$. We also restrict the analysis to the measure of safety ρ_{s}.

When the leading vehicle brakes hard, the ACC vehicle cannot obtain the necessary deceleration a_{d}, since the actuator saturates at -2.5 $\mathrm{m/s}^2$. Now, for fine-tuning the controller parameters, we would like to know the percentage of brake situations (p)for which ρ_{s} meets a pre-defined threshold. The threshold γ is simply set at 1 in this case (i.e. no collision). The safety of the system obviously decreases with a stronger deceleration a_{l}, such that the function $\rho_{\mathrm{s}}(a_{\mathrm{l}})$ is non-decreasing. Therefore, the boundary value Δ_γ, for which only just a collision is prevented, can easily be calculated numerically using an iterative algorithm as $-3.015\,\mathrm{m/s}^2$. Below this value the scenario will always result in a collision, above this value the ACC vehicle will be able to stop autonomously and prevent a collision.

Randomization of the problem Although for this example it is feasible to formulate a deterministic algorithm, in practice it can be difficult or even impossible to determine p in a deterministic way, when the dimension of Δ increases and the function ρ_Δ becomes non-convex. So instead of calculating p explicitly in deterministic sense, the function is randomized in such a way that it takes a random input Δ_i from its distribution function $f^{\mathrm{N}}(a_{\mathrm{l}})$, according to Algorithm 1.

In order to verify the performance of Algorithm 1, we execute it $M = 500$ times (each with $N = 100$). Each simulation set gives an estimate $\hat{p}_{N_j}$ for $j = 1, \ldots, M$. The distribution of the estimate is shown in the histogram in Figure 3. The accuracy and reliability for a single simulation set (each consisting of 100 simulations), can then be estimated as $\hat{\epsilon}$ and $\hat{\delta}$ respectively as follows.

The empirical mean $\hat{p}_M$ of the probability of a collision-free scenario is 0.97752, based on all $MN = 50\,000$ simulations. The variance of each individual estimate $\hat{p}_{N_j}$ can be found by the unbiased estimator for the variance $\sigma_{\mathrm{ss}}^2 = \frac{1}{M-1}\sum_{j=1}^{M}(\hat{p}_{N_j} - \hat{p}_M)^2 = 0.00020506$.

Analysis of the simulation results Suppose we desire that $\delta = 0.02$ and $\epsilon = 0.03$. Eq. (2) then gives $N_{\mathrm{ch}} = 2559$ as an upper bound on the sample complexity. From Figure 3 we see that only 10 out of 500 estimates $\hat{p}_{N_j}$ fall outside the interval $[p-\epsilon, p+\epsilon]$, which indicates an estimated reliability $\hat{\delta} = 0.02$. So the same level of accuracy and reliability has been achieved with only $N = 100$ instead of $N_{\mathrm{ch}} = 2559$ samples.

This example suggests that a certain value for δ and ϵ can be achieved with a lower number of samples than deemed necessary by the Chernoff bound. Correspondingly, by choosing $N = N_{\mathrm{ch}}$, a higher level of accuracy and reliability can be obtained. Although the degree of conservatism for this example is rather limited, it can be shown

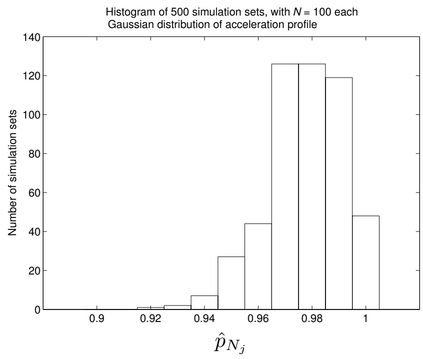

Fig. 3. Histogram of 500 estimates $\hat{p}_{N_j}$, with $N = 100$ each, where the acceleration profile is sampled from a Gaussian pdf $\mathcal{N}(0, 1.5)$.

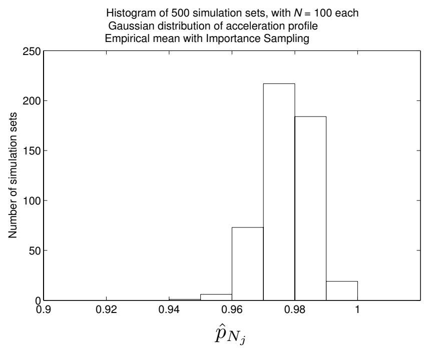

Fig. 4. Histogram of 500 estimates $\hat{p}_{N_j}$, with $N = 100$ each, where the acceleration profile is sampled from an importance sampling distribution $\varphi = -0.005a_{\rm l} + 0.05$.

that this conservatism increases with smaller values for δ and ϵ (Vidyasagar, 1998).

5.3 Example 2: Importance sampling

Suppose that we again want to estimate the probability p for the given perturbation, as specified by Example 1. The goal is then to estimate $p = \int_S J(a_{\rm l}) f^{\rm N}(a_{\rm l}) da_{\rm l} = E[J(\Delta)]$ from Eq. (5), where $\Delta \sim f^{\rm N}$. Assuming this Gaussian pdf, a simple sampling method will yield relatively few samples in the interval of interest, i.e. [-10, -3.015], as was observed in Example 2. We therefore define a more suitable φ to sample from. We choose $\varphi(a_{\rm l}) = -0.005a_{\rm l} + 0.05$ with $a_{\rm l}$ bounded on the interval $S = [-10, 10]$.

With Algorithm 2 more 'important' samples will be generated for every j-th simulation set, thus decreasing the variance of $\hat{p}_N$. This result can be seen from the histogram in Figure 4, where the empirical mean $\hat{p}_M = 0.97751$ and the variance $\sigma^2_{\rm is} = 5.92 \cdot 10^{-5}$, based on 50 000 simulations. Note that $\hat{p}_M$ is approximately equal to the situation in Example 2. However, the variance of a particular realization $\hat{p}_{N_j}$ has decreased with a factor 3.5! The inverse of this factor is the importance sampling reduction factor $\lambda_{\rm is}$, which indicates the reduction in the sample complexity necessary to achieve the same level of accuracy and reliability as the Chernoff bound (Stadler, 1993).

However, the performance of the importance sampling method heavily depends on the reliability of the pdf φ to generate random variables, and of the models used in the simulation. In addition, the sample complexity cannot be determined a priori, thus requiring the iterative loop in Algorithm 2 and the need for a suitable stopping criterion.

6. CONCLUSIONS

We have presented a methodological approach for probabilistic performance validation of advanced driver assistance systems (ADASs), and applied a randomized algorithm to a simple adaptive cruise control problem. This probabilistic approach cannot *prove* that the system is safe or reliable. However, we accept a certain risk of failure (though small), since any other conventional validation process (e.g. test drives) are also based on a probabilistic analysis. Furthermore, use can be made of a priori information on the system, thereby emphasizing interesting samples.

Ongoing research is focussed on extension of this methodological approach to more complex ADAS models, with non-convex performance functions, multiple performance criteria, and to include model validation with the VEHIL facility.

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