# ROBUST STABILITY OF A CLASS OF NONLINEAR DELAYED IMPULSIVE NEURAL NETWORKS WITH INTERVAL UNCERTAINTIES [1]

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Abstract: Many complex dynamical behaviors in the real world can be modelled by interval impulsive differential systems or interval impulsive neural networks. This paper formulates and studies a model of uncertain interval nonlinear delayed impulsive neural networks. Some fundamental issues such as the global exponential robust stability of the equilibrium of this new model are established and analyzed.


Keywords: Impulsive neural network, Nonlinearity, Time-delay, Interval uncertainty, Global robust exponential stability.

## 1. INTRODUCTION

In the last decade, tremendous efforts have been devoted to the study of theory and applications of neural networks (Baldi and Atiya, 1994; Cao and Zhou, 1998; Civalleri *et al.*, 1993; Carpenter *et al.*, 1987; Cohen and Grossberg, 1983; Fang and Kincaid, 1996; Grossberg, 1988; Guan and Chen, 1999; Guan *et al.*, 2000; Hopfield, 1984; Liao and Yu, 1998; Michel and Gray, 1990; Qiao *et al.*, 2001). Various neural network structures are built based on well-established mathematical and engineering theories and particularly fundamental principles that govern biological neural systems.

The most widely studied neural networks in the current literature may be grouped into continuous and discrete networks. However, there are still many neural networks existing in the real world with dynamics in between these two groups, more precisely with impulses (Bainov and Simeonov, 1989; Lakshmikantham *et al.*, 1989; Liu and Guan, 1996). Impulsive phenomena can be found in many fields of information science, electronics, automatic control systems, computer networking, artificial intelligence, robotics, and telecommunications, among others. Recently, a new type of neural networks — impulsive neural networks — have been constructed as an appropriate description of such special phenomena of abrupt and qualitative dynamical changes (Guan and Chen, 1999; Guan *et al.*, 2000).

In deterministic impulsive neural networks, many vital data such as the neurons fire rate and the synaptic interconnection weight are usually acquired and processed by means of statistical anal-

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ysis, in which estimation errors inevitably exist. On the other hand, parameter perturbation in neural-networks, particularly in their implementation with the VLSI technology, is also unavoidable. To handle these uncertainties, one way is to explore the ranges of such vital data as well as the bounds of such circuitry parameters, and to describe them by intervals. In addition, since delays frequently appear in both biological and artificial neural networks, the delays may slow down the rate of information transmission and may also result in instability (Baldi and Atiya, 1994; Carpenter *et al.*, 1987; Hou and Qian, 1998; Liao and Yu, 1998; Lu, 2000). In circuits, for example, qualitative analysis of delay neural networks has known to be necessary for design. These motivate the present study of dynamical properties of uncertain nonlinear impulsive neural networks with delays, and this is the first attempt for an investigation of the subject, to the best of our knowledge (Liao and Yu, 1998; Lu, 2000).

The paper is organized as follows. In Sect. 2, the uncertain nonlinear delayed impulsive neural network model is first described. Then, in Sect. 3, the concept of equilibrium of the model is introduced and its global exponential robust stability property is established. Finally, some conclusions are given in Sect. 4.

## 2. PROBLEM FORMULATION

Based on the structure of the Grossberg-Cohen-Hopfied model (Carpenter *et al.*, 1987; Cohen and Grossberg, 1983; Grossberg, 1988; Hopfield, 1982; Hopfield, 1984), the cellular neural networks (Cao and Zhou, 1998; Civalleri *et al.*, 1993), and the delayed impulsive autoassociative neural network model (Guan and Chen, 1999; Guan *et al.*, 2000), the uncertain nonlinear delayed impulsive neural network model considered in the paper is described as follows:

$$Dy_i = -a_i P_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} F_j(y_j(t)) Du_j + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) Dw_j + I_i, \qquad (1)$$

where $i = 1, \cdots, n$, $a_i > 0$ are given constants, $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$ are uncertain parameters, $\underline{a}_{ij}$, $\overline{a}_{ij}$, $\underline{b}_{ij}$, and $\overline{b}_{ij}$ are known real numbers, $i, j = 1, \cdots, n$; $\tau \geq 0$ is the time delay; $y = \mathrm{col}(y_1, \cdots, y_n) \in R^n$, $y_i$ is the state of the $i$th neuron, $I = \mathrm{col}(I_1, \cdots, I_n)$ is the input to the network, $D$ denotes the distributional derivative, $u_i, w_i : J = [t_0, +\infty) \mapsto R$ are functions of bounded variations and right-continuous on any compact subinterval of $J$, and $F_i(\cdot)$ and $G_i(\cdot)$ are integrable functions with respect to $u_i$ and $w_i$, respectively, $P_i(\cdot)$ is differentiable.

System (1) has initial conditions

$$y_i(t) = \psi_i(t), \quad t_0 - \tau \leq t \leq t_0, \quad i = 1, \ldots, n, \qquad (2)$$

where $\Psi(t) = \mathrm{col}(\psi_1(t), \cdots, \psi_n(t))$ are functions of bounded variation and right-continuous on any compact subinterval of $[t_0 - \tau, t_0]$.

*Definition 2.1.* The vector-valued function $y(t) = (y_1(t), \cdots, y_n(t))^\top \in R^n$ is said to be a solution of (1) with the initial condition (2), if it satisfies Eqs. (1) and (2) with the given parameters $a_i > 0$, $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$.

It is obviously that, in general, the solution of (1) depends on the corresponding parameters $a_i, a_{ij}$ and $b_{ij}$. In the subsequent discussions, it is always assumed that a solution of (1) exists and is unique (Guan and Liu, 1992; Pandit and Deo, 1982). In fact, the model formulation given above implies that the states $y_i$, $i = 1, 2, \cdots$, are functions of bounded variations and right-continuous on any compact subinterval of $J$, in which $Du_i$ and $Dw_i$ represent the effects of sudden changes in the states of the system at the discontinuity points of $u_i$ and $w_i$, $i = 1, \cdots, n$. They both can be identified with the usual Lebesgue-Stieltjes measure.

Without loss of generality, assume that

$$u_i(t) = t + \sum_{k=1}^{\infty} \beta_{ik} H_k(t),$$
$$w_i(t) = t + \sum_{k=1}^{\infty} \gamma_{ik} H_k(t), \quad i = 1, \cdots, n, \qquad (3)$$

where $\beta_{ik}$ and $\gamma_{ik}$ are constants, with discontinuity points $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, and $H_k(t)$ are Heaviside functions defined by

$$H_k(t) = \begin{cases} 0, & t < t_k \\ 1, & t \geq t_k. \end{cases}$$

It can be easily seen from (3) that

$$Du_i = 1 + \sum_{k=1}^{\infty} \beta_{ik} \delta(t - t_k), Dw_i = 1 + \sum_{k=1}^{\infty} \gamma_{ik} \delta(t - t_k),$$

where $\delta(t)$ is the Dirac impulsive function, which means that the state of system (1) has jumps at $t_k, k = 1, 2, \cdots$.

*Remark 2.1.* For any $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$, define

$$a_{ij}^{(0)} = \frac{1}{2}\left(\underline{a}_{ij} + \overline{a}_{ij}\right), \quad a_{ij}^{(1)} = \frac{1}{2}\left(\overline{a}_{ij} - \underline{a}_{ij}\right), \qquad (4)$$

$$b_{ij}^{(0)} = \frac{1}{2}\left(\underline{b}_{ij} + \overline{b}_{ij}\right), \quad b_{ij}^{(1)} = \frac{1}{2}\left(\overline{b}_{ij} - \underline{b}_{ij}\right). \qquad (5)$$

Then $a_{ij} = a_{ij}^{(0)} + \Delta a_{ij}$, $b_{ij} = b_{ij}^{(0)} + \Delta b_{ij}$, where $\Delta a_{ij}$ and $\Delta b_{ij}$ are uncertain parameter perturbations, which satisfy $|\Delta a_{ij}| \leq a_{ij}^{(1)}$, $|\Delta b_{ij}| \leq b_{ij}^{(1)}$.

System (1), except for its uncertainty, is a general framework for neural network models, which includes some well-known networks as its special cases. For instance, in system (1), if $\beta_{jk} = 0$ and $\gamma_{jk} = 0$, $j = 1, \cdots, n$, $k = 1, 2, \cdots$, then it reduces to

$$y_i'(t) = -a_i P_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} F_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) + I_i, \qquad (6)$$

which is a typical continuous-time nonlinear neural network with time delay. Stability of system (6), in the special case with $G_j = F_j$, has been investigated in (Lu, 2000).

Similarly, if $b_{ij} = 0$, $\beta_{jk} = 0$, and $P_i(y_i) = y_i$, $i, j = 1, \cdots, n$, $k = 1, 2, \cdots$, then system (1) reduces to

$$y_i'(t) = -a_i y_i(t) + \sum_{j=1}^{n} a_{ij} F_j(y_j(t)) + I_i, \qquad (7)$$

which is the typical continuous Grossberg-Cohen-Hopfied neural network model. Stability of system (7) has been extensively studied (Civalleri *et al.*, 1993; Cohen and Grossberg, 1983; Fang and Kincaid, 1996; Hopfield, 1982; Hopfield, 1984; Matsuoka, 1992).

Also, if $a_{ij} = 0$, $\gamma_{jk} = 0$, and $P_i(y_i) = y_i$, $i, j = 1, \cdots, n$, $k = 1, 2, \cdots$, then system (1) becomes

$$y_i'(t) = -a_i y_i(t) + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) + I_i, \qquad (8)$$

which is the typical continuous delayed Hopfield neural network model and has been studied in, for instance, (Baldi and Atiya, 1994; Gopalsamy and He, 1994; Hou and Qian, 1998; Zhang *et al.*, 1996).

In addition, if $\beta_{jk} = 0$, $\gamma_{jk} = 0$, and if the parameters $a_{ij}$, $b_{ij}$ and functions $P_i$, $F_j$, $G_j$ are appropriately chosen, then system (1) reduces to the delayed bidirectional associated memory networks (BAM)(Gopalsamy and He, 1994) and the cellular neural networks (CNN) with delay (Carpenter *et al.*, 1987; Civalleri *et al.*, 1993).

A typical characteristic of the nonlinear impulsive neural network system (1) that differs from most existing models (Guan and Chen, 1999; Guan *et al.*, 2000) is its discontinuity in the form of impulses. Therefore, to ensure that it can be successfully used to describe and to deal with various impulsive phenomena, specially some evolution processes involving impulses in the real word (Bainov and Simeonov, 1989; Guan *et al.*, 1995; Lakshmikantham *et al.*, 1989), a detailed investigation of this new model is very important.

In what follows, the concept of equilibrium of the model is introduced and its global robust exponential stability are first studied.

## 3. EQUILIBRIUM AND ITS GLOBAL ROBUST EXPONENTIAL STABILITY

For simplicity, first consider system (1) in the nominal situation, that is,

$$Dy_i = -a_i P_i(y_i) + \sum_{j=1}^{n} a_{ij} F_j(y_j) Du_j + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) Dw_j + I_i, \qquad (1)'$$

where $i = 1, \cdots, n$, $a_i, a_{ij}$, and $b_{ij}$ are given and determined parameters.

*Definition 3.1.* A solution $y(t) = (y_1(t), \cdots, y_n(t))^\top$ of system $(1)'$ is said to be an *equilibrium solution*, if it satisfies the following equations:

$$-a_i P_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} F_j(y_j(t)) Du_j + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) Dw_j + I_i = 0, \qquad (9)$$

where $t \in [t_{k-1}, t_k)$, $i = 1, \cdots, n$ and $k = 1, 2, \cdots$.

*Remark 3.1.* (i) It is easy to see that if $y(t) = (y_1(t), \cdots, y_n(t))^\top$ is an equilibrium solution of system $(1)'$, then, in general, $y(t)$ is a right-continuous piecewise-constant vector-valued function. In fact, Eq. (9) implies that, for $t \in [t_{k-1}, t_k)$

$$y_i'(t) = -a_i P_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} F_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} G_j(y_j(t-\tau)) + I_i = 0, \qquad (10)$$

since both $u_j'$ and $w_j'$ in system $(1)'$ exist on all intervals $[t_{k-1}, t_k)$. This immediately implies that $y(t)$ is a constant-valued vector in $[t_{k-1}, t_k)$, and that

$$y(t) = (y_1(t), \cdots, y_n(t))^\top := (y_1(t_{k-1}), \cdots, y_n(t_{k-1}))^\top, \qquad (11)$$

where $t \in [t_{k-1}, t_k), y_1(t_{k-1}), \cdots, y_n(t_{k-1})$ are constants. On the other hand, $y(t)$ is a solution of system $(1)'$, so that

$$
\begin{aligned}
& y_i(t_k, t_0, \Psi) - y_i(t_k - h, t_0, \Psi) \\
&= \int_{t_k-h}^{t_k} I_i ds - \int_{t_k-h}^{t_k} a_i P_i(y_i(s)) ds \\
&\quad + \int_{t_k-h}^{t_k} \sum_{j=1}^{n} a_{ij} F_j(y_j(s)) du_j(s) \\
&\quad + \int_{t_k-h}^{t_k} \sum_{j=1}^{n} b_{ij} G_j(y_j(s-\tau)) dw_j(s)\,,
\end{aligned}
$$

where $h > 0$ is sufficiently small. As $h \to 0^+$, it gives

$$
\begin{aligned}
& y_i(t_k, t_0, \Psi) - y_i(t_k^-, t_0, \Psi) \\
&= \sum_{j=1}^{n} a_{ij} F_j(y_j(t_k))\, \beta_{jk} \\
&\quad + \sum_{j=1}^{n} b_{ij} G_j(y_j(t_k - \tau))\, \gamma_{jk}\,, \qquad (12)
\end{aligned}
$$

which usually causes a jump at the discontinuity point $t_k$.

(ii) For system $(1)'$, if $\beta_{jk} = \gamma_{jk} = 0$, $j = 1, \cdots, n$, $k = 1, 2, \cdots$, then it reduces to the usual ordinary differential system without impulses. Accordingly, Definition 3.1 reduces to the usual definition of equilibrium.

(iii) Apparently, the equilibrium solution defined in Definition 3.1 is also applicable to the impulsive systems without time-delay.

It is well known that stability of equilibrium play an important role in the theory and applications of neural networks. Furthermore, unavoidably network parameter fluctuation and noise perturbation exist, so it is essential to investigate the robustness of the equilibrium stability for neural network against such uncertainties. In doing so, the equilibrium of the uncertain system is assumed to be the same as that of the unperturbed system. In this case, the equilibrium solution remains to be a special solution of the system. In this regard, an equilibrium of the interval system (1) is defined as follows.

*Definition 3.2.* The function $y^*(t) = (y_1^*(t), \cdots, y_n^*(t))^\top$ is said to be an *equilibrium solution* of system (1) if it is an equilibrium of system $(1)'$ for any parameters $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$.

As can be seen above, in general, the equilibrium solution of system (1) is a right-continuous piecewise-constant vector-valued function. But, if there exists a $y = y_0$ such that $F(y_0) = 0$, $G(y_0) = 0$, and $-A_d P(y_0) + I = 0$, then system (1) has a constant equilibrium $y^*(t) \equiv y_0$, where $A_d = \mathrm{diag}(a_1, \cdots, a_n)$, $I = (I_1, \cdots, I_n)^\top$, $P(y(\cdot)) = (P_1(y_1(\cdot)), \cdots, P_n(y_n(\cdot)))^\top$, $F(y(\cdot)) = (F_1(y_1(\cdot)), \cdots, F_n(y_n(\cdot)))^\top$, $G(y(\cdot)) = (G_1(y_1(\cdot)), \cdots, G_n(y_n(\cdot)))^\top$.

**Example 3.1** Consider system (1) with $n = 1$:

$$
\begin{aligned}
Dy = &-a_1 P_1(y(t)) + a_{11} F_1(y(t)) Du_1 \\
&+ b_{11} G_1(y(t-\tau)) Dw_1 + I_1,
\end{aligned}
$$

where $a_1 = I_1 > 0$, $a_{11} \in [\underline{a}_{11}, \overline{a}_{11}] = [-2, 3]$, $b_{11} \in [\underline{b}_{11}, \overline{b}_{11}] = [-3, 2]$, $u_1$ and $w_1$ are given by (3), and $P_1(y) = \sin y$, $F_1(y) = y + \frac{\pi}{2}\sin 3y$, $G_1(y) = \frac{4}{\pi^2} y^2 + \cos 2y$. It is easy to verify that $y = \frac{\pi}{2}$ is an equilibrium solution of the system.

*Definition 3.3.* The equilibrium $y^*(t) = (y_1^*(t), \cdots, y_n^*(t))^\top$ of system (1) is said to be *global robustly exponentially stable*, if for any parameters $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$, the equilibrium $y^*(t) = (y_1^*(t), \cdots, y_n^*(t))^\top$ of system (1) remains to be global exponentially stable.

If $y^*(t) = (y_1^*(t), \cdots, y_n^*(t))^\top$ is an equilibrium of system (1), let $x_i = y_i - y_i^*$, $i = 1, \cdots, n$. Then,

$$
\begin{aligned}
Dx_i = &-a_i p_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) Du_j \\
&+ \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau)) Dw_j, \qquad (13)
\end{aligned}
$$

with $i = 1, \cdots, n$, $a_i > 0$, $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$, where

$$
\begin{aligned}
p_i(x_i(t)) &= P_i(x_i(t) + y_i^*(t)) - P_i(y_i^*(t)), \\
f_j(x_j(t)) &= F_j(x_j(t) + y_j^*(t)) - F_j(y_j^*(t)),
\end{aligned}
$$

and

$$
\begin{aligned}
g_j(x_j(t-\tau)) = &\, G_j(x_j(t-\tau) + y_j^*(t-\tau)) \\
&- G_j(y_j^*(t-\tau)).
\end{aligned}
$$

It is then clear that the robust stability of the zero solution, $x = 0$, of system (13) corresponds to the robust stability of the equilibrium $y = y^*$ of system (1). Therefore, one may simply consider system (13) with initial conditions

$$
x_i(t) = \phi_i(t), \; t_0 - \tau \le t \le t_0, \; i = 1, \cdots, n, \quad (14)
$$

where $\Phi(t) = \mathrm{col}\big(\phi_1(t), \cdots, \phi_n(t)\big)$ are functions of bounded variation and right-continuous on any compact subinterval of $[t_0 - \tau, t_0]$.

Now, one is in a position to discuss the global robust exponential stability of the uncertain nonlinear and delayed impulsive neural network (1), or (13).

Assume that $t_k - t_{k-1} \geq \delta\tau$, $\delta > 1$, $\tau > 0$, $k = 1, 2, \cdots$. For any functions $p_i$, $f_i$, and $g_i$, satisfying

$$z\,p_i(z) \geq 0, \quad |p_i'(z)| \geq p_i^0 > 0,$$
$$|f_i(z)| \leq f_i^0 |z|, \quad z f_i(z) \geq 0,$$

and

$$g_i(z)| \leq g_i^0 |z|, \quad \forall\, z \in R, \tag{15}$$

where $p_i^0$, $f_i^0$ and $g_i^0$ are constants, $i = 1, \cdots, n$, let

$$\alpha_k = \max_{1\leq j\leq n} \left\{ \sum_{i=1}^{n} \left( \left| a_{ij}^{(0)} \right| + a_{ij}^{(1)} \right) |\beta_{jk}|\ f_j^0 \right\}, \tag{16}$$

$$\beta_k = \frac{1}{1-\alpha_k},$$

$$\gamma_k = \beta_k \max_{1\leq j\leq n} \left\{ \sum_{i=1}^{n} \left( \left| b_{ij}^{(0)} \right| + b_{ij}^{(1)} \right) |\gamma_{jk}|\ g_j^0 \right\}, \tag{17}$$

where $a_{ij}^{(0)}, a_{ij}^{(1)}, b_{ij}^{(0)}$ and $b_{ij}^{(1)}$ are defined by (4) and (5). Also, for constants $\xi_i > 0$, $i = 1, \cdots, n$, let

$$a = \min_{1\leq j\leq n} \left\{ a_j\, p_j^0 - a_\xi^+ f_j^0 \right\},$$

$$b = \max_{1\leq j\leq n} \left\{ \sum_{i=1}^{n} \frac{\xi_i}{\xi_j} \left( \left| b_{ij}^{(0)} \right| + b_{ij}^{(1)} \right) g_j^0 \right\}, \tag{18}$$

$$a_\xi = \max_{1\leq j\leq n} \left\{ \overline{a}_{jj} + \sum_{i=1, i\neq j}^{n} \frac{\xi_i}{\xi_j} \left( \left| a_{ij}^{(0)} \right| + a_{ij}^{(1)} \right) \right\},$$

$$a_\xi^+ = \max\left\{0, a_\xi\right\}. \tag{19}$$

*Theorem 3.1.* Suppose that (15) is satisfied and for $k = 1, 2, \cdots$,

(i) there exist constants $\xi_i > 0$, $i = 1, \cdots, n$, such that $a > b > 0$;
(ii) $\alpha_k < 1$;
(iii) $\beta_k + \gamma_k e^{\lambda\tau} \leq M$ for a constant $M \geq 1$, where $\lambda$ is a positive solution of $\lambda - a + b e^{\lambda\tau} = 0$.

Then, $\frac{\ln(\xi M)}{\delta\tau} - \lambda < 0$ implies that the zero solution of system (13), namely, the equilibrium $y = y^*$ of system (1), is globally robustly exponentially stable, where $\xi := \left( \max\limits_{1\leq i\leq n} \{\xi_i\} \right) \Big/ \left( \min\limits_{1\leq i\leq n} \{\xi_i\} \right)$, $\alpha_k$, $\beta_k$, $\gamma_k$, $a$ and $b$ are defined in (16), (17), and (18), respectively.

*Remark 3.2.* Apparently, the dynamical behavior of the uncertain nonlinear and delayed impulsive neural network (1), or (13), mainly depends on the bounds of perturbation intervals, $\overline{a}_{ij}$, $\underline{a}_{ij}$, $\overline{b}_{ij}$ and $\underline{b}_{ij}$, the properties of the nonlinear functions, $p_i$, $f_i$ and $g_i$, and the functions $u_i$ and $v_i$ which caused the impulsive jumps to the neural network. So, it is natural to see that some key parameters, such as $a$, $b$, $\alpha_k$, $\beta_k$ and $\gamma_k$, appear in Theorem 3.1, which are given on the basis of the aforementioned terms. In some sense, the parameters $\alpha_k$, $\beta_k$ and $\gamma_k$ characterize the impulsive effects associated with the perturbation intervals and the state variables of neural network (13) at the discontinuity points $t_k$. Since neural network (13) has both uncertain interval perturbations and impulsive perturbations at discontinuity points $t_k$, it is necessary that some conditions, such as Assumptions (ii) and (iii), are imposed on the parameters of the neural network at $t_k$, so as to guarantee the stability of the overall neural network. Similarly, parameters $a$ and $b$, as well as Assumption (i), characterize the intrinsic relations between the nonlinear functions and uncertain intervals of (13).

In addition, in Theorem 3.1, one may take $\xi_i = 1$, $i = 1, \cdots, n$, so as to draw a corresponding conclusion for which the conditions are easier to verify.

**Example 3.2** Consider system (13) with $n = 2$:

$$\begin{aligned} Dx_i = &-a_i p_i(x_i(t)) + a_{i1} f_1(x_1(t)) Du_1 \\ &+ a_{i2} f_2(x_2(t)) Du_2 + b_{i1} g_1(x_1(t-\tau)) Dw_1 \\ &+ b_{i2} g_2(x_2(t-\tau)) Dw_2,\ i = 1, 2, \end{aligned} \tag{20}$$

where $t_k - t_{k-1} \geq \delta\tau$ with $\tau = \frac{1}{2}$, $t_0 \geq 0, u_j$ and $w_j$ are given by (3) with $\beta_{1k} = \frac{1}{6}(-1)^k$, $\beta_{2k} = \frac{1}{6}(-1)^{k+1}$, $\gamma_{1k} = \frac{1}{3}(-1)^{k+1}$, and $\gamma_{2k} = \frac{1}{3}(-1)^k$, and $a_1 = 7, a_2 = 8$, $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, and $b_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$ with $[\underline{a}_{11}, \overline{a}_{11}] = [-1, 1]$, $[\underline{a}_{12}, \overline{a}_{12}] = [0, 2]$, $[\underline{a}_{21}, \overline{a}_{21}] = [1, 2]$, $[\underline{a}_{22}, \overline{a}_{22}] = [-\frac{1}{2}, 1]$, $[\underline{b}_{11}, \overline{b}_{11}] = [-1, 1]$, $[\underline{b}_{12}, \overline{b}_{12}] = [0, 2]$, $[\underline{b}_{21}, \overline{b}_{21}] = [-2, 0]$, $[\underline{b}_{22}, \overline{b}_{22}] = [0, 1]$, and $p_1(z) = 3.35z - \sin z$, $p_2(z) = 4z + \sin z$, $f_i(z) = g_i(z) = \mathrm{arctg} z$, $i = 1, 2$.

It is easy to compute from (15)-(19) that $p_1^0 = 2.35$, $p_2^0 = 3$, $f_i^0 = g_i^0 = 1$, $i = 1, 2$, $\alpha_k = \frac{1}{2}$, $\beta_k = 2$, $\gamma_k = 2$.

Take $\xi_i = 1, i = 1, 2, \cdots, n$, then $a = 16.45$, $b = 3$. Obviously, $a > b > 0$, $\alpha_k < 1$, and $\beta_k + \gamma_k e^{\lambda\tau} \leq M = 11$ with $\lambda = 3$ satisfying $\lambda - a + b e^{\lambda\tau} = 0$, that is, the conditions (i)-(iii) of Theorem 3.1 are satisfied. Therefore, when $\delta \geq 1.65$, $\frac{\ln(M)}{\delta\tau} - \lambda < -0.1$, which implies from Theorem 3.1 that the zero solution of system (20) is globally robustly exponentially stable.

## 4. CONCLUSIONS

This paper has formulated and analyzed an interval delayed nonlinear impulsive neural network model. This new model is useful for describing some evolutionary processes that have sequential abrupt changes and parameter perturbations. Such networks cannot be appropriately represented by either purely continuous or purely discrete additive networks; therefore, the new model is important for mathematical modelling and potential engineering applications in the future. Some fundamental issues such as the network equilibrium and the robust exponential stability of the equilibrium have also been studied in detail, with some explicit results derived.

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