

LOCALIZED SWITCHING RAMP-METERING CONTROL WITH QUEUE LENGTH ESTIMATION AND REGULATION AND MICROSCOPIC SIMULATION RESULTS [★]

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Abstract: In this paper, we first review a localized ramp-metering strategy for freeways that achieves the goal of reducing the spatial and temporal span of traffic congestion, using locally available information. The control scheme is composed of a switching mainline-traffic responsive ramp-metering controller that adapts to the different traffic dynamics under different congestion conditions—free-flow or congested—and a queue length regulator that yields improved performance over the currently used *ad hoc* “queue-override” scheme and prevents the queue from exceeding the ramp storage capacity. Subsequently, a queue length estimator is designed to provide feedback to the queue length regulator, using the queue-detector speed data that are available in the field. Test results on a calibrated microscopic traffic simulator demonstrate the performance and effectiveness of the switching ramp-metering controller, the queue length estimator and regulator, and the overall control strategy. The Total Vehicle and Passenger Delays are both reduced by 16%, while the Total Vehicle Time and the Total Average Vehicle Speed are improved by 5.6% and 5.8%, respectively. As a comparison, simulation results of ALINEA are also presented. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Freeway traffic congestion is a major problem in today’s metropolitan areas. It occurs regularly during commute hours. In addition, non-recurrent congestion often takes place as a result of incidents, road work, or public events. Congestion causes inefficient operation of freeways, wasting of resources, increased pollution, and intensified driver fatigue.

The 2004 Urban Mobility Report (Schrank and Lomax, 2004) finds: “Congestion has grown everywhere in areas of all sizes. Congestion occurs during longer portions of the day and delays more travelers and goods

than ever before.” In the report, it was calculated that in 2002, congestion cost Americans 3.5 billion hours of delay and 5.7 billion gallons of wasted fuel, with an equivalent monetary cost of U.S. \$63.2 billion.

On-ramp metering has been widely used as an effective strategy to increase freeway operation efficiency. It has been recommended to the U.S. Federal Highway Administration as the No. 1 tool to address the congestion problem, other than adding more capacity to transportation infrastructures (Cambridge Systematics, Inc. and Texas Transportation Institute, 2004). It has been reported that ramp metering was able to reduce delay by 101 million person-hours in 2002, approximately 5% of the congestion delay on freeways where ramp-metering was in effect (Schrank and Lomax, 2004).

In this paper, we first briefly review a set of localized ramp-metering algorithms that we have developed (Sun and Horowitz, 2005), including a switching mainline-

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traffic responsive ramp-metering controller that employs a different feedback structure, depending on whether the freeway is in a *free-flow* or *congested* mode, a queue length regulator that keeps the queue below the ramp storage capacity limit, and a localized control strategy to achieve the goal of reducing the spatial and temporal span of the congestion.

In addition, we design and validate a queue length estimator using the speed data measured by the queue detector. This estimator provides the feedback that is needed by the queue length regulator.

Test results of this set of ramp-metering algorithms, as well as those of ALINEA (Papageorgiou et al., 1991), on a calibrated microscopic traffic simulator are also presented.

2. PERFORMANCE MEASURES

In this section, some performance measures are defined for quantitative evaluation of a given freeway segment. All the quantities are defined for the time period T and the freeway segment L .

$D_{V,tot}$ Total Vehicle Distance, which is defined as the sum of the distances traveled by all the vehicles in L within T .

$T_{V,tot}$ Total Vehicle Time, which is the sum of the time that is spent by all vehicles in L within T . It includes the time spent by vehicles waiting in the on-ramp queues.

$DL_{V,tot}$ Total Vehicle Delay (also known as Congestion Delay), which is the difference between the Total Vehicle Time and the time that would be spent by all the vehicles if there were no congestion. $DL_{V,tot} = T_{V,tot} - D_{V,tot}/v_0$, where v_0 is the nominal free-flow speed.

$\bar{v}_{V,tot}$ Average Total Vehicle Speed $\bar{v}_{V,tot} = D_{V,tot}/T_{V,tot}$.

$\bar{v}_{V,ml}$ Average Mainline Vehicle Speed, which is similar to $\bar{v}_{V,tot}$ but calculated using only data from the mainline portion of the freeway.

Another set of passenger-weighted performance measures can be defined by first collecting the traffic quantities separately for the low- or high-occupancy vehicle classes, and then weighting these quantities by the average passenger number in each vehicle class when calculating the performance measures. This set of passenger-weighted performance measures include Total Passenger Distance $D_{P,tot}$, Total Passenger Time $T_{P,tot}$, Total Passenger Delay $DL_{P,tot}$, Average Total Passenger Speed $\bar{v}_{P,tot}$, and Average Mainline Passenger Speed $\bar{v}_{P,ml}$.

3. TEST SITE

A segment of Interstate 210 Westbound (I-210W) in Pasadena, California, has been selected as the test-bed for new ramp-metering algorithms. It is approximately 14 miles long, from Vernon Avenue (Mile Post 39.159) to Fair Oaks Avenue (Mile Post 25.4),

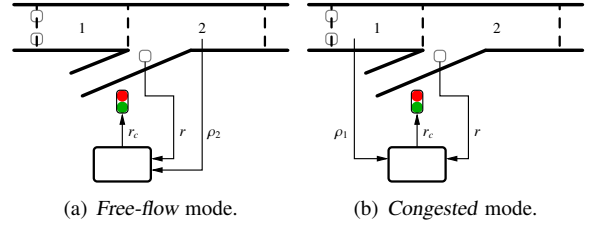


Fig. 1. Different control structures for different congestion modes.

with 20 metered on-ramps, 1 uncontrolled freeway-to-freeway connector (I-605), and 18 off-ramps. In our previous efforts, microscopic (Gomes et al., 2004) and macroscopic (Muñoz et al., 2004) traffic simulation tools have been calibrated to this test site.

4. THE SWITCHING MAINLINE-TRAFFIC RESPONSIVE RAMP-METERING CONTROLLER, QUEUE LENGTH REGULATOR, AND LOCALIZED CONTROL STRATEGY

In this section, we briefly review the switching mainline-traffic responsive ramp-metering controller, the queue length regulator, and the localized control strategy that we have developed (Sun and Horowitz, 2005).

4.1 The Switching-Mode Model and Ramp-Metering Controller

Based on the observation that traffic dynamics are different under different congestion conditions—free-flow or congested—we have piecewise linearized the cell transmission model (Daganzo, 1994, 1995) and derived a switching-mode traffic model (Muñoz et al., 2003). The properties of this switching-mode model can be summarized as follows:

- (1) In *free-flow* mode, traffic moves freely at drivers' desired speeds without restriction. Therefore, the information travels from upstream to downstream, and the downstream vehicle densities are affected by the upstream densities. As a consequence, the vehicle density at one location is observable using a *downstream* measurement, and it can be controlled by metering an *upstream* on-ramp.
- (2) In *congested* mode, traffic moves slowly and is restricted by the downstream available spaces, which means the information travels from downstream to upstream. Therefore, the upstream densities are affected by the downstream densities. The observability and controllability are opposite to those of the *free-flow* mode: the vehicle density at one location can be observed using an *upstream* measurement and can be controlled by a *downstream* ramp.

It is therefore natural to employ different feedback structures for the mainline-traffic responsive ramp-metering controller, depending on different congestion modes, as illustrated in Fig. 1. The mixture Kalman

filter (MKF) based traffic state estimator that we have developed (Sun et al., 2003, 2004) is used to estimate, in real time, the most probable congestion mode and the cell vehicle densities in a freeway section. The estimated congestion mode is used to determine the appropriate control structure, and the estimated vehicle densities are used as the feedback.

To compensate for disturbances and to accommodate the difference between the model sampling time and the metering-rate update interval, a multirate linear quadratic control with integral action (multirate LQI) approach (Sun and Horowitz, 2005) was used to synthesize the ramp-metering controller for both of the congestion modes.

In either mode, the desired metering rate is first calculated using

$$r_c(t) = r(t-1) - K(t) \begin{bmatrix} \tilde{\rho} \\ z \end{bmatrix} (t), \quad (1)$$

and then saturated using

$$r_c(t) = \min\{r_{\max}, \max\{r_{\min}, r_c(t)\}\}, \quad (2)$$

where $r(t)$ is the actual ramp flow measured by the entrance loop-detector, as shown in Fig. 2, $\tilde{\rho}(t)$ is the mainline density error, $z(t) = \tilde{\rho}(t) - \tilde{\rho}(t-1)$, and r_{\max} and r_{\min} are the established maximum and minimum metering rates. There is an anti-windup scheme in (1) that is similar to what is used in ALINEA (Papageorgiou et al., 1991) to address the metering-rate saturation problem.

In (1),

$$K(t) = \begin{cases} K_p, & \text{when } t = np \text{ for some } n \in \mathbb{Z}, \\ 0, & \text{when } t \neq np \text{ for any } n \in \mathbb{Z}, \end{cases} \quad (3)$$

where p is the ratio between the metering-rate update interval and the model sampling time, and K_p is determined by solving a periodic Riccati equation. See (Sun and Horowitz, 2005) for details.

4.2 The Queue Length Regulator

A typical configuration of loop detectors and signals on an on-ramp on a California freeway is shown in Fig. 2. To prevent the on-ramp queue from spilling over into surface streets and interfering with the street traffic, the queue length must be regulated. The “queue-override” scheme currently used on California freeways steadily increases the metering rate (e.g., 120 vehicles per hour per lane every 30 seconds) whenever the end of the queue reaches the queue detector, until the metering rate saturates to the maximum value. After the queue dissipates and drops below the queue detector location, the metering rate is reset to the value determined by the mainline-traffic responsive metering controller. This scheme is equivalent to an integral control with a saturated integrating rate and resetting. It can be easily shown that the resulting closed loop dynamics is not asymptotically stable, given that the open loop queue length dynamics is that of a simple integrator. It

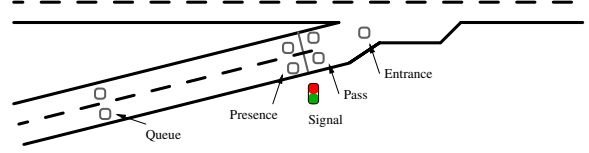


Fig. 2. A typical configuration of loop detectors and signals on an on-ramp.

has been noted (Gordon, 1996; Smaragdis and Papageorgiou, 2004) that this queue-override scheme leads to oscillatory behavior and under-utilization of on-ramp storage capacities. Gordon (1996) attempted to improve the performance of the queue-override scheme by filtering the occupancy signal and reducing the sampling time interval. Smaragdis and Papageorgiou (2004) proposed a proportional controller that relies on the on-ramp vehicle demands. However, real-time demand measurements are generally not available in the field, and such a control scheme would have to instead rely on historical demands.

If the queue length $l(t)$ could be measured, an asymptotically stable PI-controller

$$r_c(z) = \left(k_p + \frac{k_I}{z-1} \right) \tilde{l}(z) \quad (4)$$

would be able to regulate the queue length precisely at a specified value. This controller can be designed by choosing proper gains k_p and k_I , using the root-locus method on the closed-loop sensitivity function from the disturbance to the error, which is given by

$$\frac{\tilde{l}(z)}{d(z)} = \frac{T_s(z-1)}{(z-1)^2 - k_p T_s z + (k_I - k_p) T_s}, \quad (5)$$

where $\tilde{l}(t)$ is the queue length error, and $d(t)$ is the vehicle arrival rate (the demand), which is regarded as a disturbance.

The anti-windup and saturating mechanisms in (1) and (2) also need to be implemented in this queue length regulator.

4.3 The Localized Control Strategy

Localized traffic responsive strategies are desired for reasons including reduced algorithmic complexity, lower computational requirements, and higher robustness to changing traffic conditions such as unpredicted demands. We have proposed a localized metering strategy and tested it on a calibrated macroscopic traffic model (Sun and Horowitz, 2005). It is described as follows:

- (1) The set-point for the switching mainline-traffic responsive ramp-metering controller is chosen to be the critical density, i.e., the density at which congestion is about to form. This is adopted to slow down congestion shock waves propagating in the upstream direction and to speed up congestion shock waves moving downstream.
- (2) The set-point of the queue length regulator is the maximum allowed queue length. This value

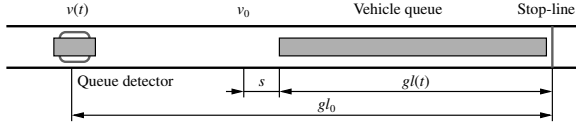


Fig. 3. A schematics for on-ramp queue length estimation.

is chosen to fully utilize the available storage capacity on the ramp and to deter short-trip travelers from using the freeway, thus making the freeway capacity available to longer-distance travelers.

- (3) The *higher* of the two rates determined by the mainline-traffic responsive metering controller and the queue length regulator is chosen to be the actual metering rate that is sent to the signal control box. This rule, which was first proposed by Smaragdis and Papageorgiou (2004), is designed to properly resolve the conflict between the objectives of these two controllers.

5. AN ON-RAMP QUEUE LENGTH ESTIMATOR

Though it has a more stable response than the queue-override scheme, the PI regulator described in Section 4.2 needs the current queue length as its feedback, which unfortunately is not available in the field currently. A suitable estimator has to be designed using available information, such as the vehicle speed measured by the queue detector.

We assume the following simplified driving behavior model for a vehicle approaching the end of the queue: the vehicle decelerates at a constant rate, $-a$, from its cruising speed to a target speed v_0 at the position where the distance from the end of the queue is s . We also assume a uniform effective vehicle length g . Let l_0 be the number of vehicle spaces from the stop line to the queue detector and $v(t)$ be the vehicle speed measured by the queue detector. See Fig. 3.

A straightforward kinematic calculation yields

$$g(l_0 - l(t)) - s = \frac{v(t)^2 - v_0^2}{2a}, \quad (6)$$

where $l(t)$ is the current queue length, in number of vehicles. From (6), we obtain

$$gl(t) = gl_0 - s + \frac{v_0^2}{2a} - \frac{v(t)^2}{2a} = c_0 - c_2 v(t)^2. \quad (7)$$

To determine the coefficients c_0 and c_2 in (7), a curve fitting was performed on the $gl(t)$ and $v(t)$ data collected using the VISSIM (PTV AG, 2004) microscopic traffic simulator. Fig. 4 shows a typical scatter plot of queue lengths versus speeds. A few points need to be noted:

- (1) When the queue is shorter than a certain length, the approaching vehicles pass the queue detector at the drivers' desired cruising speeds, which are

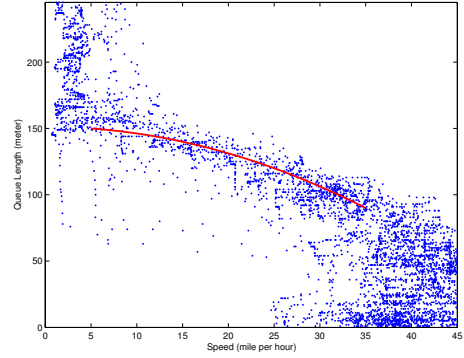


Fig. 4. A scatter plot of queue lengths vs. queue detector speeds and the least median-of-squares curve fit for one of the on-ramps.

independent of the queue length. This phenomenon corresponds to the data points at the lower-right corner of the scatter plot.

- (2) When the queue is longer than gl_0 , i.e., the queue has extended beyond the queue detector, the measured speed is also a constant, which is related to the queue discharging rate and the vehicle lengths, and is also independent of the queue length. This phenomenon corresponds to the data points at the upper-left corner of the scatter plot.
- (3) There are many outliers among the data points. Therefore, the usual least-squares curve fitting method, which is biased toward outliers, is not suitable.

For these reasons, we neglected the data points whose speeds are below v_{\min} or above v_{\max} and those whose queue lengths are below l_{\min} or above l_{\max} in the curve fitting. These values were determined by visual inspection of the scatter plots.

To increase robustness to outliers, we used the least median-of-squares (Rousseeuw, 1984) curve fitting method, instead of the usual least (sum-of-)squares. The fitted curve is also shown in Fig. 4.

After the $l-v$ curve is fitted for each on-ramp, the difference between the actual and desired queue length, which is used as the feedback to the regulator (4), is estimated as

$$\tilde{l}(t) = \begin{cases} (c_0 - gl_0 - c_2 v(t)^2)/g, & \text{if } v(t) \geq v_{\min}, \\ -kc_2(v(t)^2 - v_{\min}^2)/g, & \text{if } v(t) < v_{\min}, \end{cases} \quad (8)$$

where k is a tuning parameter.

When $v(t) < v_{\min}$, the end of the queue is very close to or beyond the queue detector, and the speed $v(t)$ measured by the queue detector is a constant, which is roughly gr_c . Therefore, (8) can be thought of as saturating \tilde{l} to $-kc_2((gr_c)^2 - v_{\min}^2)/g$, which is larger when the metering rate r_c is lower. This has a desirable effect on the regulator: The metering rate r_c will be increased more aggressively when there is more room for this increase, and more slowly when r_c is close to

Table 1. Performance measures for the I-210 test segment under different ramp-metering algorithms. Q/R means queue estimation and regulation.

	$D_{V,tot}$ (10^3 mile)	$T_{V,tot}$ (10^3 hour)	$DL_{V,tot}$ (10^3 hour)	$\bar{v}_{V,tot}$ (mph)	$\bar{v}_{V,ml}$ (mph)	$D_{P,tot}$ (10^6 mile)	$T_{P,tot}$ (10^3 hour)	$DL_{P,tot}$ (10^3 hour)	$\bar{v}_{P,tot}$ (mph)	$\bar{v}_{P,ml}$ (mph)
No metering	973	24.0	8.52	40.6	37.8	1.32	31.7	10.8	41.6	39.4
Switching + Q/R	972	22.6	7.19	43.0	40.8	1.32	29.9	9.0	44.0	42.4
Improvement	–	5.6%	16%	5.8%	7.9%	–	5.5%	16%	5.7%	7.4%
Switching	974	22.3	6.81	43.7	55.8	1.32	29.3	8.3	45.1	56.4
Improvement	–	7.1%	20%	7.7%	47%	–	7.7%	23%	8.4%	43%
ALINEA + Q/R	974	23.5	8.01	41.5	39.2	1.32	31.0	10.1	42.5	40.7
Improvement	–	2.0%	5.8%	2.2%	3.6%	–	2.1%	6.3%	2.3%	3.4%

its maximum value. In addition, this saturation value can be further tuned by changing the value of k .

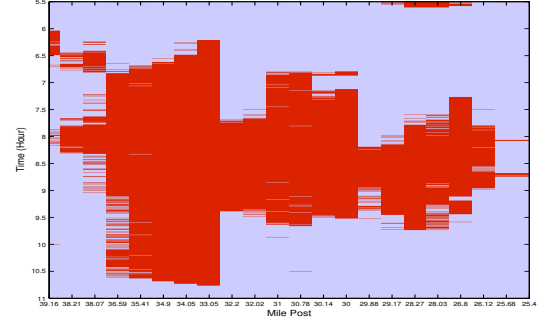
It is also worth mentioning that the coefficients c_0 and c_2 identified by the least median-of-squares fitting are very close to the nominal values predicted by using the actual distance between the stop-line and the queue detector and a nominal vehicle deceleration of 2.5 m/s^2 . Therefore, when the queue length measurements are unavailable through any means to perform a curve fitting, these nominal values can be used in the queue length estimation.

6. RESULTS

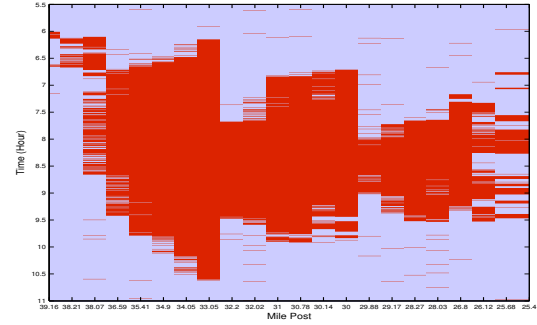
The switching mainline-traffic responsive metering controller and the queue length regulator were implemented and interfaced with the VISSIM microscopic traffic simulation model that has been calibrated to the I-210W test segment by Gomes et al. (2004). The localized control strategy described in Section 4.3 was used. Fig. 5 shows the congestion patterns, as determined by the MKF traffic state estimator (Sun et al., 2003, 2004), before and after ramp metering. In the plots, red indicates *congested* mode and blue *free-flow*. The vertical axis is time, from 5:30 to 11:00 in the morning. The horizontal axis is the mile post along the freeway, and the traffic travels from left to right. It can be seen that the localized ramp metering strategy was able to reduce the congestion, in terms of both the spatial span and the time duration.

We also implemented a modified version of ALINEA (Papageorgiou et al., 1991) and combined it with the queue length estimator and regulator that we have developed in Sections 5 and 4.2. In this modified ALINEA, we used the occupancy data measured upstream to the on-ramps, instead of those measured downstream in the original ALINEA, due to the loop-detector configuration on California freeways. Gomes (2004) has shown, using the calibrated I-210W VISSIM model, that this modified ALINEA can achieve comparable, sometimes even better, performance, when compared to the original ALINEA. Optimal ALINEA gain (7000) and set-point (27.2%) found by Gomes (2004) were used in our simulations.

Different ramp-metering algorithms, including 1) switching LQI plus queue regulation, 2) switching LQI only, without queue regulation, and 3) ALINEA plus



(a) Without ramp metering.



(b) With ramp metering.

Fig. 5. Congestion modes for the I-210W test segment under different metering scenarios, blue: *free-flow*, and red: *congested*.

queue regulation, were tested with the I-210W VISSIM model. Under each scenario, 8 simulation runs were carried out, with 8 different VISSIM random seeds. The random seed was chosen to be the second of the computer clock at the time when it was changed, to ensure its randomness.

Some of the performance measures for this freeway segment, as defined in Section 2, are listed in Table 1. The listed numbers are the averages from the 8 simulation runs for each scenario. In calculating these quantities, the average passenger numbers per one low- and high-occupancy vehicle are assumed to be 1.2 and 2.5, respectively, and the nominal free-flow speed v_0 is 63 miles per hour.

Under all the scenarios, the freeway segment served almost the same amount of demand, as measured by the Total Vehicle Distance $D_{V,tot}$ or Total Passenger Distance $D_{P,tot}$. Ramp-metering was able to reduce the congestion under all the metered scenarios. For example, with the switching LQI mainline control and

queue length regulation, the Total Vehicle Delay (also known as Congestion Delay) $DL_{V,tot}$ was reduced by 16%, while with the switching LQI mainline control only, $DL_{V,tot}$ was reduced by 20%.

When only the switching mainline-traffic responsive metering was used, without activating the queue length regulator, on-ramp queues can be accumulated to arbitrary lengths, sometimes hundreds of vehicles. In this case, almost all the congestion on the mainline was eliminated, as evidenced by the average mainline vehicle speed $\bar{v}_{v,ml}$, which was 55.8 mph. Another interesting phenomenon in this case is that the relative improvements in terms of passenger-weighted performance measures were greater than those in terms of vehicle performance measures. This is because many of the metered on-ramps on this freeway segment have designated lanes for HOVs to bypass the long queues.

It can also be seen from the numbers in Table 1 that the switching control algorithm outperforms the modified ALINEA, when both algorithms are combined with the queue length estimator and regulator.

7. CONCLUSIONS

In this paper, we first reviewed a localized ramp-metering strategy that achieves the control goal of reducing the spatial and temporal extent of the congestion, using locally available information. This control strategy works with a switching mainline-traffic responsive ramp-metering controller that adapts to the different traffic dynamics under different congestion conditions, and a PI queue length regulator that yields improved performance over the currently used “queue-override” scheme and keeps the queue under the ramp storage capacity limit. In addition, a queue length estimator was designed to provide feedback to the queue length regulator, using the queue-detector speed data that are available in the field.

Test results on the calibrated VISSIM I-210W microscopic model demonstrated the performance and effectiveness of the switching ramp-metering controller, the queue length estimator and regulator, and the overall control strategy. The Total Vehicle and Passenger Delays were both reduced by 16%, while the Total Vehicle Time and the Total Average Vehicle Speed were improved by 5.6% and 5.8%. As a comparison, simulation results of ALINEA were also presented. The switching mainline-traffic responsive control was able to outperform ALINEA, when both algorithms were combined with the same queue length estimator and regulator.

REFERENCES

- Cambridge Systematics, Inc. and Texas Transportation Institute (2004). Traffic congestion and reliability: Linking solutions to problems. Tech. rep., Federal Highway Administration. URL http://www.ops.fhwa.dot.gov/congestion_report/.
- Daganzo, C. F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological* **28**(4), 269–287.
- Daganzo, C. F. (1995). The cell transmission model, part II: Network traffic. *Transportation Research Part B: Methodological* **29**(2), 79–93.
- Gomes, G., A. D. May, and R. Horowitz (2004). Calibration of VISSIM for a congested highway. In: *The 83rd Annual Meeting of the Transportation Research Board*. Washington, D.C., USA.
- Gomes, G. C. (2004). *Optimization and Microsimulation of On-ramp Metering for Congested Freeways*. Ph.D. dissertation, University of California, Berkeley.
- Gordon, R. L. (1996). Algorithm for controlling spillback from ramp meters. *Transportation Research Record* (1554), 162–171.
- Muñoz, L., X. Sun, R. Horowitz, and L. Alvarez (2003). Traffic density estimation with the cell transmission model. In: *Proceedings of the 2003 American Control Conference*. Denver, Colorado, USA, pp. 3750–3755.
- Muñoz, L., X. Sun, D. Sun, G. Gomes, and R. Horowitz (2004). Methodological calibration of the cell transmission model. In: *Proceedings of the 2004 American Control Conference*. Boston, Massachusetts, USA, pp. 798–803.
- Papageorgiou, M., H. Hadj-Salem, and J.-M. Blosseville (1991). ALINEA: A local feedback control law for on-ramp metering. *Transportation Research Record* (1320), 58–64.
- PTV AG (2004). VISSIM. Web page. URL http://www.english.ptv.de/cgi-bin/traffic/traf_vissim.pl.
- Rousseeuw, P. J. (1984). Least median of squares regression. *Journal of the American Statistical Association* **79**(388), 871–880.
- Schrank, D. and T. Lomax (2004). The 2004 urban mobility report. Tech. rep., Texas Transportation Institute. URL <http://mobility.tamu.edu/>.
- Smaragdis, E. and M. Papageorgiou (2004). Series of new local ramp metering strategies. *Transportation Research Record* (1856), 74–86.
- Sun, X. and R. Horowitz (2005). A localized switching ramp-metering controller with a queue length regulator for congested freeways. In: *Proceedings of the 2005 American Control Conference*. Portland, Oregon, USA. To appear.
- Sun, X., L. Muñoz, and R. Horowitz (2003). Highway traffic state estimation using improved mixture Kalman filters for effective ramp metering control. In: *Proceedings of the 42nd IEEE Conference on Decision and Control*. Maui, Hawaii, USA, pp. 6333–6338.
- Sun, X., L. Muñoz, and R. Horowitz (2004). Mixture Kalman filter based highway congestion mode and vehicle density estimator and its application. In: *Proceedings of the 2004 American Control Conference*. Boston, Massachusetts, USA, pp. 2098–2103.