AN ITERATIVE METHOD FOR MULTI-OBJECTIVE DYNAMIC OUTPUT FEEDBACK SYNTHESIS

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Abstract: We address in this paper the problem of multi-objective dynamic output-feedback synthesis for continuous linear time-invariant systems. The design objective can be a mix of H_2 performance, H_∞ performance and closed-loop pole clustering. A new sufficient condition is proposed. This new condition is based on additional variables which allow the use of different Lyapunov functions for each performance criterion. It is shown that the conservatism of the standard method is perceptibly improved. The efficiency of this approach is then illustrated by some academic examples.

Keywords: multi-objective performance, dynamic output-feedback, Lyapunov functions, LMI optimization, iterative algorithm.

1. INTRODUCTION

It is well known that if stability is a minimum requirement in control synthesis, it is not sufficient in practice and some performance level has to be guaranteed. In fact, the fundamental objective of a feedback control system is to achieve different performances. This paper focuses on multi-objective synthesis via dynamic output-feedback including H_2 performance, H_∞ performance and pole clustering for continuous linear time-invariant systems.

Lyapunov methods have proven to be efficient for solving such problems. However, in most LMI characterization of different performances, the different Lyapunov variables (one for each performance requirement) are multiplied with controller variables. This makes this problem non convex and then very hard to solve. One way to get rid of this problem is to impose the same Lyapunov function for all performance characterization. This approach is known in the literature as the Lyapunov Shaping Paradigm (LSP) and has been introduced by (Scherer et al., 1997).

This restriction is one important source of the remaining conservatism encountered in the multi-objective synthesis. Recently, in order to reduce this conservatism (Ebihara et al., 2004) have proposed a less conservative method based on dilated LMI approach using non common Lyapunov functions. This approach is still conservative and can't be easily generalized for all possible performance criteria (limited to $H_2$and root clustering performances).

In this paper, a new approach is proposed to the design of locally optimal output-feedback controllers using non common Lyapunov function. This method is based on an iterative algorithm taking advantage of the degrees of freedom introduced by some additional variables.

Some numerical examples are then presented to evaluate the efficiency of the proposed approach with respect to those presented in (Ebihara et al., 2004).

Notations:

- *A^t denotes the transpose of A.*
- *$sym(A)= A + A^t$.*
- *$[*]^t BA= A^t BA$.*
- $\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^t & C \end{bmatrix}$
- *$\sigma(A)$ means the set of the eigenvalues of A.*

2. PRELIMINARIES

In this paper, we consider the continuous-time linear time-invariant system with the state-space equations:

$$(S)\begin{cases} x = Ax + B_\infty w_\infty + B_2 w_2 + Bu \\ z_\infty = C_\infty x + D_\infty w_\infty + D_{\infty 2} w_2 + D_{\infty u} u \\ z_2 = C_2 x + D_{2\infty} w_\infty + D_{\infty 2} u \\ y = Cx + D_{y\infty} w_\infty + D_{y2} w_2 \end{cases} \quad (1)$$

where $x \in IR^n$ is the state, $w_\infty \in IR^{m\infty}$ and $w_2 \in IR^{m2}$ are two disturbance vectors, $u \in IR^m$ is the input vector, $z_2 \in IR^{r2}$ and , $z_\infty \in IR^{r\infty}$ are vectors of output signals related to the performance of the control system and $y \in IR^r$ is a vector of measured output.

Let T_∞, T_2 denote the closed-loop transfer functions from, respectively w_∞ to z_∞ and w_2 to z_2.
For some dynamical output-feedback controller u=Ky, our goal is to compute a dynamical output-feedback controller:

$$\begin{cases} x_k = A_k x_k + B_k y \\ u = C_k x_k \end{cases} \quad (2)$$

that satisfies different performance objectives. The specifications include H_∞ and H_2 performance as well as closed-loop poles clustering in some sub-region of the left half-plane. The motivation for using such a combination of performance measures are as follows:

- The H_∞ performance is convenient to enforce robustness of the closed-loop with respect to unstructured model uncertainties as well as to express frequency domain specifications.
- H_2 performance is related to the RMS response to white noise and to the energy of the system impulse response.
- Poles clustering are useful to enforce some minimum decay rate or closed-loop damping. They are also useful to avoid fast dynamic and high frequency gain in the controller.

Problem1: Multi-objective synthesis

Given an H_∞ level γ, find a strictly proper full-order output-feedback dynamic controller K (2) such that :

- The H_∞ performance $\|T_\infty\|_\infty \le \gamma$ is achieved.

- The closed-loop poles must lie in a prescribed sub-region of the left-half complex plane defined as $D = H(\alpha) \cap C(c,r) \cap S(k)$ (figure 1) where:

- $H(\alpha)$ is a half-plane region defined as:
  $H(\alpha) = \{\lambda \in C : \text{Re}(\lambda) < -\alpha\} (\alpha > 0)$.
- *C(c,r)* is a disk defined as:
  $C(c,r)) = \{\lambda \in C : |\lambda - c| < r\} (c < -r < 0)$.
- *S(k)* is a conic sector region defined as:
  $S(k) = \{\lambda \in C : |\text{Im}(\lambda)| < k|\text{Re}(\lambda)|\} (k > 0)$.

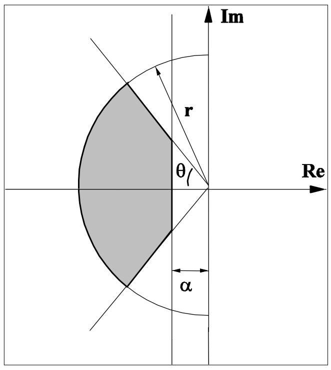

Figure 1: Region D for c=0

- The H_2 performance $\|T_2\|_2$ is minimized subject to the above two constraints.

The controller K is then a solution of the following optimization problem:

$$\begin{aligned} & \underset{K}{Min} \|T_2\|_2 \\ under \quad & \\ & \|T_\infty\|_\infty < \gamma \\ & \sigma(A_{cl}) \subset D \end{aligned} \quad (3)$$

2.1 Closed-loop formulation of the multi-objective problem

For a given dynamic output-feedback controller K, let the minimal realization be given by:

$$P_{cl} := \begin{bmatrix} A_{cl} & B_{\infty cl} & B_{2cl} \\ C_{\infty cl} & D_{\infty cl} & D_{\infty 2cl} \\ C_{2cl} & D_{2\infty cl} & 0 \end{bmatrix}$$
$$:= \begin{bmatrix} \tilde{A} + \tilde{B}K\tilde{C} & \tilde{B}_\infty + I_1 K \tilde{D}_{y\infty} & \tilde{B}_2 + I_1 K \tilde{D}_{y2} \\ \tilde{C}_\infty + \tilde{D}_{\infty u} K I_2 & \tilde{D}_\infty & \tilde{D}_{\infty 2} \\ \tilde{C}_2 + \tilde{D}_{2u} K I_2 & \tilde{D}_{2\infty} & 0 \end{bmatrix} \quad (4)$$

where:

$$\tilde{A}=\begin{bmatrix} A & 0_{n,n} \\ 0_{n,n} & 0_{n,n} \end{bmatrix};\tilde{B}=\begin{bmatrix} B & 0_{n,n} \\ 0_{n,m} & I_{n,n} \end{bmatrix};\tilde{B}_2=\begin{bmatrix} B_2 \\ 0_{n,m_2} \end{bmatrix}$$

$$\tilde{D}_{y2}=\begin{bmatrix} D_{y2} \\ 0_{n,m} \end{bmatrix};\tilde{B}_\infty=\begin{bmatrix} B_\infty \\ 0_{n,m_\infty} \end{bmatrix};\tilde{D}_{y\infty}=\begin{bmatrix} D_{y\infty} \\ 0_{n,m} \end{bmatrix};\tilde{C}_2=\begin{bmatrix} C_2 & 0_{r_2,n} \end{bmatrix}$$

$$\tilde{C}_\infty=\begin{bmatrix} C_\infty & 0_{r_\infty,n} \end{bmatrix};\tilde{D}_{2u}=\begin{bmatrix} D_{2u} & 0_{r_2,n} \end{bmatrix};\tilde{D}_{\infty u}=\begin{bmatrix} D_{\infty u} & 0_{r_\infty,n} \end{bmatrix}$$

$$\tilde{C}=\begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix};\tilde{D}_{2\infty}=D_{2\infty};\tilde{D}_\infty=D_\infty;\tilde{D}_{\infty 2}=D_{\infty 2}$$

$$K=\begin{bmatrix} 0_{m,r} & C_k \\ B_k & A_k \end{bmatrix};I_1=\begin{bmatrix} 0_{n,m} & 0_{n,n} \\ 0_{n,m} & I_{n,n} \end{bmatrix};I_2=\begin{bmatrix} 0_{r,n} & 0_{r,n} \\ 0_{n,n} & I_{n,n} \end{bmatrix}$$

Let us define the following closed-loop transfer matrices:

$$T_\infty(K)=\begin{bmatrix} A_{cl} & B_{\infty cl} \\ C_{\infty cl} & D_{\infty cl} \end{bmatrix} \tag{5}$$

and

$$T_2(K)=\begin{bmatrix} A_{cl} & B_{2cl} \\ C_{2cl} & 0 \end{bmatrix} \tag{6}$$

A necessary and sufficient condition for the existence of an optimal multi-objective H2/H∞/D-stability controller may be given:

Lemma 1 (Scherer et al., 1997)
The following conditions are equivalent:
1- Problem 1 has a solution.
2- There exist symmetric positive definite matrices X_2, X_∞, X_α, X_c, X_s and a dynamic output-feedback K solution of the non-convex optimization problem:

$$\min_{X_2,X_\infty,X_\alpha,X_c,X_s,K} trace(B_{2cl}^t X_2 B_{2cl})$$

Under

H_2 Performance

$$A_{cl}^t X_2+X_2 A_{cl}+C_{2cl}^t C_{2cl}<0 \tag{7}$$

H_∞ Performance

$$\begin{bmatrix} sym(A_{cl}^t X_\infty)+C_{\infty cl}^t C_{\infty cl} & X_\infty B_{\infty cl}+C_{\infty cl}^t D_{\infty cl} \\ * & D_{\infty cl}^t D_{\infty cl}-\gamma^2 I \end{bmatrix}<0 \tag{8}$$

D-stability Performance

α-stability region:

$$A_{cl}X_\alpha+X_\alpha A_{cl}^t+2\alpha X_\alpha<0 \tag{9}$$

Circular region:

$$A_{cl}X_c+X_c A_{cl}^t-\frac{c^2-r^2}{c}X_c-\frac{1}{c}A_{cl}X_c A_{cl}^t<0 \tag{10}$$

Conic sector region:

$$\begin{bmatrix} k(A_{cl}X_s+X_s A_{cl}^t) & A_{cl}X_s-X_s A_{cl}^t \\ * & k(A_{cl}X_s+X_s A_{cl}^t) \end{bmatrix}<0 \tag{11}$$

This direct formulation of the multi-objective problem leads to a non-convex optimization problem known as a BMI problem (Bilinear Matrix Inequality) which is a generalization of the notion of LMI (Linear Matrix Inequality). Unfortunately, BMIs are much harder to solve than LMIs due to the non-convexity. One way to convexify this formulation is the use of the Lyapunov Shaping Paradigm which imposes the same Lyapunov function for the different performances at the expense of some conservatism.

3. AN ITERATIVE APPROACH FOR THE MULTI-OBJECTIVE SYNTHESIS

This section presents the main results of the paper. It is based on a new bilinear parameterization of the dynamic output-feedback controller.

Theorem 1
The following conditions are equivalent:
1- There exist a controller K solution of optimization Problem 1.

2- There exist symmetric positive definite matrices X_2, X_∞, X_α, X_c, X_s and a dynamic output-feedback K solution of the optimization problem :

$$\min(\beta)$$

under

H_2 Performance

$$trace(T)<\beta \tag{12}$$

$$\begin{bmatrix} -T & 0 \\ * & X_2 \end{bmatrix}+sym(\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}\begin{bmatrix} \tilde{B}_2+I_2K\tilde{D}_{y2} & -1 \end{bmatrix})<0 \tag{13}$$

$$\begin{bmatrix} 0 & \tilde{C}_2+\tilde{D}_{2u}KI_1 & X_2 \\ * & -1_{r2} & 0 \\ * & * & 0 \end{bmatrix}+sym(\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}\begin{bmatrix} \tilde{A}+\tilde{B}K\tilde{C} & 0 & -1 \end{bmatrix})<0 \tag{14}$$

H_∞ Performance

$$\begin{bmatrix} 0 & 0 & (\tilde{C}_\infty+\tilde{D}_{\infty u}KI_2)' & X_\infty \\ * & -\gamma 1 & \tilde{D}_\infty^t & 0 \\ * & * & -\gamma 1 & 0 \\ * & * & * & 0 \end{bmatrix}+$$

$$sym(\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}[\tilde{A}+\tilde{B}K\tilde{C} \quad \tilde{B}_\infty+I_1K\tilde{D}_{y\infty} \quad 0 \quad -1])<0 \tag{15}$$

D-stability Performance

α-stability region:

$$\begin{bmatrix} sym(X_d\tilde{A})+2\alpha X_d & X_d\tilde{B} \\ * & 0 \end{bmatrix}+sym(\begin{bmatrix} G_1 \\ G_2 \end{bmatrix}\begin{bmatrix} K\tilde{C} & -1 \end{bmatrix})<0 \tag{16}$$

Circular region:

$$\begin{bmatrix} -\frac{c^2-r^2}{c}X_c & X_c \\ * & -\frac{1}{c}X_c \end{bmatrix} + sym(\begin{bmatrix} H_1 \\ H_2 \end{bmatrix}\begin{bmatrix} (\tilde{A}+\tilde{B}K\tilde{C})^t & -1 \end{bmatrix}) < 0 \qquad (17)$$

Conic sector region:

$$\begin{bmatrix} ksym(\tilde{A}X_s) & \tilde{A}X_s - X_s\tilde{A}^t & kX_s\tilde{C}^t & -X_s\tilde{C}^t \\ * & \tilde{A}X_s - X_s\tilde{A}^t & X_s\tilde{C}^t & -X_s\tilde{C}^t \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} +$$

$$sym(\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix}\begin{bmatrix} 0 & K^t\tilde{B}^t & 0 & -1 \end{bmatrix}) + sym(\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ 0 \end{bmatrix}\begin{bmatrix} K^t\tilde{B}^t & 0 & -1 & 0 \end{bmatrix}) < 0 \qquad (18)$$

Proof:

Note that inequalities (7), (8), (9), (10) and (11) can be written as follow:

$$\begin{bmatrix} 1 & \Phi^t(K) \end{bmatrix} Q(X) \begin{bmatrix} 1 \\ \Phi(K) \end{bmatrix}$$

Where ϕ, Q and X are defined as follow :

Performances	$Q(X)$	$\Phi^t(K)$
H_2	$\begin{bmatrix} 0 & \tilde{C}_2+\tilde{D}_{2u}KI_1 & X_2 \\ * & -1_{r2} & 0 \\ * & * & 0 \end{bmatrix}$	$\begin{bmatrix} (\tilde{A}+\tilde{B}K\tilde{C})^t \\ 0 \end{bmatrix}$
H_∞	$\begin{bmatrix} 0 & 0 & (\tilde{C}_\infty+\tilde{D}_{\infty u}KI_2)' & X_\infty \\ * & -\gamma 1 & \tilde{D}_\infty^t & 0 \\ * & * & -\gamma 1 & 0 \\ * & * & * & 0 \end{bmatrix}$	$\begin{bmatrix} (\tilde{A}+\tilde{B}K\tilde{C})^t \\ (\tilde{B}_\infty+I_1K\tilde{D}_{y\infty})^t \\ 0 \end{bmatrix}$
α-stability	$\begin{bmatrix} sym(X_d\tilde{A})+2\alpha X_d & X_d\tilde{B} \\ * & 0 \end{bmatrix}$	$(K\tilde{C})^t$
D-stability: Circular region	$\begin{bmatrix} -\frac{c^2-r^2}{c}X_c & X_c \\ * & -\frac{1}{c}X_c \end{bmatrix}$	$\tilde{A}+\tilde{B}K\tilde{C}$
D-stability: Conic sector	$\begin{bmatrix} 0 & -X_s\tilde{A}^t & kX_s \\ * & sym(k\tilde{A}X_s) & X_s \\ * & * & 0 \end{bmatrix}$	$\begin{bmatrix} (\tilde{A}+\tilde{B}K\tilde{C}) \\ 0 \end{bmatrix}$

Then applying the elimination Lemma (Skelton et al. 1997) we have inequalities (14), (15), (16), (17) and (18).

A cross-decomposition algorithm

Solution of the problem (1) implies to solve a bilinear matrix inequality (BMI) problem. We propose in this paper an iterative algorithm similar to coordinate-descent algorithm. The method consists in performing the optimization by alternatively fixing certain variables so that the problem is convex in the remaining ones. Thanks to the extra variables introduced by the elimination Lemma (Skelton et al. 1997), a natural decomposition among the decision variables is quite natural.

Algorithm

1. Initialization step (k=1): Choose a stabilizing dynamic output-feedback controller K.
2. step k (first part): Solve the optimization LMI problem:

$$\beta_{k,1} = \min_{\substack{X_2,X_\infty,X_\alpha,X_c,X_s,T \\ F_1,F_2,E_1,E_2,E_3,D_1,D_2,D_3 \\ D_4,G_1,G_2,H_1,H_2,J_1,J_2,J_3}} (\beta)$$

Under

(12), (13), (14), (15), (16), (17) and (18).

F_i, G_i, H_i, E_j, D_j, J_j (i=1..2, j=1..3) are frozen.

3. step k (second part): Solve the optimization LMI problem:

$$\beta_{k,2} = \min_{\substack{X_2,X_\infty,X_\alpha, \\ X_c,X_s,T,K}} (\beta)$$

Under

(12), (13), (14), (15), (16), (17) and (18).

K is frozen.

4. Final step: If $\beta_{k,1} - \beta_{k,2} < \varepsilon$ then stop.

$$K^* = K$$

and $$\beta^* = \beta_{k,2}$$

Otherwise k ← k+1 and go to step 2.

4. NUMERICAL EXAMPLES

Two different examples are proposed to illustrate the reduction of conservatism obtained by the new proposed method for multi-objective design.

Example 1: Multi-objective H_2/D-stability problem

The first example is borrowed from (Ebihara et al. 2004) and it concerns the multi-objective H_2/D-stability problem. The state-space equations are given by:

$$\begin{cases} \dot{x} = \begin{bmatrix} -0.32 & 0.04 & 0.01 \\ 0.45 & 0.99 & 0.64 \\ -0.76 & -0.37 & -0.40 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} w + \begin{bmatrix} 0.42 \\ 0.74 \\ 0.31 \end{bmatrix} u \\ z = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u \\ y = \begin{bmatrix} 0.72 & 0.85 & 0.88 \end{bmatrix} x + 2w \end{cases}$$

The goal is to design a full-order dynamic output-feedback controller K minimizing $\|T_{zw}\|_2$ subject to the D-stability constraint $\sigma(A_{cl}) \subset H(0.3)$.

This problem is solved using the Lyapunov Shaping Paradigm (LSP) and the dilated LMI approach (DLA) respectively developed in (Scherer et al., 1997) and in (Ebihara et al., 2004). The results are then compared to the new approach proposed in theorem 2. In table 1, we show both the resulting upper bound and the actual cost. The corresponding closed-loop poles are recalled in table 2.

Table 1 the resulting H_2 cost

Approach	Upper bound	Actual cost
Lyapunov Shaping Paradigm	115.62	79.09
Dilated LMI approach (DLA)	73.60	68.49
New approach (BMI)	67.24	63.87

Table 2 the closed loop poles

LSP	DLA	BMI
-0.99 + 0.67i	-0.76 + 0.79i	-1.34 + 1.11i
-0.99 - 0.67i	-0.76 - 0.79i	-1.34 - 1.11i
-0.34 + 0.21i	-0.34 + 0.26i	-0.38 + 0.34i
-0.34 + 0.21i	-0.34 - 0.26i	-0.38 - 0.34i
-1.09	-0.63	-0.31 + 0.29i
-1.15	-1.15	-0.31 - 0.29i

It is clearly shown here that the approach developed in theorem 1 achieves the desired performance and yields upper bounds that are lower than those developed in (Scherer et al., 1997) and in (Ebihara et al., 2004).

Example 2: Multi-objective H_2/H_∞ problem

This example is borrowed from (Scherer et al., 1997) and it concerns the multi-objective H_2/H_∞ problem. The state representation is given by:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} w + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x + 2w \\ z_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ z_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \end{cases}$$

We are interested here in the H_∞ performance from w to z_∞ and the H_2 performance from w to z_2.
The optimal H2 performance from w to z_2 is 7.748 and is achieved for the controller:

$$K_2(s) = \frac{-5.7275(s+5.168)(s-0.2711)}{(s+5.164)(s^2+3.669s+9.933)}$$

for this controller the H_∞ is $\gamma = 23.586$.
The problem considered here is:
Given an H_∞ level γ, find a full order strictly proper dynamic-controller K(s) such that:
- The H_2 performance $\left\| T_{wz_2} \right\|_2$ is minimized.
- The H_∞ performance $\left\| T_\infty \right\|_\infty \le 23.6$ is achieved.

We know that the optimal solution of this problem is $K_2(s)$. Hence, we can test the performance of our approach against this known optimal solution:
Solving this problem using the Lyapunov Shaping Paradigm (LSP), we obtain the following result:

- the H_2 performance cost is 8.956 which is 15% higher than the optimal value 7.748 due to the use of a common Lyapunov function.
- the corresponding controller is:

$$K_{LSP}(s) = \frac{-7.5924(s+5.271)(s-0.05601)}{(s+5.272)(s^2+4.253s+10.25)}$$

Now we solve the same problem using algorithm -1- and choosing K_{LSP} as initialization. We obtain the following result:

- The H_2 performance cost is 7.981 which is very near the optimal value 7.748.
- the corresponding controller is:

$$K_{BMI}(s) = \frac{-7.7558s^2 - 40.6398s - 1.7233}{s^3 + 9.6109s^2 + 33.4880s + 55.2163}$$

5. CONCLUSION

A new iterative method involving a bilinear matrix inequality for multi-objective dynamic output-feedback synthesis has been proposed. This new approach features the properties of computational efficiency, guaranteed convergence to a local optimum, and applicability to a very wide range of problems. The design objectives considered are H2, H_∞, and pole-placement constraints. Preliminary results obtained with academic examples shows its efficiency.
Nevertheless, some prospective improvements may be scheduled especially its extension to the case of reduced-order dynamic output feedback.

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