POSITION CONTROL OF A LEVITATING MAGNETIC ACTUATOR APPLICATIONS TO MICROSYSTEMS

Jiri Stepanek[1,2], Hervé Rostaing[2], Suzanne Lesecq[3], Jérôme Delamare[2], Orphée Cugat[2]

[1]Corresponding author:
Jiri.Stepanek@leg.ensieg.inpg.fr
[2]Laboratoire d'Electrotechnique de Grenoble, France
BP 46, 38402 Saint Martin d'Hères Cedex
Tel: +33 (0)4 76 82 71 54 Fax : +33 (0)4 76 82 63 00
[3]Laboratoire d'Automatique de Grenoble, France
BP 46, F-38402 Saint Martin d'Hères Cedex
Tel: +33 (0)4 76 82 62 25 Fax : +33 (0)4 76 82 63 88

Abstract: This paper deals with a novel magnetic microactuator. Its moving part is a permanent magnet in unstable levitation between two stable positions, and thus its central position must be precisely controlled. A cm-scale prototype was built to test the dynamic behaviour of the device. The quaternion algebra is used to describe the 3D movement of the moving magnet. A PD controller for position control is determined by a pole placement; it is then tested on the complex model and the theoretical results are satisfactory.

Keywords: Actuators, Electromagnetic device, Mechanical manipulator, Microsystem, Multi-degree-of-freedom system, Moment of inertia, PD Controller, Position control, Positioning system, Quaternion.

1. INTRODUCTION

With the growing world of MEMS, new challenges are appearing in the field of robotics. The potential applications of our device are for instance the precise micro-manipulation in biotechnology or in other fields (Cugat *et al*., 2003; Rostaing *et al.*, 2004). In this case the requirements are µm-scale precision and a "high" reaction speed. The control of such a micro-actuator raises problems of position sensing, robustness against pertubations, and controller speed.

This article presents such a magnetic micro-actuator. The application (device and position sensor) is first presented, the simplified and complete description of the system is given; the controller synthesis and some simulated results are shown in the end.

2. APPLICATION

The structure shown in Fig. 1, 2 and 3 is used for micro-positioning, and is protected by an international patent (Delamare *et al*., 2001). The actuating part is a free-moving magnet (2), which is captive of the magnetic field of two neighboring fixed magnets (1).

Here, all three magnets are magnetized in the same direction. The mobile magnet has two stable positions, against either one of the fixed magnets. Commutation between the stable positions is forced by current pulses through the conductors (3).

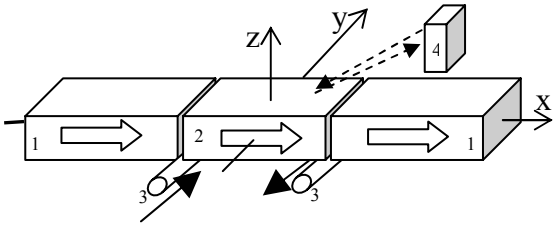

Fig. 1. Magnetic micro-actuator

The position of the mobile magnet should be controlled anywhere between these two stable positions, including the unstable equilibrium point in the middle of the device, where any small perturbation will provoke its attraction back to one of the stable positions. We control this position by modulating the current in the conductors, using feedback from the position sensor (4).

In order to achieve real-time position control, it is first necessary to determine its dynamic behaviour by calculating the 3D forces and torques which affect the mobile magnet in any position and/or orientation (Stepanek *et al*, 2004).

2.1 Microrelay realisation

A µm-scale prototype is actually fabricated in the Laboratoire d'Electronique de Technologie de l'Information (LETI) in Grenoble, France. The actuator operates in bistable mode as a micro-relay (Fig. 2).

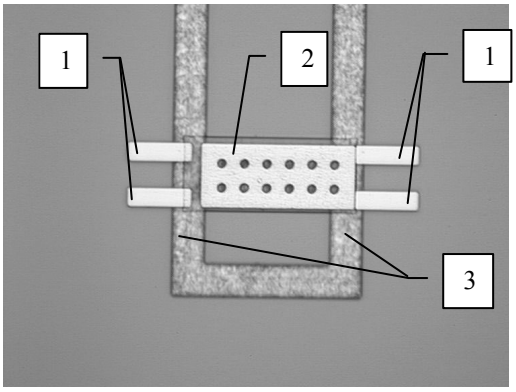

Fig. 2. Realisation of µ-relay - LETI

The dimensions of the device are 50 x 200 µm.

2.2 Macroscopic prototype

A cm-scale prototype has been built to validate the functionality and validate the model of the complete system. The prototype has two major parts: the actuator itself (Fig. 3) and a position sensor.

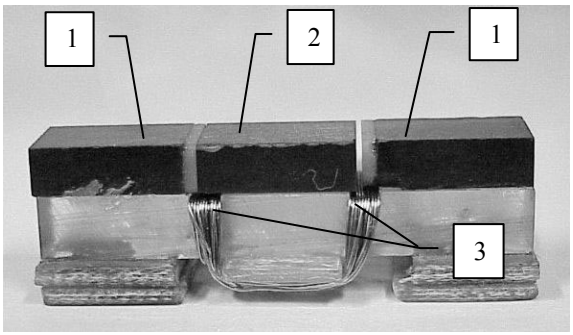

Fig. 3. Macroscopic prototype of the micro-actuator

The overall length of the prototype is 4 cm. The conductor is made by 30 loops of copper wire. The magnets are standard ferrites, and mobile magnet in the middle can move in 0.5 mm. Here the gap is widened with plastic stoppers so as to adjust for the strength of the magnets at this large scale.

2.3 Position sensor

The position of the mobile magnet is captured with a differential capacitive position sensor (Fig. 4). Each half-sensor is composed of a pair of fixed electrodes (1) facing a mobile electrode (2).

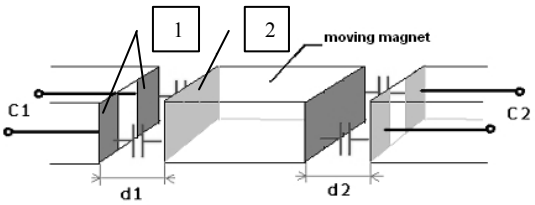

Fig. 4. Principle of the position sensor

The electronic circuit shown on Fig. 5 produces a signal proportional to the position of the moving magnet.

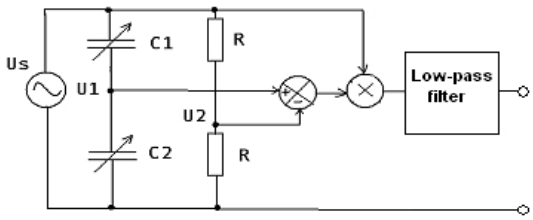

Fig. 5. Electrical diagram of the position sensor

The overall set-up is shown on Fig. 6. One of the fixed magnets (1) and the moving magnet (2) stuck on the adjustable console (5) can be seen. This configuration allows us to characterise the sensor's performance. The position signal is captured with the split electrodes (4a) and processed by the electronics (4b). The present sensor has a good resolution (100 mV / 100 µm), but still exhibits some linearity problems. Work is in progress on its improvement.

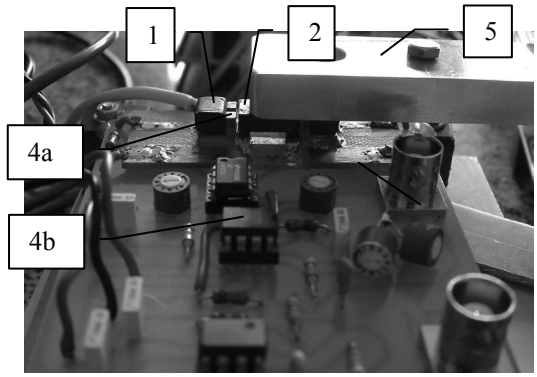

Fig. 6. Actuator & sensor set-up

3. SIMPLIFIED MECHANICAL SYSTEM DESCRIPTION

The micro-actuator will be levitating in the magnetic field. It will possess six degrees of freedom. As there is a risk of possible mechanic oscillations, the complete translation / rotation movements must be modelled.

As the complete model is non-linear and too complex and cannot be fully described analytically (only linearised), a simplified description of the actuator is used to synthetise the controller and make the first tests. The developed corrector will be then tested on the complex model described in section 4, to verify all the performance.

Let us first study the system without rotations and moving only along the *x* direction.

The overall force exerted on the moving magnet is composed of the forces from the fixed magnets and from the coil. Dieppedale *et al*. (2004) showed that in the middle position, the composed force from the symmetrical fixed magnet is proportional to the moving magnet displacement. The force from the coil is proportional to the current in the coil:

$$F_x = k_1 x + k_2 I \quad (1)$$

with:

x.. the moving magnet's position
I.. the current in the coil

The constants k_1 and k_2 are the electromagnetic constants which can be easily estimated with electromagnetic calculus.

The differential equation describing the dynamics is:

$$m\ddot{x} = k_1 x + k_2 I \quad (2)$$

where *m* is the moving magnet's mass.

The resulting transfer function is given by:

$$T.F_{M.S.} = \frac{x}{I} = \frac{\frac{k_2}{k_1}}{\frac{m}{k_1}p^2 - 1} = \frac{k_2}{k_1} \cdot \frac{1}{(\tau^2 p^2 - 1)} \quad (3)$$

(3) corresponds to a second order unstable system. The phase is 180^0 for all frequencies.

4. COMPLEX ACTUATOR MODELING

Among different approaches, the Euler angles or the quaternion formulation can be chosen in order to model a free three dimensions (3D) rotation. However, in this work, the quaternion approach has been chosen to avoid the Gimbal lock effect (Guiziou, 2002; Chou, 1992).

Let us define a fixed coordinate system (Xs, Ys, Zs), called Ro and a moving coordinate system (Xi, Yi, Zi) as specified in Fig. 7.

The levitating magnet is made of a homogeneous material in the form of a rectangular parallelepiped. Its dimensions are length *a*, width *b* and height *c*.

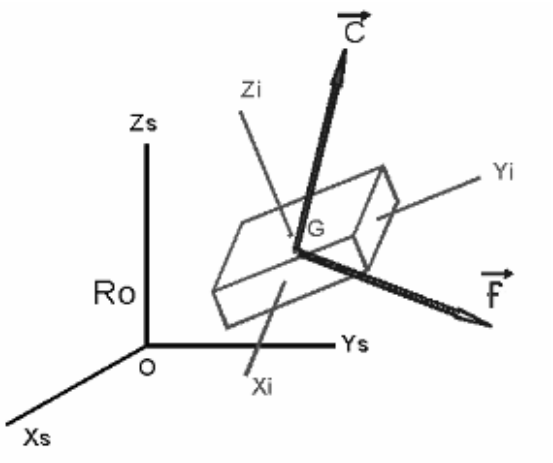

Fig. 7. Coordinate systems

The inertial moment tensor in the moving coordinates can be calculated as:

$$Ii = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} = \frac{1}{m}\begin{pmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2+a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix} \quad (4)$$

with *m* the mass of the moving magnet.

4.1. Dynamics: introduction

The dynamic system contains four unknown vectors, namely:

- the translation position vector $\vec{R}$ (3 components)
- the translation speed vector $\vec{V}$ (3 components)
- the quaternion $\vec{Q}$ (4 components)
- the angular speed $\vec{\Omega}$ (3 components)

Their components are respectively:

$$\vec{R}=\begin{pmatrix}x\\y\\z\end{pmatrix}\quad \vec{V}=\begin{pmatrix}vx\\vy\\vz\end{pmatrix}\quad \vec{Q}=\begin{pmatrix}q0\\q1\\q2\\q3\end{pmatrix}\quad \vec{\Omega}=\begin{pmatrix}p\\q\\r\end{pmatrix} \quad (5)$$

4.2. Force and torque calculation

The calculation is based on the magnetostatic calculus of forces exerted on a magnet in a magnetic field. If the magnet exact position, rotation and the current in the coil are known, the magnetic force and torque can be found. The magnetic force is formulated in the fixed coordinate system, the torque must be recalculated in the moving coordinate system.

Note that the calculus based upon the "simple" magnetostatic hypothesis; the magneto-dynamics effects on the systems are not considered because the Eddy currents can be neglected.

The calculus of the force $\vec{F}$ and the torque $\vec{\Gamma}$ can be seen as a "black box", function of the magnet position and rotation. They are not described in this paper and the reader can consult Stepanek *et al*. (2003); (2004).

$\vec{f}$ and $\vec{C}$ are the normalised force and torque:

$$\vec{F}=\vec{F}(R,Q,I)=\begin{pmatrix}F_1\\F_2\\F_3\end{pmatrix}\quad \vec{f}=\frac{\vec{F}}{m}=\begin{pmatrix}\frac{F_1}{m}\\\frac{F_2}{m}\\\frac{F_3}{m}\end{pmatrix} \quad (6)$$

$$\vec{\Gamma}=\vec{\Gamma}(R,Q,I)=\begin{pmatrix}\Gamma_1\\\Gamma_2\\\Gamma_3\end{pmatrix}\quad \vec{C}=\begin{pmatrix}C_1\\C_2\\C_3\end{pmatrix}=\begin{pmatrix}\frac{\Gamma_1}{I_1}\\\frac{\Gamma_2}{I_2}\\\frac{\Gamma_3}{I}\end{pmatrix} \quad (7)$$

4.3. Translation equations

The translation equations (8) and (9) are easy to understand. They obey Newton's law, using the initial conditions (10), (11):

$$\frac{d\vec{R}}{dt}=\vec{v} \quad (8) \qquad \frac{d\vec{v}}{dt}=\vec{f} \quad (9)$$

$$\vec{R}_0=\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix} \quad (10) \qquad \vec{v}_0=\begin{pmatrix}vx_0\\vy_0\\vz_0\end{pmatrix} \quad (11)$$

4.4. Rotation equations – movement around the centre of gravity

The problem to be solved is given by:

$$\vec{\Gamma}=I_i\cdot\frac{d\vec{\Omega}}{dt}+\vec{\Omega}\times Ii\cdot\Omega \quad (12)$$

The equation (12) can be rewritten:

$$\frac{d\vec{\Omega}}{dt}=C-\begin{pmatrix}\frac{I_3-I_2}{I_1}qr\\\frac{I_1-I_3}{I_2}pr\\\frac{I_2-I_1}{I_3}pq\end{pmatrix} \quad (13)$$

where the initial conditions are (14) :

$$\vec{\Omega}_0=\begin{pmatrix}p_0\\q_0\\r_0\end{pmatrix} \quad (14)$$

4.5. Quaternions

The quaternion $\vec{Q}$ represents the geometric rotation transforming the basis R_0 into the basis R. The quaternion multiplication is used in order to determine the elements of one vector in both coordinate systems.

The quaternion evolution has to be calculated in parallel with the instantaneous angular speed (Giuziou, 2002). These computations with initial conditions give:

$$\frac{d\vec{Q}}{dt}=\frac{1}{2}\cdot\begin{pmatrix}0&-p&-q&-r\\p&0&r&-q\\q&-r&0&p\\r&q&-p&0\end{pmatrix} \quad (15)$$

$$\vec{Q}_0=\begin{pmatrix}q0_0\\q1_0\\q2_0\\q3_0\end{pmatrix} \quad (16)$$

Note that the quaternion must be unitary in order to represent a rotation. Therefore, it must satisfy:

$$\sqrt{q0^2+q1^2+q2^2+q3^2}=1 \quad (17)$$

4.6. Complete system

The applied torque depends on the translation and the rotation of the moving magnet in the fixed field. No equation can be uncoupled, and the problem will

contain 13 differential equations (Eqs. (8), (9), (13), (15)) with 13 unknowns namely:

$$(x, y, z, vx, vy, vz, p, q, r, q0, q1, q2, q3)$$

One algebraic constraint (17) must also be dealt with. The complete system is numerically solved with a Matlab-Simulink block-diagram. A 4th order fixed-step Runge-Kutta integration scheme has been chosen. Note that a variable-step method based on an explicit Runge-Kutta (4, 5) formula (ODE45 in Matlab) has also been tested. The fixed step has been carefully chosen thanks to the results obtained with the variable-step method.

5. CLOSED LOOP SYSTEM

Different works on bistable levitating systems control still exist. Faure *et al.* (1999) showed a corrector synthesis for a magnetic bearing where the controller zeros are chosen thanks to the system poles.

In this paper, the controller synthesis is made by the pole-placement technique (DeLarminat, 2002). The denominator of the closed-loop transfer function in then simplified to a 3rd order non-oscillating system.

The closed loop from the Fig. 8 will be used for the synthesis.

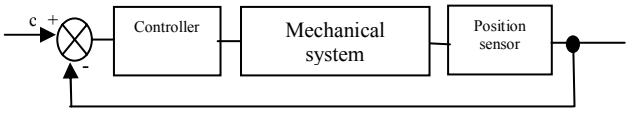

Fig. 8. Generalised closed loop system

5.1. Controller choice and determination

The role of the controller is firstly to regulate the moving magnet of the system in the middle position. It will also compensate the perturbations. On a further level, the controller should also be able to stabilise the moving magnet in any required position on the x axis (Fig.1). The other axes are not controlled in this configuration.

For the present application, a (filtered) PD controller can satisfy these constraints. It can reduce the 180° phase of the system. Its transfer function is given by:

$$T.F._{REG} = \frac{k_0 \cdot (1+\tau_1 p)}{(1+\tau_2 p)} \cdot \frac{k_c}{1+\tau_c p} \quad (18)$$

Note that this transfer function is composed by the (filtered) PD controller and a first-order system which represents the limitation of the controller to supply the electric current at high frequencies. Actually, $\tau_c << \tau_2$.

5.2. Position sensor

The x position sensor has been described in section 2.3. Its transfer function is given by:

$$T.F._{PS} = \frac{k_f}{(1+\tau_f p)^2} \quad (19)$$

Note that its characteristics satisfy $\tau_f << \tau_2$.

6. PD CONTROLLER SYNTHESIS

6.1. Controller for the simplified mechanical model

The description of the simplified mechanical system is used for the controller synthesis.
The closed-loop system transfer function is calculated with the controller transfer function (18), the mechanical system transfer function (3) and the position sensor transfer function (19). The denominator of the closed loop transfer function is a 6th order polynom:

$$Den_{C.L.}(p) = K_{Den} \cdot \{a_0 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 + a_5 p^5 + a_6 p^6\} \quad (20)$$

where:

$$\begin{aligned}
K_{Den} &= \frac{1}{k_0 k_c k_2 k_f - k_1} \\
a_0 &= k_0 k_c k_2 k_f - k_1 \\
a_1 &= -2k_1\tau_f - \tau_c k_1 - \tau_2 k_1 + k_0 k_c k_2 k_f \tau_1 \\
a_2 &= \tau^2 k_1 - k_1 \tau_f^2 - 2\tau_c k_1 \tau_f - \tau_2 \tau_c k_1 \\
a_3 &= 2\tau^2 k_1 \tau_f + \tau_c \tau^2 k_1 - \tau_c k_1 \tau_f^2 + \tau_2 \tau^2 k_1 - \tau_2 k_1 \tau_f^2 - 2\tau_2 \tau_c k_1 \tau_f \\
a_4 &= \tau^2 k_1 \tau_f^2 + 2\tau_c \tau^2 k_1 \tau_f + 2\tau_2 \tau^2 k_1 \tau_f + \tau_2 \tau_c \tau^2 k_1 - \tau_2 \tau_c k_1 \tau_f^2 \\
a_5 &= \tau_c \tau^2 k_1 \tau_f^2 + \tau_2 \tau^2 k_1 \tau_f^2 + 2k_1 \tau^2 \tau_2 \tau_f \tau_c \\
a_6 &= \tau_2 \tau_c \tau^2 k_1 \tau_f^2
\end{aligned} \quad (21)$$

As stated previously, $Den_{C.L.}$ is simplified to as 3rd order polynom with:

$$Den_{P.P.} = (\alpha p + 1)^3 \quad (22)$$

Because of the application context, an overshoot on the position of the magnet is not suitable, the response time being of second order interest. This main requirement can justify the choice of (22).
Using and optimisation solver, (for instance the MATLAB function *fsolve*), the unknown coefficients defining the controller k_0, τ_1 and τ_2 are computed for any chosen α. Actually, the validity of the simplification hypothesis (21) depends on the numerical value set for α. Therefore, after the optimisation step, the validity of the hypothesis (22) must be *a posteriori* checked.

6.2. Application of the obtained controller onto the complex actuator model

The controller obtained is now tested on the complete quaternion model to validate the behaviour of the whole actuator. Some results are shown on Fig. 9 :

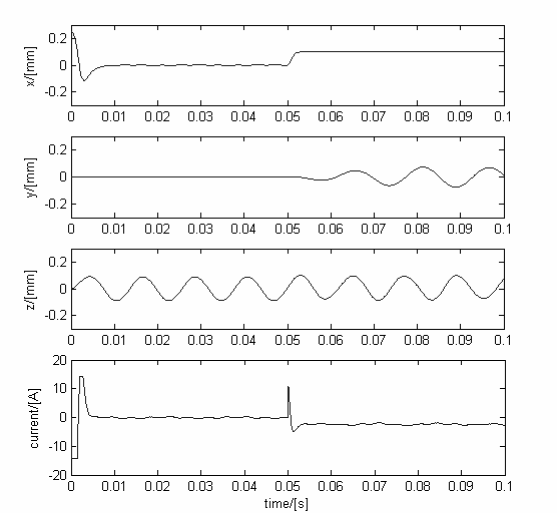

Fig. 9. Simulation results : X, Y and Z translations and control current versus time

The position on the x-axis is successfully controlled from 0mm; at t = 50 ms a step of 0.1mm has been applied. The actuator oscillates in the z-axis direction with an amplitude of 0.1 mm. The oscillation is generated by coupled forces created by conductor 3, (Fig. 1.) and are not attenuated because there is no friction in the model. However, in practice it may be dampened by air. A similar phenomenon can be observed in the y-axis since 50ms.

An x-position overshoot can be seen on Fig. 9. It is due to two non-linear events: current saturation and position saturation. There is no overshoot in the step response at 50ms.

The rotation movement is represented in Fig. 10. Quaternions make it difficult to imagine the actuator behaviour from the graphs – but it is clearly visible on animations. The actuator exercises a complicated "Lissajoux-type" periodic rotation around its centre of gravity.

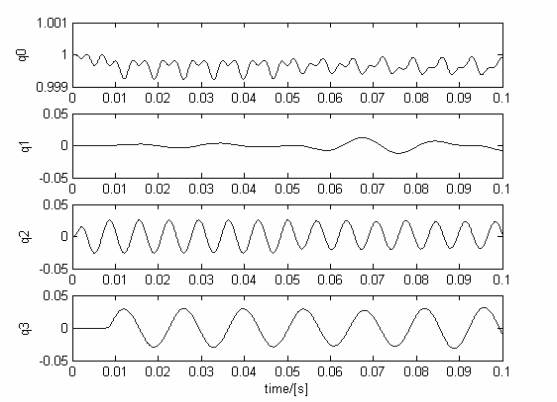

Fig. 10. Simulation results – quaternions vs time

7. CONCLUSION

The complete description of the kinematics and dynamics of the novel levitating magnetic micro-actuator has been shown. The modelling is made on a cm-scale prototype, but can be easily applied on the micrometric integrated device. The complex actuator movement can be simulated with this tool.

A PD controller was then established to control the actuator movement. A simplified model was used for the synthesis. The resulting controller was tested on the quaternion model and the results are promising.

The controller has been realised in electronics and tested on the magnetic system. The mobile magnet is successfully maintained in levitation.

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