THE NON-CONVENTIONAL SAMPLING PATTERN AS A DESIGN PARAMETER FOR PID CONTROLLERS

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Abstract: This paper deals with the possibility of considering the non-conventional sampling pattern as a design parameter in a control system where the controller is a PID. Once the PID is designed and placed in a single rate control system, the goal is to obtain similar behaviours for two different stages: the first considers a non-conventional control system with the same PID, and the second, a single rate control system but varying the initial PID parameters.

Keywords: Multirate, PID Controllers, Poles and Zeros.

1. INTRODUCTION. APPROACH OF THE PROBLEM.

A multirate sampling system is a system on two or more variables are updated at different frequencies. Typically, a global period T_0 is considered and, inside it, different samplings (at different rates for each sampler) are spaced in a regular way.

A particular case of multirate sampling is the cyclic one. In this case, variables are sampled at irregular intervals, considering also the existence of a global period T_0 and cyclic repetition. In (Cuenca *et al.*, 2004) and (Salt *et al.*, 2004) a modelling methodology and a design technique for this kind of sampled systems are respectively introduced.

Both, multirate sampling and the particular case of cyclic sampling, are non-conventional ways to carry out system sampling. Their application can appear in a wide range of situations. For instance:

- Time sharing computer by means of several detection services (Jury and Mullin, 1959).
- Aerospace applications (Halevi and Ray, 1988), robotic applications (Tsao and Hutchinson, 1994), chemical process control (Morant *et al.*, 1986), computer hard disc control (Baek and Lee 1999).
- Missing and scarced data (Albertos *et al.*, 1999).
- Distributed and multiprocessors control systems (Hovestädt, 1991).
- Real-time control systems (Salt *et al.*, 2000).
- Multivariable control systems (Vélez, 2000).

The main goal of this paper is to make possible that the non-conventional sampling pattern can be considered as a design parameter. In order to achieve it, firstly a PID is designed to work in a single rate control system. Once its behaviour is observed, the idea is to change the sampling pattern going towards a non-conventional one. It is easy to think, the system performance will vary, and then, the aim is to achieve this new performance by means of the single rate structure, but modifying the initial PID parameters (kp-td-ti). In short, both stages (the non-conventional and the modified single rate) can achieve similar behaviours.

As it is obvious, the behaviour of the single rate stage can be obtained by means of a classical single rate control structure. Nevertheless, in order to obtain the behaviour for the non-conventional control stage, the structure of the figure 1 must be considered. This diagram presents a MRIC (Multirate Input Control) structure, where, each T_0 instants of time, N (multiplicity) actions are injected to the process and only 1 output is taken from it.

In more detail, the control system of figure 1 works in the following way: the PID controller, $G_R(s)$, and the process, $G_p(s)$, are preceded of a ZOH (Zero Order Hold) device, which is defined at t0 period (it is known as intersampling period, and it coincides normally with the greatest common divisor of the different periods of the system). Moreover, the output of the system is taken at T_0 period (it is known as metaperiod, and it coincides normally with the least common multiple of the different periods of the system). In the same way, the reference and the error signals are sampled at T_0 period. Finally, in the input of the process a non-conventional pattern is established, so that a non-uniform succession of control actions within the metaperiod can be injected to the process and their effect over the system response observed. This response can be also deduced examining the poles and zeros location in the Z plane.

So, the control and sampling structure of figure 1 makes possible to carry out different sampling patterns for the non-conventional stage, which will be compared with the single rate one, changing in this case the initial parameters of the PID controller.

The present study is organised in two subsections. In the first one, a series of empirical tuning rules are presented, and in the second one, a later considerations about the poles and zeros location are commented.

Finally, to perform the study, it is necessary to consider a working methodology, (Cuenca, 2004), based on the use of the Kranc Operators, (Thompson, 1986), which permits modelling the control system, and finally, simulating its behaviour, (Salt *et al*., 2003).

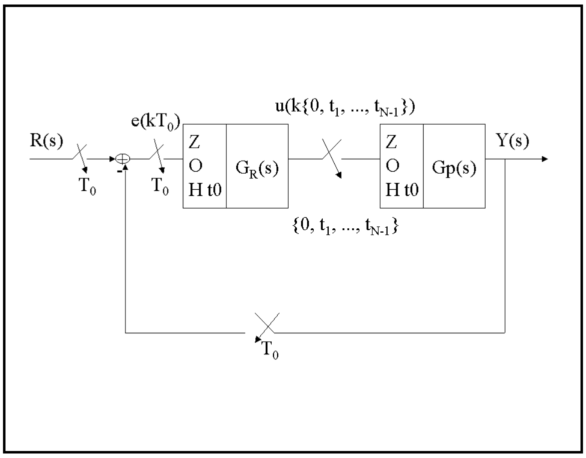

Fig. 1. Control and sampling structure for the non-conventional stage.

2. STUDY OF THE PROBLEM.

2.1 Empirical tuning rules.

In this subsection the goal is to achieve a series of empirical rules, which permit, varying the initial parameters kp-td-ti of the PID controller, to obtain similar control system responses for single rate and non-conventional stages.

In order to carry out the study, the following example will be chosen:

- Process:

$$G_p(s) = \left(\frac{1.5}{s^2 + 2s + 0.75} \right) \qquad (1)$$

- PID controller (with derivative filter):

$$G_R(s) = 8 \cdot \left(1 + \frac{1}{3.2s} + \frac{0.2s}{0.1s + 1} \right) \qquad (2)$$

- Metaperiod and intersampling period:

$$T_0 = 0.18 \text{ sec}; \quad t0 = 0.015 \text{ sec} \qquad (3)$$

In this study, the control system is modelled and implemented in order to obtain different Poles and Zeros Maps (PZM) at T_0 period for each considered sampling scheme in the input sampler of the process. Moreover, to achieve the pursued goal, a grid together the different PZM obtained is included. This grid is composed by different points, which establish the location of the dominant pole of the single rate system inside a range of values for kp-td-ti. In this case, this dominant pole is a conjugated complex one. Concretely, its positive imaginary side will be taken into account.

The proposed analysis in the PZM can be observed in a general way in the figure 2. Here, different closed loop poles and zeros (and the open loop poles) for the different sampling patterns can be seen. In the figure 3, a more detailed perspective for the positive imaginary side of the dominant pole is shown. In this case, the grid intersects non-conventional locations for N=2 and single rate ones with kp-td-ti modified. Concretely, kp varies inside the range [5.5 – 8] by means of increments of 0.25, td inside the range [0.18 – 0.23] by means of increments of 0.01, and ti inside the range [2 – 3.5] by means of increments of 0.5. The sense of increment for the three parameters is marked in the figures, so that for each value of kp (there are 11) can be seen the variation of the location of the pole according to the different values of td (there are 6) and ti (there are 4). Finally, in the figure, the notation [0] makes reference to the single rate sampling at period T_0 for the input sampler of the process, and [0,t] makes reference to the non-conventional sampling in the instants 0 and t of each metaperiod T_0 for the same sampler. Each boldface cross mark is related to the non-conventional pattern indicated close of the mark.

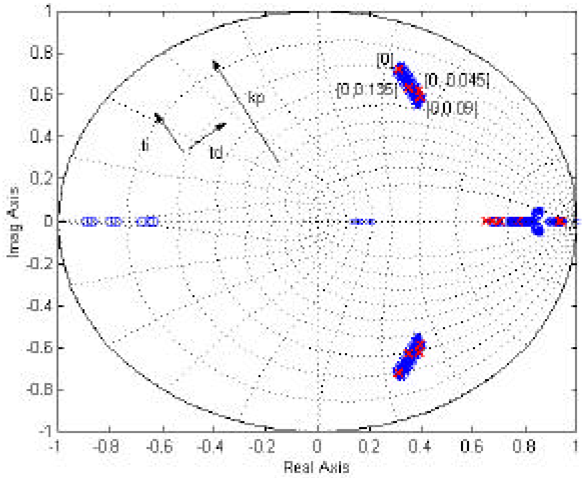

Fig. 2. General PZM with grid.

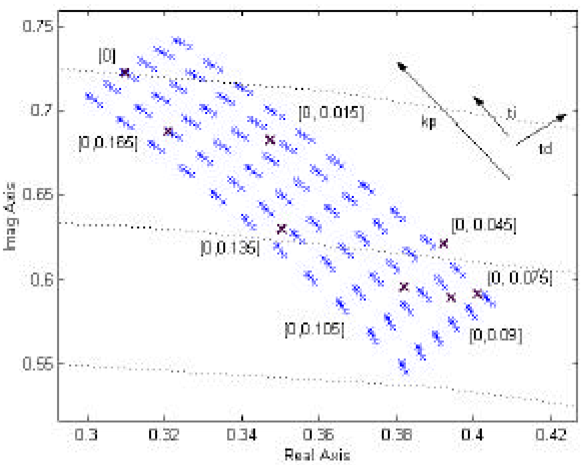

Fig. 3. Detailed PZM with grid.

Starting from the figure 3, to make similar the single rate response and the non-conventional one (N=2), and, moreover, if the single rate structure wants to be maintained, then the initial kp-td-ti parameters (kp=8, td=0.2, ti=3.2) must be varied. This variation should follow the next empirical rules:

- kp should be reduced. This fact can be explained by the following argument: with this non-conventional pattern (N=2), the controller injects twice actions to the process, and the energy to be supplied must be shared out between them.

- td should be augmented in general for the non-conventional cases where the sampling is produced in the first side of the metaperiod (that is, from the instant 0 to the instant T_0/N), and td should be reduced in general for the contrary cases (sampling in the second side of the metaperiod). So, a similar overdamping coefficient can be obtained for complementary cases (for example, [0, 0.015] and [0, 0.165], or [0, 0.045] and [0, 0.135], etc), since, normally, this cases have a similar response.

- ti carries out the fine adjustment of the system response. Empirically it is not possible to deduce a clear tuning rule, since kp and ti vary in the same sense, and kp does it in a higher way (as shown figure 3).

2.2 Later considerations about the rest of poles and zeros location.

Once the empirical tuning rules have been deduced (starting from the dominant pole), which can assure a similar behaviour for modified single rate stages and non-conventional (in this case, with N=2) ones, and thus, allowing to establish the sampling pattern as a design parameter for PID controllers, now it is important to note an interesting aspect relative to the rest of poles and zeros of the non-conventional control system.

This aspect emphasises in the following fact: although the dominant pole is well tuning for both stages, if the location of the rest of closed loop poles and zeros differ significantly, a not so accurate final system response could be obtained.

The following example can illustrate this remark. So, remembering the previous subsection, a first stage is analysed, where it has been chosen the non-conventional pattern [0, 0.165] and the modified single rate for kp=7.5, ti=3.25 and td=0.19. In figure 4 both cases are represented by means of their PZM, so that, the non-conventional case is represented by little X-shaped cross and circles, and the single rate by bigger ones. The dominant pole (conjugated complex) clearly coincides for both cases, and, in the same way, practically the rest of closed loop poles and zeros. Thus, the system response is very similar for both cases (figure 5), and even, the control actions (notice only a peak in the final of the metaperiod for the non-conventional case due to the chosen pattern).

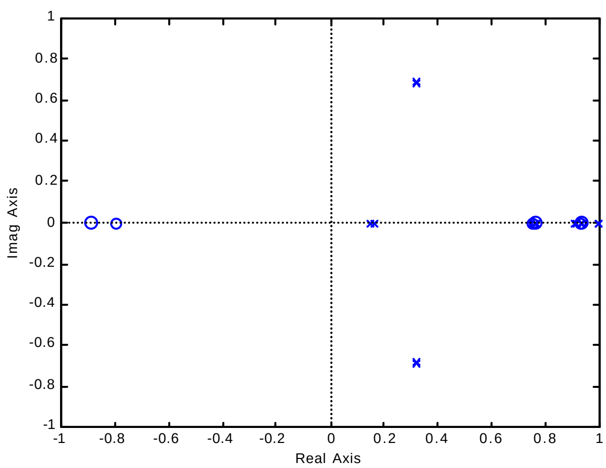

Fig. 4. PZM for modified single rate case and for non-conventional case (stage 1).

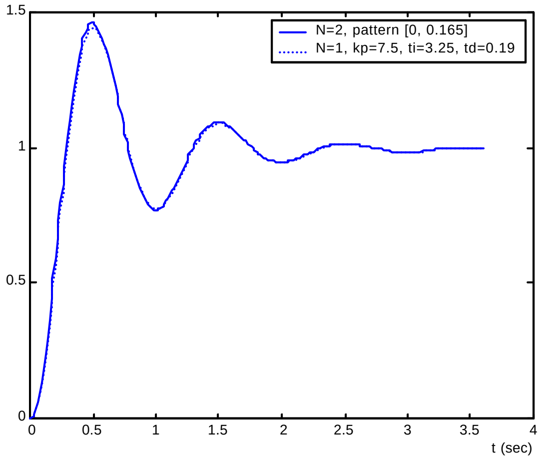

Fig. 5. Response for modified single rate case and for non-conventional case (stage 1).

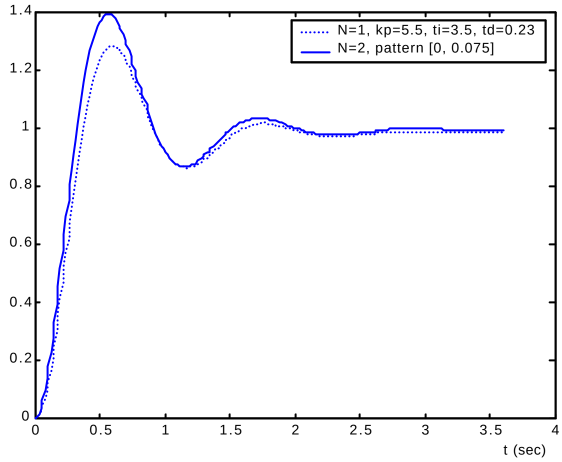

Fig. 8. Response for modified single rate case and for non-conventional case (stage 2).

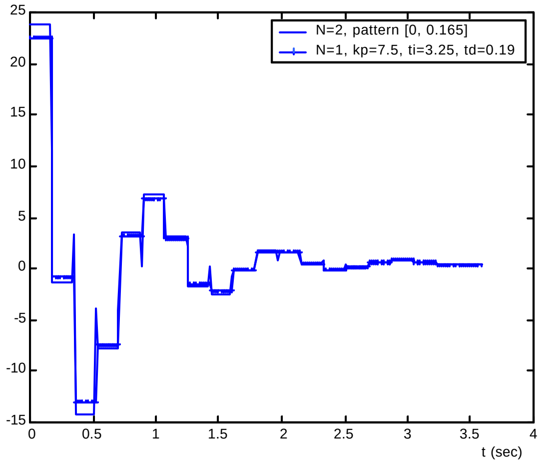

Fig. 6. Control actions for modified single rate case and for non-conventional case (stage 1).

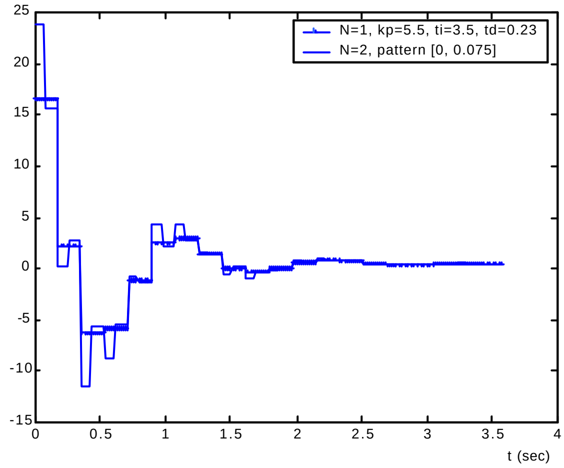

Fig. 9. Control actions for modified single rate case and for non-conventional case (stage 2).

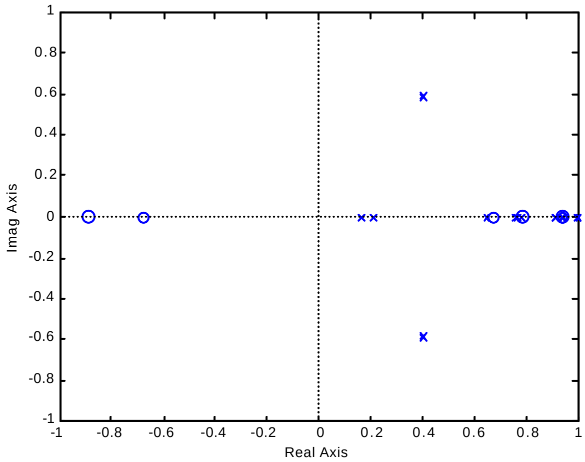

Fig. 7. PZM for modified single rate case and for non-conventional case (stage 2)

Nevertheless, a second stage is studied, where now the non-conventional pattern is [0, 0.075] and the modified single rate has kp=5.5, ti=3.5 and td=0.23. The consequent PZM are represented in figure 7 (with the same notation than figure 4). In this stage, although the dominant pole coincides practically in both cases, the rest of poles and zeros differ clearly. The consequence is a not so accurate system response for both cases (figure 8). This fact can be explained observing the significant differences between control actions of this stage (figure 9).

Thus, besides the dominant pole, it is very important to take into account the location of the rest of poles and zeros of the non-conventional control system in order to make similar both responses. In future works, a quota, which indicates how different can become the location of these poles and zeros between both cases, will be searched.

3. CONCLUSIONS.

In the present paper, the possibility of considering the sampling pattern as a design parameter for PID controllers is analysed. In order to achieve it, it is important to take into account the location of the dominant closed loop poles of the system. Anyway, it is important to observe always the location of the rest of closed loop poles and zeros of the system, since they could influence in the overall dynamic response.

After establishing a series of empirical tuning rules, the study shows it is possible to obtain similar control system responses for non-conventional stages and modified single rate ones. The consequent main benefits are: to be able to improve the single rate control system behaviour (this improved behaviour corresponds to the achieved one by sampling the system in a non-conventional way) and to establish a clear relation between sampling pattern and PID parameters.

Finally, the study is carried out for one concrete example, but its conclusions can be easily generalised to other examples.

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