PROCESS INTEGRATED DESIGN WITHIN A MODEL PREDICTIVE CONTROL FRAMEWORK

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Abstract:
In this work the Integrated Design of the activated sludge process in a wastewater treatment plant has been performed, including a linear multivariable predictive controller with constraints. In the Integrated Design procedure, the process parameters are obtained simultaneously with the parameters of the control system by solving a multiobjective constrained non-linear optimization problem, taking into account investment and operation costs. The mathematical optimization for tuning all parameters is tackled in two iterative steps. First, plant parameters are obtained using a sequential quadratic programming (SQP) method, and secondly, a type of random search method is used to tune the controller parameters (horizons and weights) . Due to the difficulty to measure some variables, there has been also developed a Kalman Filter for state estimation.

Keywords: process Integrated Design, predictive control, controllability, nonlinear optimization, stochastic optimization

1. INTRODUCTION

The traditional mode of designing processes has been the use of heuristic knowledge concentrated on determining the economically optimal process configuration among many possible alternatives. After the configuration is selected, the process parameters and a steady state working point are evaluated in order to satisfy operational requirements and reduce investment costs. In this procedure, there is no consideration about the operability and controllability, resulting plants very difficult to control. Once the process has been designed, the following step is the selection of the controller structure and tuning. The design and control of processes are tasks performed sequentially, and examination of controllability occurs only after the optimal process configuration and parameters are known.

Integrated Design methodology allows for the evaluation of the plant parameters and control system at the same time, making the designed system more controllable (Fisher, 1988; Luyben, 1993). At design stage, controllability indicators are evaluated together with economic considerations, in order to give an optimum plant. This problem is stated mathematically as a NLP/DAE multiobjective optimization problem with non-linear constraints. Many works apply Integrated Design techniques, particularly to chemical process design, such as distillation systems or reactors, stressing the interactions of design and control (Ross, 2001; Gil, 2001). These works also tackle process structure selection by solving a synthesis problem. A comprehensive review of advances in the area is given by Sakizlis (2004).

Some good examples of Integrated Design applied to the activated sludge process are given by Francisco (2003), where PI controllers and the plant were obtained, including linear matrix inequality (LMI) constraints to state stability conditions and some desired closed-loop behaviour, and by Vega (1999), that presents a study of Integrated Design with PI controllers applied to different plant structures. Despite of the complicated dynamics of the process under design, works adding advanced controllers to the Integrated Design procedure have not been reported in the literature and it could be a good way to improve control performance. In this work, model predictive control (MPC) has been selected as advanced control method because of the existence of several successful applications in activated sludge control (Vega, 1999; Nejjari, 1999; Sotomayor, 2002) and the easiness to deal with constraints.

One important issue in Integrated Design is the tuning of controller parameters. Usually the tuning of these parameters has been performed using expert knowledge and a trial and error procedure. However, some works deal with automatic tuning of MPC. Ali (1993) proposed an off-line procedure for tuning the algorithm parameters of a nonlinear predictive controller specifying time-domain performance criteria. Results are good, but the tuning of integer parameters such as horizons is performed using a non intelligent grid search. For linear MPC, Al-Ghazzawi (2001) has developed an on-line tuning strategy based on the linear approximation between the closed-loop predicted output and the MPC tuning parameters, but without considering output constraints on the on-line optimization step.

At the view of previous works, the main contributions of this work are the following. First, a new method for optimal automatic tuning of linear MPC parameters taking into account input and output constraints has been developed and tried for linear plants and the activated sludge process. This tuning method uses a specific random search based on the optimization algorithm of Solis (1981) for MPC integer parameters tuning. The second contribution is to develop Integrated Design techniques in order to perform at the same time the design of the optimal plant for activated sludge process and the optimal linear MPC for this process. This strategy has been tested in one simulated example based on a real wastewater treatment plant.

In addition to costs, other performance specifications were considered in the Integrated Design procedure, such as the Integral Square Error (ISE) or the integral of changes in the manipulated variables. The methodology proposed here is a general one, and any other dynamic performance criteria can be considered. The use of linear models also allows for the specification of convex performance criteria within an LMI framework.

The paper is organized as follows. First, the activated sludge process is presented and the way to implement an MPC for this process is explained. Secondly, a method for automatic tuning of the MPC is presented and applied to a linear plant and the activated sludge process. Then, the Integrated Design procedure is stated and solved for the activated sludge process, showing some results and ending with conclusions.

2. DESCRIPTION OF THE ACTIVATED SLUDGE PROCESS AND MODEL PREDICTIVE CONTROLLER

2.1. Plant description

For applying Integrated Design methodology, a wastewater treatment plant has been selected. The plant layout is represented in Figure 1, which is a simplified model of a real plant. It consists of one aeration tank and one secondary settler. The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor, that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor.

The whole set of variables is presented in Figure 1. Generically, "*x*" is used for the biomass concentrations (mg/l), "*s*" for the organic substrate concentrations (mg/l), "*c*" for the oxygen concentrations (mg/l) and "*q*" for flow rates (m^3/h).

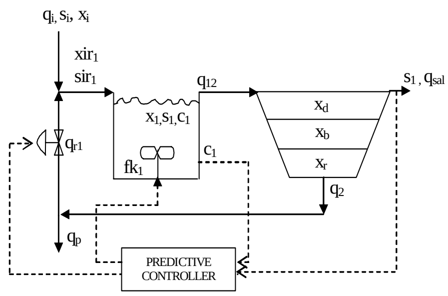

Fig. 1: Selected plant for Integrated Design

A first principles model of the system is obtained by considering mass balances of oxygen, biomass and organic substrate in the whole plant, together with the equilibrium equations for the flows of water and sludge. Note that three layers of different and increasing biomass concentration are considered in the settler. This model has been linearized to use it as internal model in the MPC studied.

2.2. Control problem

The control of this process aims to keep the substrate at the output (s_1) below a legal value despite the large variations of the flow rate and the substrate concentration of the incoming water (q_i and s_i). Another control objective is to keep dissolved oxygen concentration (c_1) around 2 mg/l, concentration that is necessary for the proper working of activated sludge process.

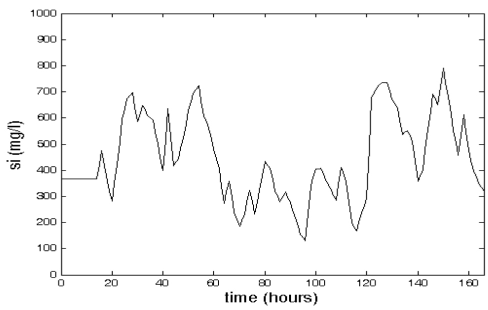

Fig. 2: Substrate disturbances at the influent

One of the main problems when trying to control the plant properly are the input disturbances q_i and s_i. The set of disturbances for designing the plant (Figure 2) has been taken out from a real wastewater treatment plant and it has been used as system input in dynamic simulations.

The general structure of a multivariable controller applied to the activated sludge process can be seen in figure 3. Two manipulated variables are considered: recycling flow (qr_1) and aeration factor (fk_1), and also three outputs: biomass (x_1) , oxygen and substrate in the reactor. In this case the biomass is not controlled, it is only a constrained variable between two limits for a good performance of the process.

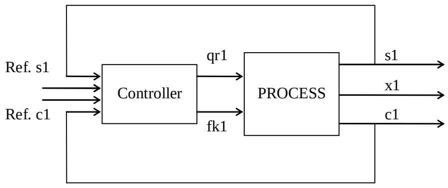

Fig. 3: General controller structure

2.3. Model predictive controller applied to the process

A standard multivariable MPC has been considered to apply the automatic tuning procedure and the Integrated Design methodology proposed in this paper. It calculates manipulated variables by solving the following on-line constrained optimization problem subject to constraints on inputs, predicted outputs and changes in manipulated variables.

$$\min_{\Delta u} V(k) = \sum_{i=Hw}^{Hp} W_y \cdot (\hat{y}(k+i\,|\,k) - r(k+i\,|\,k))^2 + \sum_{i=0}^{Hu-1} W_u \cdot (\Delta\hat{u}(k+i\,|\,k))^2 \qquad (1)$$

where k denotes the current sampling point, $\hat{y}(k+i\,|\,k)$ is the predicted output at time $k+i$, depending of measurements up to time k, $r(k+i\,|\,k)$ is the reference trajectory, $\Delta\hat{u}$ are the changes in the manipulated variables, H_p is the upper prediction horizon, H_w is the lower prediction horizon, H_u is the control horizon, W_u is a diagonal matrix representing the weights of the change of manipulated variables and W_y is a diagonal matrix representing the weights of the errors of set-points tracking.

The MPC prediction model is a linear discrete state space model of the plant obtained linearizing the model equations. The reference trajectories approach the set-point trajectories exponentially from the current output values, with T_{ref} as the 'time constant' of the exponentials.

In the activated sludge process, substrate concentration in the biological reactor is difficult to measure on-line. For this reason a Kalman Filter estimator was incorporated to the MPC algorithm, considering only x_1 and c_1 as measured outputs. Another specific issue for our process is that when there is no way to satisfy constraints, for example, due to very large disturbances, soft constraints technique is used to keep the controller feasible.

3. OPTIMAL AUTOMATIC TUNING OF MPC

3.1. MPC tuning parameters

The main tuning parameters are those affecting the behaviour of the closed loop combination of plant and MPC. The most important are the weights W_u in the controller cost function, the prediction and control horizons (H_p, H_u), and T_{ref} in the reference trajectories. Note that when working with a multivariable controller, weights in the cost function are matrices, so several different values will be tuned.

3.2. Optimization problem

The automatic tuning procedure of MPC parameters W_u, H_p and H_c, is based on the minimization of dynamic performance indexes like integral square error (ISE) and the integral of control variations (CTR), defined by the following equations:

$$ISE = \int_{t=0}^{T\max} (s_{1r} - s_1)^2 \cdot dt + \beta \int_{t=0}^{T\max} (c_{1r} - c_1)^2 \qquad (2)$$

$$CTR = \int_{t=0}^{T\max} (\Delta qr_1)^2 \cdot dt + \beta \int_{t=0}^{T\max} (\Delta fk_1)^2 \qquad (3)$$

where c_{1r} is the dissolved oxygen reference and s_{1r} the substrate reference. Due to the different magnitudes of variables, some normalization factors β are included in the equations.

The tuning procedure consists of minimizing the following function:

$$f_2(c) = w_1 ISE + w_2 CTR + \alpha \qquad (4)$$

where $c = (H_p, H_c, W_u)$, and w_1, w_2 are suitable weights for optimization. Parameter α is a penalty factor added to f_2 when the controller obtained in the iterative procedure is infeasible.

3.3. Algorithm description

The main difficulty to solve this optimization problem is that controller horizons are integer values, so classical optimization algorithms cannot be used. Therefore, a modified random search method based on Solis algorithm (1981) has been proposed, for optimizing not only the horizons but also the controller weights W_u. The algorithm steps are the following:

I. One initial point for all controller parameters $c(0) = (H_p, H_c, W_u)$ is selected. Initial variances for random vectors of Gaussian distributions are selected. The initial Gaussian centre vector for the real part is $b(0) = 0$.
II. Two random vectors of Gaussian distributions are generated, $\xi_i(k)$ integer (for the horizons) and $\xi_r(k)$ real (for the weights) with Gaussian centre $b(k)$. Index k is the current iteration.

III. New points are obtained by adding and removing $\xi_i(k), \xi_r(k)$ to the current point. Variables limits are checked.

IV. Cost function f_2 is evaluated at the original point and at new points, and the algorithm chooses the point with the smallest cost value. Vector $b(k)$ is also modified to improve convergence.

V. The variance for generating the random vectors in step II is decreased. If some convergence criteria is satisfied, stop the algorithm, otherwise return to step II and make $k=k+1$.

3.4. Tuning results

In order to test the procedure, several cases have been studied. For discrete models, the sampling period is T=1 and parameter value H_w=1 was fixed. For all controllers, set point for substrate is 55 mg/l and for dissolved oxygen is 2 mg/l. Note that more than follow closely this values, the real aim of the controller is to keep substrate output below a fixed limit and dissolved oxygen over a certain value. Matrices W_y and W_u are properly scaled for better performance.

- *Results considering linear MPC without constraints applied to a linear system*

First of all, some simple results of MPC without constraints tuning are shown, to evaluate the proposed algorithm (Table 1). The controller has been applied to a multivariable linear system. Perfect set-point tracking is obtained when control variations are not penalized (Figure 4).

Table 1: Tuning results with unconstrained MPC

Weigths in f_2	w_1= 1;w_2= 0	w_1= 1;w_2= 0.04
W_u	[0 0]	[1.3 6.5]
T_{ref}	0	7.1
H_p, H_c	3 , 3	24 , 12
Figures	Solid line	Dash dotted line

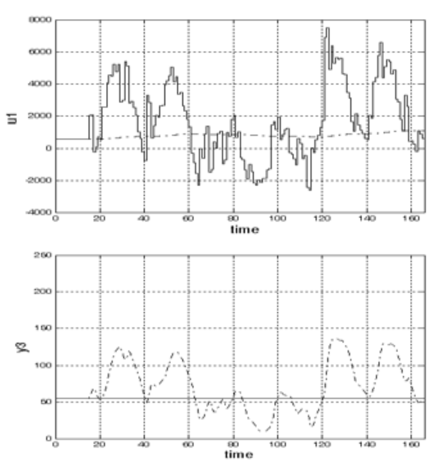

Fig. 4: Results with unconstrained MPC

- *Results considering linear MPC with constraints applied to a linear plant*

Now results when an MPC with constraints is considered for tuning are presented (Table 2). A linearized model of the activated sludge process is included as internal model of the controller and as plant model. In this case disturbances are assumed to be unmeasured. Constraints included in the controller are (soft constraints for s_1, x_1 and c_1):

$$\begin{array}{lll} 0 < s_1 < 125 & 0 < qr_1 < 3500 & 0 < \Delta qr_1 < 500 \\ 0 < x_1 < 3000 & 0 < fk_1 < 1 & 0 < \Delta fk_1 < 500 \\ 0 < c_1 < 10 & & \end{array} \quad (5)$$

Table 2: Tuning results with constrained MPC

Weigths in f_2	w_1= 1;w_2= 0	w_1= 1;w_2= 0.2
W_u	[0 0]	[5.03 2.87]
T_{ref}	0	1.26
H_p, H_c	17 , 7	22 , 10
Figures	Solid line	Dash dotted line

As can be seen in figure 5, results are obviously worse than using the unconstrained controller, obtaining better set-point tracking when control variations are not penalized.

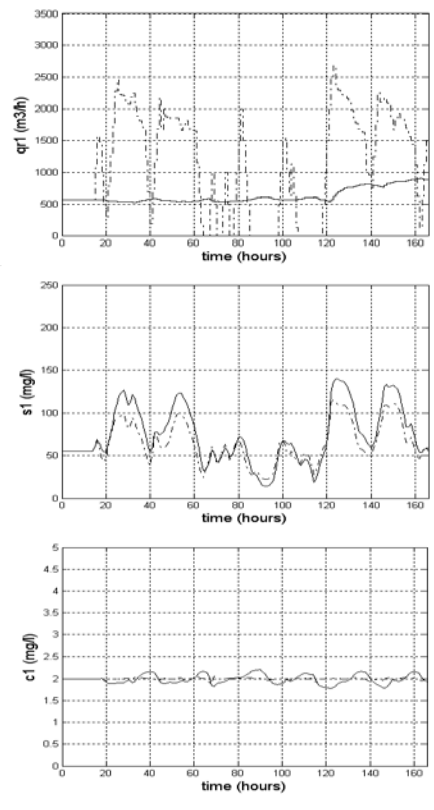

Fig. 5: Results with constrained MPC

- *Results considering linear MPC with constraints applied to the activated sludge process*

In this case, the plant model is the real nonlinear model of the process. Results are shown in Table 3 and Figure 6. Constraints included in the controller are the same that in the previous point, excepting for:

$$0 < \Delta qr_1 < 1e6 \qquad 0 < \Delta fk_1 < 1e6 \qquad (6)$$

Table 3: Tuning results with constrained MPC applied to the activated sludge process

Weigths in f_2	w_1= 1;w_2= 0	w_1= 1;w_2= 1
W_u	[2.54 3.46]	[5.35 2.25]
T_{ref}	9.73	5.47
H_p, H_c	10 , 2	10 , 6
Figures	Dash dotted line	Solid line

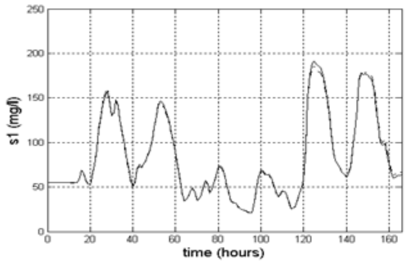

Fig. 6: Results of constrained MPC applied to the activated sludge process

As can be seen in figure 6, there is only a little difference between the tuning taking into account changes in the manipulated variables ($w_2 \neq 0$) and giving no weight corresponding to that changes ($w_2 = 0$). Control signals are also similar in both cases. Then, for this plant the ISE does not seem to be the best index for designing the controller. Anyway, when solving Integrated Design it gives good results because the plant parameters will also change.

4. INTEGRATED DESIGN OF PLANT AND MPC

The Integrated Design problem consists of determining simultaneously the plant and controller parameters and a steady state working point, while the investment and operation costs are minimized. Non-linearities of the plant, inclusion of dynamic simulations, relatively high number of variables, increase the complexity of the problem and make necessary the use of an iterative two steps optimization approach. In the first step the MPC is tuned using the method exposed in 3, and in the second step the plant is designed according to the procedure explained below.

4.1. Design of the plant

The plant design step for Integrated Design consists of minimizing the following cost function, representing construction and operation costs, where V_1 and A_d are the volume of the reactor and the cross-sectional area of the settler, fk_1 is the aeration factor in the reactor and q_2 is the total recycling flow.

$$f_1(x) = w_1 \cdot V_1^2 + w_2 \cdot A_d^2 + w_3 \cdot fk_1^2 + w_4 \cdot q_2^2 \tag{7}$$

subject to lower and upper bounds for optimization variables (x) and nonlinear constraints representing the physical, process and controllability constraints. The numbers w_i ($i = 1,\ldots,4$) are the corresponding weights for each term. Some constraints are:

- *Residence time* and *mass load* in the aeration tanks:

$$2.5 \le \frac{V_1}{q_{12}} \le 5 \;;\; 0.001 \le \frac{q_i s_i + q_{r1} s_2}{V_1 x_1} \le 0.06 \tag{8}$$

- Limits in hydraulic capacity and sludge age in the settler and limits in the relationship between the input, recycled and purge flow rates:

$$\frac{q_{12}}{A_d} \le 1.5 \,;\, 3 \le \frac{V_1 x_1 + A_d l_r x_r}{q_p x_r 24} \le 10 \tag{9}$$

$$0.03 \le \frac{q_p}{q_2} \le 0.07 \,;\, 0.5 \le \frac{q_2}{q_i} \le 0.9 \tag{10}$$

- Constraints on the non-linear differential equations of the plant model to obtain a solution close to a steady state (ε close to zero). For example, the constraint for substrate in the first reactor is:

$$\left|\frac{ds_1}{dt}\right| = \left|-\mu \frac{s_1 x_1}{k_s + s_1} + f_{kd} k_d \frac{x_1^2}{s_1} + f_{kd} k_c x_1 + \frac{q_{12}}{V_1}\left(sir_1 - s_1\right)\right| \le \varepsilon \tag{11}$$

- Constraint over the ISE norm

$$ISE_1 = \int_{t=0}^{T\max} \left(s_{1r} - s_1\right)^2 \cdot dt < \beta \tag{12}$$

where T_{max} =166 hours is the simulation time and the value of β is fixed for each controller. This constraint is included to obtain a plant with some closed loop disturbance rejection capability, independently of the controller implemented.

4.2. Two steps optimization algorithm

The algorithm for solving the nonlinear optimization problem generated tackles the problem in an iterative two step approach. The first step performs the controller tuning, and the second step the plant design. Firstly, with an initial fixed plant, the controller is designed. Once the controller is designed, the plant is optimized with the controller obtained in the previous step. Then the controller is designed again, but using the new plant parameters obtained before. The loop is finished when convergence criteria is reached. (see figure 7)

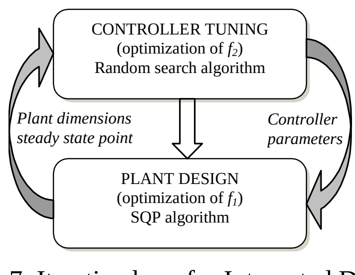

Fig. 7: Iterative loop for Integrated Design

For optimization of f_1 cost function, all decision variables are real numbers, so the SQP method (MATLAB Optimization Toolbox) has been used. For optimization of f_2 the procedure explained in 3.3 has been used.

4.3. Integrated Design results

The activated sludge process has a response delay when applying input variations. For some plant dimensions, the linearized model shows right-half plane zeros producing inverse response. Because of this, the lower prediction horizon H_w, has been set here to 3, allowing a delay of 2 hours before the controller starts to penalize set-point deviations of substrate. Results are shown in table 4 and figure 8. Integrated Design improvement can be deduced from substrate variations, that in this case are much lower that in figure 6. Due to the high computational demand, the sampling period here is T=2 hours.

Table 4: Integrated Design results

Weigths in f_2	w_1= 1;w_2= 0	w_1= 1;w_2= 2
W_u	[0.06 0]	[18.57 25.67]
T_{ref}	2.72	21.80
H_p, H_c	21 , 4	16 , 4
V_1	8167	8910
A_d	3229	3193
Figures	Dash-dotted line	Solid line

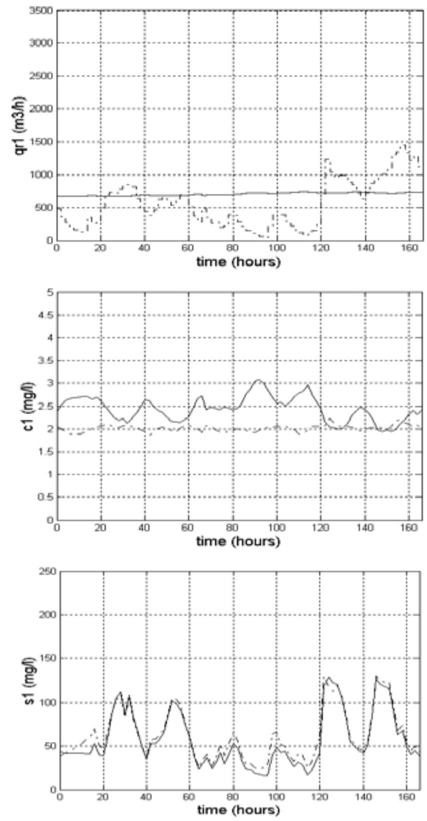

Fig. 8: Integrated Design results

5. CONCLUSIONS

In this paper an Integrated Design procedure to obtain one optimal plant for the activated sludge process and its MPC tuning parameters has been developed. The design procedure shown here produces better controllable plants that the classical procedure. The responses for closed loop design with MPC show clearly a good behaviour for interest variables. When Integrated Design procedure is solved, the designed plant is able to fulfil disturbance rejection requirements with optimum cost units. This is an important result because one can obtain an optimum plant with lower construction costs and good disturbance rejection. Note also that no further MPC tuning is needed because the optimization gives also its optimum parameters. The solved problem guarantees that the dynamic non-linear model of the plant is satisfied, as well as the operation and process constraints. This Integrated Design methodology also allows for the easy inclusion of other indexes for tuning controller parameters, such as LMI conditions.

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