ON DISCONTINUOUS ROBUST STATIC OUTPUT FEEDBACK CONTROL

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Abstract: A robust stabilisation problem using static output feedback is studied for a class of linear systems with nonlinear disturbances. Two discontinuous output feedback control schemes are synthesised: one is based on sliding mode control and the other based on Lyapunov techniques. The output information appearing in the nonlinear bounds is separated from the unmeasurable part and is directly used in the control design to enhance robustness. The study in this paper shows the effects on system stability from assuming bounds on the uncertainties.
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1. INTRODUCTION

Discontinuous control has been studied by many scientists and engineers (Emelyanov *et al.*, 1992; Edwards and Spurgeon, 1998). One of the main characteristics is that discontinuous control exhibits good robust performance in the presence of uncertainty. Moreover, sometimes the real system can not be stabilised by continuous control (Brocket, 1983). This shows that the study of discontinuous control is valuable and necessary not only theoretically but also from the applications perspective.

Real engineering systems unavoidably suffer from a variety of uncertainties. In order to ensure the system exhibits good performance in the presence of disturbances it is necessary to design a robust controller to reject the uncertainties. In much of the existing work, it is required that the uncertainty has a linear bound/linear growth condition of the form $a\|x\| + b$. However, in practical systems, the bound may take a nonlinear form (Chen and Pandey, 1990). Although sometimes it is possible to use a large linear bound to replace the nonlinear one, this undoubtedly produces some conservatism and also unnecessary control consumption.

In static output feedback design, nearly all work requires that the uncertainty bound is a function of the output (see, e.g (Emelyanov *et al.*, 1992; Żak and Hui, 1993; Benabdallah and Hammami, 2001; Edwards and Spurgeon, 1995)). Recently, some authors have tried to extend the linear bounds on the uncertainties to nonlinear bounds (Yan *et al.*, 1998; Yan *et al.*, 2004; Yan *et al.*, 2005). In sliding mode control, the matched uncertainty is allowed to have nonlinear bounds since the sliding motion is insensitive to matched uncertainty (Edwards and Spurgeon, 1995). However, the case of nonlinear bounds on the mismatched uncertainty has rarely been considered and in nearly all existing associated work, the bounds on the mismatched uncertainty are not fully exploited for controller design.

Output feedback control is much more complicated than state feedback control since only partial information is available. Indeed the fundamental question of the existence of a static output feedback controller for a triple (A, B, C) is still open even in the scalar case (Syrmos *et al.*, 1997). The focus of this paper is the study of robustness, and two discontinuous output feedback control schemes are presented for a class of systems with nonlinear disturbances. Using the approach given by (Edwards and Spurgeon, 1998), a sliding mode control scheme based on (Yan *et al.*, 2004) is given where the mismatched uncertainty has a nonlinear bound which is allowed to be a function of the state variables. A Lyapunov controller is presented which improves on the work in (Yan *et al.*, 1998). An approach for dealing with the nonlinear bounds on both the matched and mismatched uncertainty is proposed when the bounds can not be expressed as a function of the output alone. By identifying the known information in the nonlinear bounds and using it directly in the control design, the robustness is enhanced and the conservatism is reduced. Some discussions and remarks are presented to show a comparison between the two approaches. It is shown that the effect of the uncertainty on system stability is very closely connected with whether the uncertainty is matched or not and whether its bound can be expressed as a function of the output.

Notation: In the sequel, for a matrix A, $\mathrm{Im}(A)$ denotes the image (or range) of A and A^M will be used to denote its Moore-Penrose inverse. The set of real numbers will be denoted by $\mathcal{R}$, $\mathcal{R}^+$ denotes the set $\{t \mid t \geq 0\}$, and $\mathcal{R}^{n\times m}$ represents the $n \times m$ matrix set with its elements defined in $\mathcal{R}$. The Lipschitz constant of a function f in its domain of definition will be written $\mathcal{L}_f$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

2. PRELIMINARIES

Consider an uncertain linear system

$$\dot{x}(t) = Ax(t) + B(u + G(x,t)) + F(x,t) \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

where $x \in \Omega \subset \mathcal{R}^n$, $u \in \mathcal{R}^m$, $y \in \mathcal{R}^p$ with $m \leq p < n$ are the state variable, input and output respectively; (A, B, C) are constant matrices of appropriate dimension with B and C both being of full rank; $G(x,t)$ and $F(x,t)$ are the matched and mismatched uncertainties respectively, which are continuous in their arguments.

It is assumed that there exist known continuous functions ϕ_1, ϕ_2, ψ_1 and ψ_2 in $\Omega \times \mathcal{R}^+$ such that for $(x,t) \in \Omega \times \mathcal{R}^+$

$$\|G(x,t)\| \leq \phi_1(x,t) + \phi_2(y,t) \quad (3)$$

$$\|F(x,t)\| \leq \psi_1(x,t) + \psi_2(y,t) \quad (4)$$

where ϕ_1 and ψ_1 are Lipshitz in $x \in \Omega$ and uniformly about $t \in \mathcal{R}^+$. Furthermore $\phi_1(0,t) = \phi_2(0,t) = \psi_1(0,t) = \psi_2(0,t) = 0$ for all $t \in \mathcal{R}^+$ which implies that $x = 0$ is an equilibrium point despite the disturbances.

Remark 1. This paper focuses on the study of discontinuous control, and thus the right hand side of equation (1) is not continuous due to the discontinuouity of the control u. As a result, the classical solution of the equation no longer exists. In this case, the solution of the equation is defined in the Filippov sense (Filippov, 1983). This is assumed throughout the paper.

Remark 2. From (3) and (4), it is observed that both the matched and mismatched uncertainty are allowed to have general nonlinear bounds, and this framework includes all previous work as special cases. Furthermore, the structure of the uncertainties are not necessarily known. Only their bounds are assumed to be known and will be used in the control design.

Definition 1. The domain Ω_q $(0 \in \Omega_q \subset \Omega)$ is called a stabilised domain of system (1)–(2) if in Ω_q there exist a Lyapunov function $V(x,t)$ and a control $u(x,t)$ such that the time derivative of $V(x,t)$ along the corresponding closed-loop system is negative definite in Ω_q.

It should be pointed out that the stabilised domain defined here is different from the domain of attraction (stability region) of the closed-loop system.

The following assumptions will be imposed on the system (1)–(2):

Assumption A1. $\mathrm{rank}(CB) = m$.

Under Assumption A1, from (Edwards and Spurgeon, 1998), there exists a nonsingular linear coordinate transformation such that the triple (A, B, C) with respect to the new coordinates has the structure

$$\begin{bmatrix} \widetilde{A}_1 & \widetilde{A}_2 \\ \widetilde{A}_3 & \widetilde{A}_4 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ \widetilde{B}_2 \end{bmatrix}, \quad \begin{bmatrix} 0 & \widetilde{C}_2 \end{bmatrix} \quad (5)$$

where $\widetilde{A}_1 \in \mathcal{R}^{(n-m)\times(n-m)}$, $\widetilde{B}_2 \in \mathcal{R}^{m\times m}$ and $\widetilde{C}_2 \in \mathcal{R}^{p\times p}$.

Assumption A2. The triple $(\widetilde{A}_1, \widetilde{A}_2, \bar{C}_2)$ with $\bar{C}_2 = [0_{(p-m)\times(n-p)} \quad I_{p-m}]$ is output feedback stabilisable.

From Assumption A2, there exists a matrix K such that $\widetilde{A}_1 + \widetilde{A}_2 K \bar{C}_2$ is stable. Therefore, for $Q_1 > 0$, the following Lyapunov equation has a unique solution $P_1 > 0$

$$A_1^\tau P_1 + P_1 A_1 = -Q_1. \qquad (6)$$

Remark 3. Assumptions A1 and A2 are used to guarantee that there exists an output sliding surface for the triple (A, B, C) (Edwards and Spurgeon, 1998).

Assumption B1. There exists a matrix L such that $A - BLC$ is stable.

From Assumption B1, it follows that for $Q_2 > 0$, the Lyapunov equation

$$(A - BLC)^\tau P_2 + P_2(A - BLC) = -Q_2 \qquad (7)$$

has unique solution $P_2 > 0$.

Assumption B2. There exists a matrix H such that $B^\tau P_2 = HC$.

Assumption B3. $\mathcal{Y} \subseteq \mathrm{Im}((H^\tau)^M)$ where H is defined in Assumption B2 and $\mathcal{Y} \equiv: \{y \mid y = Cx, x \in \mathcal{R}^n\}$ is the system output space.

In (Edwards and Spurgeon, 1998) it is argued that a necessary condition for the existence of an output sliding surface which provides a stable sliding motion with a unique equivalent control is that the triple (A, B, C) is minimum phase and relative degree one. From (Gu, 1990), it is seen that under the assumptions $m = p$ and $\mathrm{rank}(CB) = m$, Assumptions B1 and B2 are equivalent to the requirement that (A, B, C) is minimum phase. The two sets of assumptions A and B have been independently stated to be consistent with the original work of (Edwards and Spurgeon, 1998) and (Yan *et al.*, 1998).

Lemma 1. In the case when $m = p$, Assumptions B1-B3 are satisfied if Assumptions B1-B2 are satisfied and H is nonsingular.

Proof: It is only required to prove that Assumption B3 is satisfied.

From the nonsingularity of H, it is observed that $(H^\tau)^{-1} = (H^\tau)^M$, and for any $x \in \mathcal{R}^n$

$$y = Cx = (H^\tau)^{-1} H^\tau Cx = (H^\tau)^M (H^\tau Cx)$$

This shows that $y \in \mathrm{Im}((H^\tau)^M)$. Hence the conclusion follows. #

Lemma 1 shows that the conditions used in (Yan *et al.*, 1998) are stronger than the ones used here.

3. DISCONTINUOUS CONTROL DESIGN

In this section, two static output feedback controllers are proposed for the system (1)–(2).

3.1 Sliding Mode Control Design

A control strategy for system (1)–(2) will be developed where the sliding surface given by (Edwards and Spurgeon, 1998) is employed.

Under Assumptions A1 and A2, it follows from Remark 3 that there exists a sliding surface

$$\sigma(x) \equiv: Sx \equiv: ECx = 0 \qquad (8)$$

such that the nominal system (1)–(2) when restricted to (8) gives a stable sliding motion.

Now the objective is to find the sliding mode dynamics and study its stability in the presence of uncertainty. From (Edwards and Spurgeon, 1998) there exists a nonsingular coordinate transformation $z = Tx$ such that in the new coordinates z, system (1)–(2) has the following form

$$\dot{z} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} z + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \left(u + G(T^{-1}z, t)\right) + TF(T^{-1}z, t), \qquad (9)$$

$$y = \begin{bmatrix} 0 & C_2 \end{bmatrix} z \qquad (10)$$

where $A_1 = \widetilde{A}_1 + \widetilde{A}_2 K \bar{C}_2$ is stable; $B_2 \in \mathcal{R}^{m\times m}$ and $C_2 \in \mathcal{R}^{p\times p}$ are both nonsingular. Furthermore, it can be shown that (Edwards and Spurgeon, 1995)

$$E \begin{bmatrix} 0 & C_2 \end{bmatrix} = \begin{bmatrix} 0 & E_2 \end{bmatrix} \qquad (11)$$

where $E_2 \in \mathcal{R}^{m\times m}$ is nonsingular. In addition the fact that

$$C = \begin{bmatrix} 0 & C_2 \end{bmatrix} T = C_2 \begin{bmatrix} 0 & I_p \end{bmatrix} T \qquad (12)$$

will be used.

In order to fully use the system structure, the following partitions are introduced

$$T \equiv: \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \qquad C_2 = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix} \qquad (13)$$

$$T^{-1} \equiv: \begin{bmatrix} W_1 & W_2 \end{bmatrix} \qquad (14)$$

where $T_1 \in \mathcal{R}^{(n-m)\times n}$, $W_1 \in \mathcal{R}^{n\times(n-m)}$ and $C_{21} \in \mathcal{R}^{p\times(p-m)}$. Then, system (9)–(10) can be rewritten as

$$\dot{z}_1 = A_1 z_1 + A_2 z_2 + T_1 F(T^{-1}z, t) \qquad (15)$$

$$\dot{z}_2 = A_3 z_1 + A_4 z_2 + B_2 \left[u + G(T^{-1}z, t)\right] + T_2 F(T^{-1}z, t) \qquad (16)$$

$$y = C_{21} z_{12} + C_{22} z_2 \qquad (17)$$

where $z = \mathrm{col}(z_1, z_2)$ with $z_1 \in \mathcal{R}^{n-m}$ and $z_1 = \mathrm{col}(z_{11}, z_{12})$ with $z_{11} \in \mathcal{R}^{n-p}$ and $z_{12} \in \mathcal{R}^{p-m}$.

Consider the sliding surface (8) in the new coordinate system. From (11),

$$Sx = E\,[\,0 \quad C_2\,]\,z = [\,0 \quad E_2\,]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = E_2 z_2$$

where E_2 is nonsingular. The sliding surface (8) in the new coordinates z becomes

$$z_2 = 0 \tag{18}$$

When system (15)-(17) is restricted to sliding surface (18), the sliding motion is described by

$$\dot{z}_1 = A_1 z_1 + T_1 F(W_1 z_1, t) \tag{19}$$

Obviously, the mismatched uncertainty F affects the dynamics of the sliding mode and may destroy its stability. It is necessary to impose some constraints so that the stability of the sliding mode dynamics are guaranteed. The following conclusion can now be presented:

Proposition 1. Consider system (1)–(2). Under Assumptions A1 and A2, the sliding motion in (19) is asymptotically stable if there exists a domain Ω_1 of the origin in Ω such that for $x \in \Omega_1 \setminus \{0\}$ and $t \in \mathcal{R}^+$

$$\|z_1^\tau P_1 T_1\|(\psi_1(W_1 z_1, t) + \psi_2(C_{21} z_{12}, t)) < \frac{1}{2} z_1^\tau Q_1 z_1 \tag{20}$$

where P_1 and Q_1 satisfy (6); and C_{21} and T_1 are given by (13)–(14).

Proof: Using (17) and (4), it is observed that

$$\|F(W_1 z_1, t)\| \le \psi_1(W_1 z_1, t) + \psi_2(C_{21} z_{12}, t) \tag{21}$$

Then, for system (19), consider the Lyapunov function candidate $V_1(z_1) = z_1^\tau P_1 z_1$ with P_1 satisfying (6). By direct computation, it follows from (20) that the time derivative of V_1 along the trajectories of systems (19) is negative definite. Hence, the conclusion follows. #

Next it is required to design an output feedback sliding mode control such that the system state is driven to the sliding surface (8). Write

$$Tx = \begin{bmatrix} (Tx)_1 \\ (Tx)_2 \end{bmatrix}$$

with $(Tx)_1 \in \mathcal{R}^{n-p}$ and $T^{-1} = [M_1 \quad M_2]$ with $M_1 \in \mathcal{R}^{n\times(n-p)}$. Then, from (12)

$$x = T^{-1}\begin{bmatrix} (Tx)_1 \\ C_2^{-1} y \end{bmatrix} = M_1 (Tx)_1 + M_2 C_2^{-1} y \tag{22}$$

Assume that $\|M_1(Tx)_1\|$ is bounded in Ω and that $\mu \equiv: \sup_{x\in\Omega}\{\|M_1(Tx)_1\|\}$. The following control is proposed

$$\begin{aligned} u = &-(SB)^{-1} SAM_2 C_2^{-1} y - (SB)^{-1}\frac{Ey}{\|Ey\|}\Big\{ \\ &\|SB\|\left(\phi_1(M_2 C_2^{-1} y, t) + \phi_2(y,t)\right) + \|S\| \\ &\cdot\left(\psi_1(M_2 C_2^{-1} y, t) + \psi_2(y,t)\right) + k\Big\} \end{aligned} \tag{23}$$

where k is the control gain to be designed later.

Proposition 2. If $\|M_1(Tx)_1\|$ is bounded in Ω, then, under Assumptions A1 and A2, the control (23) drives system (1)–(2) to the sliding surface (8) and maintains motion on it if in the domain Ω the control gain function k satisfies

$$k \ge (\|SB\|\mathcal{L}_{\phi_1} + \|S\|\mathcal{L}_{\psi_1} + \|SA\|)\mu + \beta \tag{24}$$

where $\mu \equiv: \sup_{x\in\Omega}\{\|M_1(Tx)_1\|\}$ and β is a positive constant.

Proof: From $\sigma(x) = Sx = Ey$ and (22), it is observed that

$$\begin{aligned} \dot{\sigma}(x) = &SAM_1(Tx)_1 + SAM_2 C_2^{-1} y + SBu \\ &+ SBG(x,t) + SF(x,t) \end{aligned} \tag{25}$$

By applying the control u in (23) to (25), it follows from (3) and (4):

$$\begin{aligned} &\sigma^\tau(x)\dot{\sigma}(x) \\ &\le \|\sigma(x)\|\Big(\|SA\|\,\|M_1(Tx)_1\| + \|SB\|\,\|G(x,t)\| \\ &\quad + \|S\|\,\|F(x,t)\| - \|SB\|(\phi_1(M_2 C_2^{-1} y, t)) \\ &\quad + \phi_2(y,t) - \|S\|(\psi_1(M_2 C_2^{-1} y, t) + \psi_2(y,t)) \\ &\quad - k\Big) \\ &\le \|\sigma(x)\|\Big((\|SB\|(\phi_1(x,t) - \phi_1(M_2 C_2^{-1} y, t)) \\ &\quad + \|S\|(\psi_1(x,t) - \psi_1(M_2 C_2^{-1} y, t)) \\ &\quad + \|SA\|\,\|M_1(Tx)_1\| - k\Big) \\ &\le -\beta\|\sigma(x)\| \end{aligned} \tag{26}$$

where (24) is used in the last implication. Hence, the result follows. #

Remark 4. Compared with the work of (Żak and Hui, 1993; Edwards and Spurgeon, 1995), mismatched uncertainty is considered in this paper. Unlike (Yan *et al.*, 2004), the bound on the mismatched uncertainty is used to design the control to enhance robustness. Therefore, the control scheme presented here is less conservative.

3.2 Lyapunov Control Design

In this section, a control scheme based on Lyapunov techniques is presented using Assumptions B1-B3.

Consider the control law

$$u = -Ly - \frac{\varepsilon_1}{2}Hy - \varepsilon_2\|P_2M_2C_2^{-1}\|^2(H^\tau)^M y + v_1(y,t) + v_2(y,t) \quad (27)$$

where ε_1 and ε_2 are both positive constants, and v_1 and v_2 are respectively defined by

$$v_1(y,t) = \begin{cases} -\dfrac{Hy}{\|Hy\|}(\phi_1(M_2C_2^{-1}y,t) + \phi_2(y,t)), & Hy \neq 0 \\ 0, & Hy = 0 \end{cases} \quad (28)$$

$$v_2(y,t) = \begin{cases} -\dfrac{(H^\tau)^M y}{\varepsilon_2\|y\|^2}(\psi_1^2(M_2C_2^{-1}y,t) + \psi_2^2(y,t)), & y \neq 0 \\ 0, & y = 0 \end{cases} \quad (29)$$

The following conclusion can be presented:

Proposition 3. Under Assumptions B1-B3, system (1)–(2) is asymptotically stabilized by control (27)–(29) if there exists a neighborhood Ω_2 of the origin in Ω such that for any $x \in \Omega_2 \setminus \{0\}$

$$x^\tau Q_2 x - \Big(\frac{1}{\varepsilon_1}\mathcal{L}_{\phi_1}^2 + \frac{2}{\varepsilon_2}\mathcal{L}_{\psi_1^2}\Big)\|M_1(Tx)_1\|^2 - 2\varepsilon_2\|P_2M_1(Tx)_1\|^2 < 0 \quad (30)$$

for some positive constants ε_1 and ε_2.

Proof: The closed-loop system from applying the control (27)–(29) to (1) is described by

$$\dot{x} = (A - BLC)x - B\Big(\frac{\varepsilon_1}{2}Hy + \varepsilon_2\|P_2M_2C_2^{-1}\|^2 \cdot(H^\tau)^M y - v_1(y,t) - v_2(y,t) - G(x,t)\Big) + F(x,t) \quad (31)$$

The objective now is to prove system (31) is stable. Consider the Lyapunov candidate $V = x^\tau P_2 x$. By using (7), the time derivative along the trajectories of system (31) is given by

$$\begin{aligned}\dot{V}\,|_{(31)} &= -x^\tau Q_2 x + 2x^\tau P_2 B\Big(-\frac{\varepsilon_1}{2}Hy - \varepsilon_2\|P_2M_2C_2^{-1}\|^2(H^\tau)^M y + v_1(y,t) + G(x,t) + v_2(y,t)\Big) + 2x^\tau P_2F(x,t) \\ &= -x^\tau Q_2 x + 2y^\tau H^\tau\Big(-\frac{\varepsilon_1}{2}Hy + v_1(y,t) + G(x,t)\Big) + 2\Big(y^\tau H^\tau[-\varepsilon_2\|P_2M_2C_2^{-1}\|^2 \cdot(H^\tau)^M y + v_2(y,t)] + x^\tau P_2F(x,t)\Big) \end{aligned} \quad (32)$$

where Assumption B2 is used above. From (3), (22) and (28), it follows that

$$\begin{aligned} & y^\tau H^\tau\Big(-\frac{\varepsilon_1}{2}Hy + v_1(y,t) + G(x,t)\Big) \\ & \leq -\frac{\varepsilon_1}{2}\|Hy\|^2 - (Hy)^\tau\frac{Hy}{\|Hy\|}[\phi_1(M_2C_2^{-1}y,t) + \phi_2(y,t)] + \|Hy\|\,[\phi_1(x,t) + \phi_2(y,t)] \\ & \leq -\frac{\varepsilon_1}{2}\|Hy\|^2 + \|Hy\|\mathcal{L}_{\phi_1}\|M_1(Tx)_1\| \\ & \leq \frac{1}{2\varepsilon_1}\mathcal{L}_{\phi_1}^2\|M_1(Tx)_1\|^2 \end{aligned} \quad (33)$$

where Young's inequality $2ab \leq \varepsilon_1 a^2 + \frac{1}{\varepsilon_1}b^2$ is used above. From Assumption B3, it follows that $H^\tau(H^\tau)^M y = y$ and from (4)

$$\begin{aligned} & x^\tau P_2F(x,t) \\ & \leq \frac{\varepsilon_2}{2}\|P_2x\|^2 + \frac{1}{2\varepsilon_2}(\psi_1(x,t) + \psi_2(y,t))^2 \\ & \leq \varepsilon_2[\|P_2M_1(Tx)_1\|^2 + \|P_2M_2C_2^{-1}\|^2\|y\|^2] + \frac{1}{\varepsilon_2}\left(\psi_2^2(y,t) + \psi_1^2(x,t)\right) \end{aligned} \quad (34)$$

Then from (29) and (34):

$$\begin{aligned} & y^\tau H^\tau\left(-\varepsilon_2\|P_2M_2C_2^{-1}\|^2(H^\tau)^M y + v_2(y,t)\right) + x^\tau P_2F(x,t) \\ & \leq -\varepsilon_2\|P_2M_2C_2^{-1}\|^2\|y\|^2 - y^\tau H^\tau\frac{(H^\tau)^M y}{\varepsilon_2\|y\|^2} \cdot\left(\psi_1^2(M_2C_2^{-1}y,t) + \psi_2^2(y,t)\right) \\ & \quad + \varepsilon_2\|P_2M_1(Tx)_1\|^2 + \varepsilon_2\|P_2M_2C_2^{-1}\|^2\|y\|^2 + \frac{1}{\varepsilon_2}\left(\psi_2^2(y,t) + \psi_1^2(x,t)\right) \\ & \leq \varepsilon_2\|P_2M_1(Tx)_1\|^2 + \frac{1}{\varepsilon_2}\mathcal{L}_{\psi_1^2}\|M_1(Tx)_1\|^2 \end{aligned} \quad (35)$$

Substituting (33) and (35) into (32), it follows from (30) that $\dot{V}\,|_{(31)}$ is negative definite in the domain Ω_2. Hence the conclusion follows #.

Remark 5. The controller designed above is based on the work in (Yan *et al.*, 1998) which is only applicable to square systems and requires H in Assumption B2 to be nonsingular. However, in this paper, these conditions have been weakened. Furthermore, unlike (Yan *et al.*, 1998), the nonlinear bounds, particularly the bound on the mismatched uncertainty, is used in the control design to enhance robustness. Thus the work developed here includes (Yan *et al.*, 1998) as a special case.

4. DISCUSSIONS

4.1 Robustness

In the sliding mode, the system dynamics are completely robust to matched uncertainty, but the matched uncertainty affects the reaching phase if the uncertainty bound is a function of the state variables and only a subset of state information is available. The mismatched uncertainty affects both the sliding motion and the reaching phase.

If the bound on the uncertainty is a function of the output then the reachability condition can always be satisfied by designing appropriate control gains no matter whether the uncertainty is matched or mis-matched.

Using Lyapunov techniques, the matched uncertainty can be cancelled completely if its bound can be expressed as a function of the output by designing an appropriate control. However, this is not true for the mismatched case.

In the two control strategies proposed, the effects of uncertainty, generally speaking, can not be cancelled completely if the bounds are a function of the states instead of just the outputs. This is reasonable since only output feedback is allowed. An approach to deal with such uncertainty has been presented and shown to enhance the robustness.

4.2 Stabilised Domain

Comparing (20) with (30), it can be observed that with the sliding mode approach, the stability condition only involves the sliding motion variables and has nothing to do with any other variables. In fact, the stability condition (20) is only required to be satisfied on the sliding surface (8), and what is more, it has nothing to do with matched uncertainty. However, for the Lyapunov design, the stability condition (30) is closely connected with all state variables and the matched uncertainty may destroy the system stability if the bounds are not functions of the outputs. Therefore, the latter is more conservative in this regard. However, reachability is necessary to apply the sliding mode technique. It should be pointed out that in sliding mode control, the reachability condition is always satisfied in any bounded region if sufficient control effort is available.

The study shows that a larger stabilised domain can be obtained using sliding mode control than Lyapunov control if a global result is not available.

5. CONCLUSION

This paper has presented two control strategies to stabilise a class of nonlinear systems with nonlinear disturbances using only static output feedback. Based on the method given by Edwards and Spurgeon, a sliding surface was proposed and an associated sliding mode control scheme presented. A Lyapunov control scheme based on (Yan *et al.*, 1998) has also been proposed which extends the approach used in (Yan *et al.*, 1998) to a wider class of systems. A new approach for dealing with the uncertainty has been given when the bounds cannot be expressed as functions of the outputs. This study shows that both approaches produce good robustness. A comparison between the two approaches has been given.

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