SAFE ADAPTIVE SWITCHING THROUGH INFINITE CONTROLLER SET: STABILITY AND CONVERGENCE

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Abstract: A primary goal of adaptive control is to achieve stability and asymptotically optimal performance, given the feasibility of adaptive control problem—defined as the existence of a stabilizing solution in a continuously parametrized controller set. A solution is proposed called *safe adaptive control*, which robustly achieves this goal without any assumptions other than feasibility. Specifically, a list of the required properties of the cost function is formulated. The paper builds on the previous results in Stefanovic *et al.* (2004) and Morse *et al.* (1992). The previous results are generalized here by allowing the class of candidate controllers to be infinite. The problem is motivated by a model-mismatch stability failure associated with a multitude of adaptive control schemes.

Keywords: Adaptive Control, Convergence, Robustness, Stability, Switching

1. INTRODUCTION

The book *Adaptive Control* (Åström and Wittenmark, 1995) begins in the following way: "In everyday language, 'to adapt' means to change a behavior to conform to new circumstances. Intuitively, an adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances".

Whether it is conventional, continuous adaptive tuning or more recent adaptive switching, adaptive control has an inherent property that it *orders controllers based on evidence found in data.* Any adaptive algorithm can thus be associated with a cost function, dependent on available data, that it minimizes, though this may not be explicitly present. The differences among adaptive schemes arise in part due to specific algorithms employed to approximately compute cost-minimizing controllers. And, major differences also arise due to the extent to which additional assumptions are tied with this cost function. The cost function needs to be chosen to reflect control goals. Thus, an important issue is the precise definition of the goal of adaptive control, which has been used variously. The perspective adopted in this paper hinges on the notion of feasibility of adaptive control. An adaptive control problem is said to be feasible if the plant is stabilizable and at least one (*a priori* unknown) stabilizing controller exists in the candidate controller set that achieves the specified control goal for the given plant. Given feasibility, the view adopted in this paper of *a primary goal of adaptive control* is to recognize when the accumulated experimental data shows that a controller fails to achieve desired stability and performance objectives. If a destabilizing controller happens to be the currently active one, adaptive control should eventually switch it out of the loop, and replace it with an optimal one. An optimal

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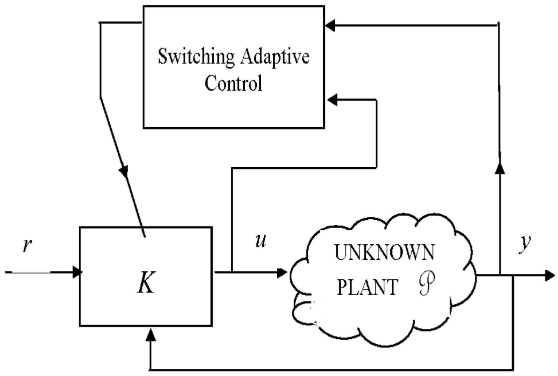

Fig. 1. Switching adaptive control system $\sum$

controller is one that optimizes the cost function given the currently available evidence. This perspective renders the adaptive control problem in a form of a standard constrained optimization.

Following the work in (Safonov and Tsao, 1997), further progress was made in (Stefanovic *et al.*, 2004) which identifies sufficient conditions for ensuring stability and convergence to a robustly stabilizing controller, given control problem feasibility, with a focus on a *finite* candidate controller set. The results of this paper widen the previous theoretical ground by allowing the class of candidate controllers to be *infinite*. This property is essential when the uncertainties are so large that no set of finitely many controllers is likely to suffice in achieving the control goal. It is shown that, under some mild additional assumptions on the cost function (designer-based, not plant-dependent), stability of the closed loop switched system is assured, as well as the convergence to a stabilizing controller in finitely many steps. Related work can be found in *e.g.* (Morse *et al.*, 1992), (Hespanha *et al.*, 2003).

The paper is organized as follows. Preliminary facts are given in § 2, followed by the main result in § 3. Then, § 4 presents an example of the cost function satisfying sufficient conditions for stability and finiteness of switches, while § 5 provides a simple simulation verification of the proposed theory.

2. PRELIMINARIES

Let $\mathbf{Z}$ be the set of all possible output signals $z = [u, y]$ reproducible by switching adaptive system $\Sigma : \mathfrak{L}_{2e} \longrightarrow \mathfrak{L}_{2e}$ in Figure 1. Let $z_{data} = [y_{data}, u_{data}] \in \mathbf{Z}$ represent the output signals recorded (hypothetically) in one single, infinite duration, experiment. At any time τ, $P_\tau z_{data}$ is the actually available data obtained using the projection operator that truncates a signal after $t = \tau$, where $t, \tau \in \mathbf{T} = \mathbb{R}_+$.

Unless otherwise noted, it is assumed, throughout the paper, that all components of the system under consideration have zero-input zero-output property, so that when system Σ is undisturbed ($(r, d, n) = \mathbf{0}$), the pair $(y, u) = (0, 0)$ is an equilibrium solution.

An infinite set $\mathbf{K}$ (*e.g.* containing a continuum) of candidate controllers is considered. The finite controller set results will be derived as a special case. The parameterization of $\mathbf{K}$, denoted Θ_K, will initially be taken to be a subset of $\mathbb{R}^n$; the more general case of infinite dimensional spaces will be discussed in Comment 3.

Definition 1. The adaptive control problem is said to be *feasible* if a candidate controller set $\mathbf{K}$ contains at least one controller that achieves stability and performance goals.

Definition 2. A controller K is said to be *feasible* if it satisfies given performance and stability constraints.

Assumption 1. (*Feasibility assumption*). The adaptive control problem is feasible.

Comment 1. It is not known *a priori* which $K \in \mathbf{K}$ is feasible.

Definition 3. An L_2*-norm* of a truncated signal $x(t)$ is given as $||x||_t = \sqrt{\int_0^t ||x(\tau)||^2 d\tau}$, where $||x(t)||$ stands for the Euclidean norm of x at time t. The Euclidean norm of the parameterization $\theta_K \in \mathbb{R}^n$ of the controller K is denoted $||\theta_K||$.

Definition 4. (Safonov, 1980) A system $\Sigma : \mathfrak{L}_{2e} \longrightarrow \mathfrak{L}_{2e}$ with input w and output z is said to be *stable* if there exists a function $\phi \in \mathcal{K}$ (class $\mathcal{K}$) such that $\forall w \in \mathfrak{L}_{2e}, w \neq 0$:

$$\limsup_{\tau \to \infty} ||z||_\tau \leq \phi(\limsup_{\tau \to \infty} ||w||_\tau)$$

Otherwise, Σ is said to be *unstable.* If ϕ exists and is linear, Σ is said to be *finite-gain stable.*

Specializing to the system in Figure 1, stability of the closed loop system Σ means $\limsup_{\tau\to\infty} ||[y, u]||_\tau \leq \phi(\limsup_{\tau\to\infty} ||r||_\tau)$, for some $\phi \in \mathcal{K}$ and $\forall r \in \mathfrak{L}_{2e}$, $r \neq 0$.

Definition 5. (Safonov and Tsao, 1997). For every $K \in \mathbf{K}$, a *fictitious reference signal* $\tilde{r}_K(z_{data})$ is defined to be an element of

$$\tilde{R}(K, z_{data}) \doteq \{r | K \begin{bmatrix} r \\ y \end{bmatrix} = u, \ z_{data} = \begin{bmatrix} u \\ y \end{bmatrix}\}.$$

In other words, $\tilde{r}_K(z_{data})$ is a hypothetical reference signal that would have exactly reproduced the measured data z_{data} had the controller K been in the loop for the entire time period over which the data z_{data} was collected.

Definition 6. Given $K \in \mathbf{K}$ and measured data z_{data}, *stability* of the system given in Figure 1 is said to be *falsified* by data z_{data} if, for some $\tilde{r}_K(z_{data}) \in \tilde{R}(K, z_{data})$,

$$\limsup_{\tau\to\infty} \frac{||z_{data}||_\tau}{||\tilde{r}_K||_\tau} = \infty$$

Otherwise, it is said to be *unfalsified*.

Definition 7. The *cost functional* $V(K, z, t)$ is a causal mapping

$$V : \mathbf{K} \times \mathbf{Z} \times \mathbf{T} \to \mathbb{R}_+ \cup \{\infty\}$$

Definition 8. (Bertsekas, 1999) A functional $f : \mathbb{R}^n \to \mathbb{R}$ is said to be *coercive* if $\lim f(x) = \infty$ when $||x|| \to \infty$, $x \in \mathbb{R}^n$.

Definition 9. The *true cost* $V_{true} : \mathbf{K} \to \mathbb{R}_+ \cup \{\infty\}$ is defined as $V_{true}(K) = \sup_{z\in\mathbf{Z},\tau\in\mathbf{T}} V(K, z, \tau)$

Definition 10. A *robust optimal controller* K_{RSP} is one that stabilizes (in the sense of the Def. 4) the given plant and minimizes the true cost $V_{true}(K)$.

Definition 11. A system is said to be *cost detectable* if, whenever stability of the system in Figure 1 with $K \in \mathbf{K}$ in the loop is falsified by data z_{data}, then $\lim_{\tau\to\infty} V(K, z_{data}, \tau) = \infty$.

In the following, $\{t_k\}_{k=1}^{\bar{N}}$, $\bar{N} \in \mathbb{N} \cup \{\infty\}$ denotes an ordered sequence of times ($t_{k+1} > t_k$, $\forall k = 1, ..., \bar{N}$). Also, $\mathcal{V} \doteq \{V_{z,t} : z \in \mathbf{Z}, t \in \mathbf{T}\} : \mathbf{K} \to \mathbb{R}_+$ denotes a family of functionals with the common domain $\mathbf{K}$, with $V_{z,t}(K) \doteq V(K, z, t)$. The theory will make use of the following level set in the controller space: $\mathbf{L} \doteq \{K \in \mathbf{K} | V_{z,t}(K) \leq V_{true}(K_{RSP}), V \in \mathcal{V}\}$.

Definition 12. (Rudin, 1976). If $E \subset X$, and f is a function defined on X, the *restriction* of f to E is the function g whose domain of definition is E such that $g(p) = f(p)$ for $p \in E$.

With the family of functionals $\mathcal{V}$ with a common domain $\mathbf{K}$, a restriction to the set $\mathbf{L} \subseteq \mathbf{K}$ is associated, defined as a family of functionals $\mathcal{W} \doteq \{W_{z,t}(K) : z \in \mathbf{Z}, t \in \mathbf{T}\}$ with a common domain $\mathbf{L}$.

Consider now the cost minimization hysteresis switching algorithm reported in (Morse *et al.*, 1992), together with the cost functional $V(K, z, t)$ (see Figure 2). The algorithm returns, at each time instant τ, a controller $\hat{K}_\tau$ which is the active controller in the loop.

ε-HYSTERESIS SWITCHING ALGORITHM A1
(Morse *et al.*, 1992)

(1) Initialize: Let $t = 0$, $\tau = 0$; choose $\varepsilon > 0$.
Let $\hat{K}_t \in \mathbf{K}$ be the first controller in the loop.
(2) $\tau \leftarrow \tau + 1$.
If $V(\hat{K}_t, z, \tau) > \min_{K\in\mathbf{K}} V(K, z, \tau) + \varepsilon$ then $t \leftarrow \tau$ and $\hat{K}_t \leftarrow \arg\min_{K\in\mathbf{K}} V(K, z, \tau)$.

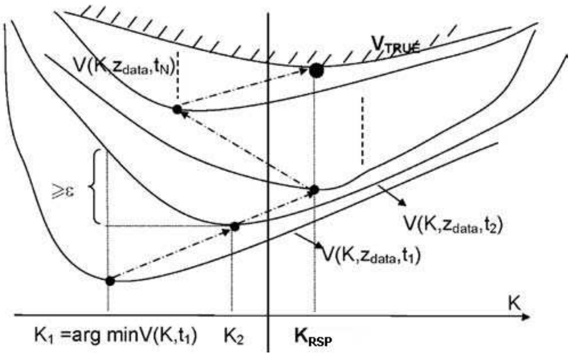

Fig. 2. Cost vs. control gain time snapshots.

(3) $\hat{K}_\tau \leftarrow \hat{K}_t$;
(4) go to 2.

In the above algorithm, the hysteresis step ε serves to limit the number of switches, and so prevents the possibility of switching limit cycle type of instability.

In conjunction with Algorithm A1 and the cost function $V(K, z, t)$, and the particular z_{data} from one experiment, $\{t_k\}_{k=1}^{\bar{N}}$, $\bar{N} \in \mathbb{N} \cup \{\infty\}$ denotes the ordered sequence of switching times. K_k is the controller switched in the loop at time t_k, $k = 1, ..., \bar{N}$, that remains in the loop until some time $t_{k+1} > t_k$, when K_{k+1} is switched in the loop according to A1. $\hat{K}_t$ is the currently active controller at time t. Thus, $\hat{K}_t = K_k$ on $t \in [t_k, t_{k+1})$.

Definition 13. (Wheeden and Zygmund, 1977). Let $\mathbf{S}$ be a topological space. A family $\mathcal{F} \doteq \{f_\alpha : \alpha \in \mathbf{A}\}$ of complex functionals with a common domain $\mathbf{S}$ is said to be *equicontinuous at a point* $x \in \mathbf{S}$ if for every $\epsilon > 0$ there exists an open neighborhood $N(x)$ such that $\forall y \in N(x)$, $\forall \alpha \in \mathbf{A}$, $|f_\alpha(x) - f_\alpha(y)| < \epsilon$. The family is said to be *equicontinuous on* $\mathbf{S}$ if it is equicontinuous at each $x \in \mathbf{S}$. $\mathcal{F}$ is said to be *uniformly equicontinuous* on $\mathbf{S}$ if $\forall \epsilon > 0$, $\exists \delta = \delta(\epsilon) > 0$ such that $\forall x, y \in \mathbf{S}$, $\forall \alpha \in \mathbf{A}$, $y \in N_\delta(x) \Rightarrow |f_\alpha(x) - f_\alpha(y)| < \epsilon$, where N_δ denotes an open neighborhood of size δ.

Lemma 1. If $(\mathbf{S}, d)$ is a compact metric space, then any family $\mathcal{F} \doteq \{f_\alpha : \alpha \in \mathbf{A}\}$ that is equicontinuous on $\mathbf{S}$ is uniformly equicontinuous on $\mathbf{S}$.

PROOF. See Stefanovic and Safonov (2005). ■

3. MAIN RESULT

The main results on stability and finiteness of switches are developed in the sequel.

Lemma 2. Consider the feedback adaptive control system Σ in Figure 1, together with the hysteresis switching algorithm A1. Suppose there are finitely many switches. If the adaptive control problem is feasible (Def. 1), and the associated cost functional $V(K, z, t)$ is continuous in time and satisfies the following properties:

- Cost-detectable (Def. 11)
- Monotone increasing in time

then stability of the switched system $\sum$ is unfalsified and, moreover, system response $z(t)$ with the final controller satisfies the performance inequality

$$V(K_N, z, \tau) \leq V_{true}(K_{RSP}) + \epsilon \ \forall \tau.$$

PROOF. It suffices to consider the final controller K_N. Denote the last switching time instant t_N. Then, by the definition of $V_{true}(K_N)$ (Def. 9), and feasibility of the control problem (Def. 1), it follows that for all $t \geq t_N$,

$$V(K_N, z_{data}, t) < \varepsilon + \min_K V(K, z_{data}, t) < \varepsilon + V_{true}(K_{RSP}) \ < \infty. \quad (1)$$

Further, by monotonicity in t of $V(K, z, t)$, it follows that (1) holds for all $t \in \mathbf{T}$. Due to the cost-detectability, stability of Σ with K_N is not falsified by z_{data}, that is, $\limsup_{\tau\to\infty} \frac{||z_{data}||_\tau}{||\tilde{r}_{K_N}||_\tau} < \infty$. ■

Lemma 3. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous and coercive function on $\mathbb{R}^n$. Then for any scalar $\alpha \in \mathbb{R}$, the level set $L(\alpha) \doteq \{x \in \mathbb{R}^n \mid f(x) \leq \alpha\}$ is compact.

PROOF. Since $L(\alpha) \subset \mathbb{R}^n$, we show that $L(\alpha)$ is closed and bounded: Let $\{x_m\} \subseteq L(\alpha)$ be a convergent sequence, and $\bar{x} \doteq \lim_{m\to\infty} x_m$. Since f is continuous, $f(\bar{x}) = \lim_{m\to\infty} f(x_m)$. Also, $f(x_m) \leq \alpha$, $\forall m \in \mathbb{N}$. Then, $f(\bar{x}) = \lim_{m\to\infty} f(x_m) \leq \lim_{m\to\infty} \alpha = \alpha$, so $\bar{x} \in L(\alpha)$. Hence, $L(\alpha)$ is closed. To show that is $L(\alpha)$ is bounded, proceed by contradiction. Assume that $L(\alpha)$ is not bounded; then there exists a sequence $\{y_m\} \subseteq L(\alpha)$ such that $\lim_{m\to\infty} ||[y_m|| = \infty$. Since f is coercive, $\lim_{m\to\infty} f(y_m) = \infty$; in particular, $\exists N \in \mathbb{N}$ such that $\forall k \geq N \ f(y_k) > \alpha$, for any fixed $\alpha \in \mathbb{R}$. Then, $\{y_m\} \not\subset L(\alpha)$, which contradicts the above assumption. Thus, $L(\alpha)$ is closed and bounded in $\mathbb{R}^n$, therefore compact. ■

Lemma 4. Consider the feedback adaptive control system in Figure 1, together with the switching algorithm A1. If the adaptive control problem is feasible (Def. 1), and the associated cost functional $V(K, z, t)$ is cost-detectable and monotone increasing in time and, in addition,

- For all $\tau \in \mathbf{T}, z \in \mathbf{Z}$, $V(K, z, t)$ is *coercive* on $\mathbf{K} \subseteq \mathbb{R}^n$ (i.e. $\lim_{||K||\to\infty} V(K, z, \tau) = \infty$)
- The family $\mathcal{W} \doteq \{W_{z,t}(K) : z \in \mathbf{Z}, t \in \mathbf{T}\}$ of restricted cost functionals with a common domain $\mathbf{L}$ is equicontinuous on $\mathbf{L}$,

then the number of switches is uniformly bounded above for all $z \in \mathbf{Z}$ by some $\bar{N} \in \mathbb{N}$.

PROOF. Due to Lemma 3, the level set $\mathbf{L}$ is compact. Then, the family $\mathcal{W} \doteq \{W_{z,t}(K) : z \in \mathbf{Z}, t \in \mathbf{T}\}$ is uniformly equicontinuous on $\mathbf{L}$ (see Lemma 1), *i.e.* for a hysteresis step ϵ, $\exists \delta > 0$ such that for all $z \in \mathbf{Z}$, $t \in \mathbf{T}$, $K_1, K_2 \in \mathbf{L}$, $||K_1 - K_2|| < 2\delta \Rightarrow |W_{z,t}(K_1) - W_{z,t}(K_2)| < \epsilon$ (*i.e.* $\delta = \delta(\epsilon)$ is common to all $K \in \mathbf{L}$ and all $z \in \mathbf{Z}, t \in \mathbf{T}$). Since $\mathbf{L}$ is compact, there exists a finite open cover $\mathcal{C}_N = \{B_\delta(K_i)\}_{i=1}^N$, with $K_i \in \mathbb{R}^n, i = 1, \ldots, N$ such that $\mathbf{L} \subset \bigcup_{i=1}^N B_\delta(K_i)$, where N depends on the chosen hysteresis step ϵ (this is a direct consequence of the definition of a compact set). Let $\hat{K}_{t_j}$ be the controller switched into the loop at the time t_j, and the corresponding minimum cost achieved is $\tilde{V} \doteq \min_{K\in\mathbf{K}} V(K, z, t_j)$. Consider that at the time $t_{j+1} > t_j$ a switch occurs at the same cost level $\tilde{V}$, i.e. $\tilde{V} = \min_{K\in\mathbf{K}} V(K, z, t_{j+1})$ where $V(\hat{K}_{t_j}, z, t_{j+1}) > \min_{K\in\mathbf{K}} V(K, z, t_{j+1}) + \epsilon$. Therefore, $\hat{K}_{t_j}$ is falsified, and so are all the controllers $K \in B_{2\delta}(\hat{K}_{t_j})$. Let I_j be the index set of the as-yet-unfalsified δ-balls of controllers at the time t_j. Since $\hat{K}_{t_j} \in B_\delta(K_i)$, for some $i \in \bar{I} \subset I_j$ also falsified are all the controllers $K \in B_\delta(K_i) \supset \hat{K}_{t_j}$, so that $I_{j+1} = I_j \setminus \{i\}$, *i.e.* I_j is updated according to the following algorithm (j is the index of the switching time t_j):

Unfalsified index set algorithm:

(1) Initialize: Let $j = 0$, $I_0 = \{1, \ldots, N\}$.
(2) $j \leftarrow j + 1$. If $I_{j-1} = \emptyset$: Set $I_j = \{1, \ldots, N\}$ // Optimal cost increases
Else
$I_j = I_{j-1} \setminus \{i\}$,where $i \in I_{j-1}$ is such that $B_\delta(K_i) \supset \hat{K}_{t_{j-1}}$.
(3) go to (2);

Thus, the number of possible switches to a single cost level is upper-bounded by N, the number of δ-balls in the cover of $\mathbf{L}$. The next switch, if any, must occur to a cost level higher than $\tilde{V}$, due to the monotonicity of V. Then, according to algorithm A1, $|V(\tilde{K}_{t_{j+N+1}}, z, t_{j+N+1}) - \tilde{V}| > \epsilon$, with $d(\tilde{K}_{t_{j+N+1}}, \tilde{K}_{t_k}) < 2\delta$, $j \leq k \leq j + N$ and $V(\tilde{K}_{t_k}, z, t_k) = \tilde{V}$. Combining the two bounds, the overall number of switches is thus upper-bounded by:

$$\bar{N} \doteq N \frac{V_{true}(K_{RSP}) - \min_{K\in\mathbf{K}} V(K,z,0)}{\epsilon}$$

■

The finite controller set case is obtained as a special case of the Lemma 4, with N being the number of candidate controllers instead of the number of δ-balls in the cover of $\mathbf{L}$. The main result follows.

Theorem 1. Consider the feedback adaptive control system Σ in Figure 1, together with the hysteresis switching algorithm A1. Suppose that the adaptive control problem is feasible (Def. 1), and the associated cost functional $V(K, z, t)$ is continuous in time and satisfies the conditions of Lemma 4. Then, the system is stable. Moreover, for each z, the system converges after finitely many switches to controller K_N that satisfies the performance inequality

$$V(K_N, z, \tau) \leq V_{true}(K_{RSP}) + \epsilon \quad \text{for all } \tau. \quad (2)$$

PROOF. Invoking Lemma 4 proves that there are finitely many switches. Then, Lemma 2 shows that the adaptive controller stabilizes and that (2) holds. ■

Comment 2. Note that, due to the coerciveness of V, $\min_{K\in\mathbf{K}} V(K, z, 0)$ is bounded below (by a nonnegative number, if the range of V is a subset of $\mathbb{R}_+$), for all $z \in \mathbf{Z}$.

Comment 3. The parameterization of $\mathbf{K}$ can be more general than $\Theta_K \subseteq \mathbb{R}^n$; in fact, it can belong to an arbitrary infinite dimensional space; however $\mathbf{K}$ has to be compact in that case, in order to ensure uniform equicontinuity property.

The switching ceases after finitely many steps for all $z \in \mathbf{Z}$. The values of the cost minima are monotone increasing and bounded above by $V_{true}(K_{RSP})$. With sufficient richness of the system input (external reference signal, disturbance or noise signals) the cost will approach $V_{true}(K_{RSP}) \pm \epsilon$.

4. COST FUNCTION EXAMPLE

Consider (a not necessarily zero-input zero-output) system $\Sigma : \mathfrak{L}_{2e} \to \mathfrak{L}_{2e}$ in Figure 1. Choose a cost functional:

$$V(K, z, t) = \max_{\tau\leq t} \frac{||y||_\tau^2 + ||u||_\tau^2}{||\tilde{r}_K||_\tau^2 + \alpha} + \beta + \gamma||\theta_K||^2 \quad (3)$$

where α, β, γ are arbitrary positive numbers. α is used in order to prevent $V = \frac{const}{0}$ when $\tilde{r} = 0$ or $\tilde{r} = y = u = 0$, β ensures $V > 0$ even when $||\theta_K|| \equiv 0$, and γ scales the importance of $||\theta_K||^2$. Such a cost function satisfies the required properties of Theorem 1. The reader is referred to (Stefanovic and Safonov, 2005) for verification of stability and finiteness of switches of the proposed cost function.

5. SIMULATION EXAMPLE

Assume that a true, unknown plant transfer function is given by $G^*(s) = \frac{s-1}{s(s+1)}$. It is desired that the output follows the output of the reference model $G_{ref} = \frac{1}{s+1}$. Presumed given is the set of candidate controllers: $C_1(s) = -\frac{s+1}{s+2.6}$, $C_2(s) = \frac{-s+1}{0.3s+1}$ and $C_3(s) = -\frac{s+1}{-s+2.6}$. A non-switched analysis (true plant in feedback with each of the controllers separately) shows that C_1 is stabilizing, while C_2 and C_3 are not. Next, a simulation was performed of a switched system, where A1 was used to select optimal controller, and a cost function was chosen to be a combination of the

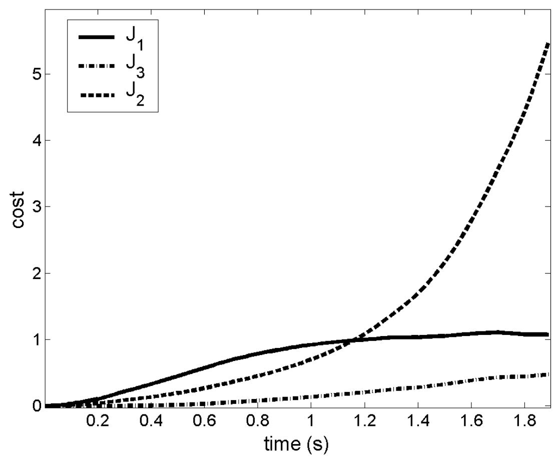

Fig. 3. Current values of the cost (4) for each controller.

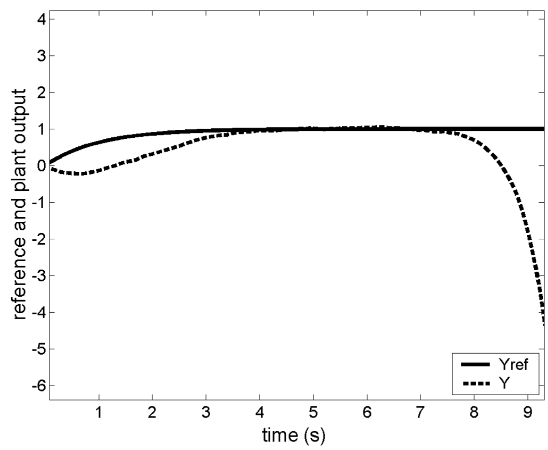

Fig. 4. Switching using cost function (4). Reference and plant outputs.

instantaneous error and a weighted accumulated error (Narendra and Balakrishnan, 1997)

$$J_j(t) = \tilde{e}_j^2(t) + \int_0^t e^{-\lambda(t-\tau)} \tilde{e}_j^2(\tau) d\tau, j = 1, 2, 3 \quad (4)$$

where $\tilde{e}_j$ is the fictitious error of the j^{th} controller, defined as $\tilde{e}_j = \tilde{y}_j - y$, and $\tilde{y}_j = G_{ref}\tilde{r}_j$ and $\tilde{r}_j = y + K_j^{-1}u$.

The stabilizing controller C_1 was initially placed in the loop, and the switching was allowed after five seconds. Figures 3 and 4 show the simulation results. The algorithm using cost function (4) discards the stabilizing controller and latches onto a destabilizing one, despite evidence found in data. For details, see (Stefanovic and Safonov, 2005). Next, a simulation was performed using a 'good' cost function (according to Theorem 1):

$$V(K, z, t) = \max_{\tau\in[0,t]} \frac{||u||_\tau^2 + ||\tilde{e}_K||_\tau^2}{||\tilde{r}_K||_\tau^2 + \alpha} \quad (5)$$

The corresponding simulations results are shown in Figure 5. The initial controller was chosen to be C_3 (a destabilizing one).

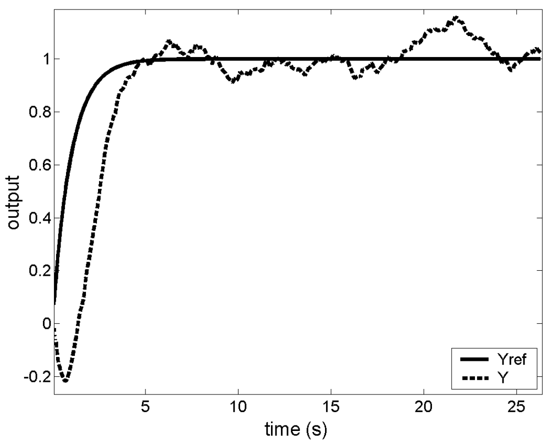

Fig. 5. Switching using cost function (5). Reference and plant outputs.

6. CONCLUSION

The goal of stabilizing an uncertain plant by means of switching through an infinite candidate controller set is solved in the paper, provided that feasibility (defined as the existence of at least one stabilizing solution in the candidate controller set) holds. Sufficient conditions are derived on the data-driven cost function to ensure stability and performance. An upper bound on the number of switches for a general continuum controller set case is calculated. The result is a solution to the problem of model mismatch instability that has long been the focus of the research efforts in adaptive control.

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REFERENCES

Åström, K. J. and B. Wittenmark (1995). *Adaptive Control.* Addison-Wesley. Reading, MA.

Bertsekas, D. P. (1999). *Nonlinear Programming.* Athena Scientific. Nashua, NH.

Hespanha, J. P., D. Liberzon and A. S. Morse (2003). Hysteresis-based switching algorithms for supervisory control of uncertain systems. *Automatica* **39**, 263–272.

Morse, A. S., D. Q. Mayne and G. C. Goodwin (1992). Applications of hysteresis switching in parameter adaptive control. *IEEE Transactions on Automatic Control* **37**, 1343–1354.

Narendra, K. S. and J. Balakrishnan (1997). Adaptive control using multiple models. *IEEE Transactions on Automatic Control* **42**, 171–187.

Rudin, W. (1976). *Principles of Mathematical Analysis.* McGraw-Hill, Inc.

Safonov, M. G. (1980). *Stability and Robustness of Multivariable Feedback Systems.* MIT Press. Cambridge, MA.

Safonov, M. G. and T. Tsao (1997). The unfalsified control concept and learning. *IEEE Transactions on Automatic Control* **42**, 843–847.

Stefanovic, M. and M. G. Safonov (2005). Safe adaptive switching control: Stability and convergence. To be submitted to *IEEE Trans. on Automatic Control.* `http://www-scf.usc.edu/~mstefano/ieeetac05.pdf.`

Stefanovic, M., R. Wang and M. G. Safonov (2004). Stability and convergence in adaptive systems. In: *Proceedings of the American Control Conference.* Vol. 2. Boston, MA. pp. 1923 – 1928.

Wheeden, R. L. and A. Zygmund (1977). *Measure and Integral: An Introduction to Real Analysis.* Marcel Dekker. New York, NY.