A SET OF OBSERVERS FOR A CLASS OF NONLINEAR SYSTEMS

M. Farza [*] **M. M'Saad** [*] **M. Sekher** [*]

[*] *GREYC, UMR 6072 CNRS, Université de Caen, ENSICAEN, 6 Bd Maréchal Juin, 14050 Caen Cedex, FRANCE.*

mfarza@greyc.ensicaen.fr

Abstract: A state observer is proposed for a class of multi-output nonlinear systems. The gain of this observer involves a design function that has to satisfy some mild conditions which are given. Different expressions of such a function are proposed. Of particular interest, it is shown that high gain observers and sliding mode like observers can be derived by considering particular expressions of the design function. A simulation example is given in order to compare the performance of a high gain observer and a sliding mode like observer obtained through two different choices of the design function.

Keywords: Nonlinear system, Nonlinear observer, High gain observer, Sliding mode observer.

1. INTRODUCTION

In spite of the intensive research efforts made during the last two decades, the observer synthesis for MIMO nonlinear systems still be an open problem (Gauthier and Bornard, 1981; Nijmeijer, 1981; Krener and Isidori, 2003; Krener and Respondek, 1985; Xia and Gao, 1989; Gauthier *et al.*, 1992; G. Ciccarella and Germani, 1993; Bornard and Hammouri, 1991; Gauthier and Kupka, 1994; Hou and Pugh, 1999; Hammouri and Farza, 2003)). This paper deals with the design of observers for a special class of MIMO nonlinear systems satisfying some regularity assumptions. The main characteristics of the proposed observer lie in its simplicity and its capability to give rise to different observers among which high gain observers and sliding mode observers. Indeed, the gain of the proposed observer involves a design function that has to satisfy some mild conditions which are given. Different expressions of the design function are proposed and it is shown that high gain observers (Bornard and Hammouri, 1991; Gauthier *et al.*, 1992; Hammouri and Farza, 2003; Farza *et al.*, 2004) and sliding mode like observers (Utkin, 1992; Drakunov, 1992; Drakunov and Utkin, 1995; Filipescu *et al.*, 2003) can be derived by considering particular expressions of the design function.

This paper is organized as follows. In the next section, one introduces the class of nonlinear systems which will be the basis of the observer design. Section 3 is devoted to the observer design: a state transformation is introduced and the equations of the proposed observer are firstly given in the new coordinates before being generated in the original ones. In section 4, different expressions of the observer gain are proposed and one shows that some of these expressions give rise to state observers which structures are similar to those of high gain observers and sliding mode observers. In section 5, a simulation example is given in order to compare the performance of a high gain observer and a sliding mode like observer obtained through two different choices of the observer gain expression.

2. PROBLEM FORMULATION

Consider MIMO systems of the form :

$$\begin{cases} \dot{x} = f(u,x) + \bar{\varepsilon}(t) \\ y = \bar{C}x = x^1 \end{cases} \quad (1)$$

with $x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}$; $f(u,x) = \begin{pmatrix} f^1(u,x^1,x^2) \\ f^2(u,x^1,x^2,x^3) \\ \vdots \\ f^{q-1}(u,x) \\ f^q(u,x) \end{pmatrix}$;

$$\bar{\varepsilon}(t) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \varepsilon(t) \end{pmatrix} \ \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{n_q} \end{pmatrix}$$

$$\bar{C} = [I_{n_1}, 0_{n_1 \times n_2}, 0_{n_1 \times n_3}, \ldots, 0_{n_1 \times n_q}] \quad (2)$$

where the state $x \in \mathbb{R}^n$ with $x^k \in \mathbb{R}^{n_k}$, $k = 1, \ldots, q$ and $p = n_1 \geq n_2 \geq \ldots \geq n_q$, $\sum_{k=1}^{q} n_k = n$; the input $u(t) \in \mathcal{U}$ the set of bounded absolutely continuous functions with bounded derivatives from $\mathbb{R}^+$ into U a compact subset of $\mathbb{R}^s$; $f(u,x) \in \mathbb{R}^n$ with $f^k(u,x) \in \mathbb{R}^{n_k}$; $\bar{\varepsilon}(t) \in \mathbb{R}^n$ where $\varepsilon(t) \in \mathbb{R}^{n_q}$ with each ε_i, $i = 1, \ldots, n_q$ being an unknown bounded real-valued function which may depend on x, u, uncertain parameters, etc. Our objective consists in designing state observers for system (1). Such a design necessitates some assumptions which will be stated in due course. At this step, one assumes the following:

(A1) Each function $f^k(u,x)$, $k = 1, \ldots, q-1$ satisfies the following rank condition:

$$Rank\left(\frac{\partial f^k}{\partial x^{k+1}}(u,x)\right) = n_{k+1} \ \ \forall x \in \mathbb{R}^n; \forall u \in U$$

Moreover $\exists \alpha, \beta > 0$ such that for all $k \in \{1, \ldots, q-1\}$, $\forall x \in \mathbb{R}^n$, $\forall u \in U$,

$$\alpha^2 I_{n_{k+1}} \leq \left(\frac{\partial f^k}{\partial x^{k+1}}(u,x)\right)^T \frac{\partial f^k}{\partial x^{k+1}}(u,x) \leq \beta^2 I_{n_{k+1}}$$

where $I_{n_{k+1}}$ is the $(n_{k+1}) \times (n_{k+1})$ identity matrix.

(A2) For $1 \leq k \leq q-1$, the function $x^{k+1} \mapsto f^k(u, x^1, \ldots, x^k, x^{k+1})$ is one to one from $\mathbb{R}^{n_{k+1}}$ into $\mathbb{R}^{n_k}$.

(A3) The function $\varepsilon(t)$ is uniformly bounded by $\delta > 0$.

When $\varepsilon = 0$, system (1) is identical to that considered in (Hammouri and Farza, 2003) and it characterizes a subclass of locally U-uniformly observable systems. In (Farza *et al.*, 2004), the authors considered a subclass of systems which involve the same uncertain term, $\varepsilon(t)$, as (1). In the sequel, one shall use a strategy of observer design similar to that adopted in (Hammouri and Farza, 2003; Farza *et al.*, 2004) to construct a state observer for a class of MIMO nonlinear systems including systems considered in the just mentioned works.

3. OBSERVERS DESIGN

One shall firstly introduce an appropriate state transformation allowing to easily design the proposed observers. Then, the equations of these observers will be derived in the new coordinates before being given in the original ones.

3.1 State transformation

Consider the following change of coordinates:

$$\Phi : \mathbb{R}^n \longrightarrow \mathbb{R}^{n_1 q},\ x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \mapsto z = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^q \end{pmatrix} =$$

$$\Phi(u,x) = \begin{pmatrix} x^1 \\ f^1(u,x^1,x^2) \\ \frac{\partial f^1}{\partial x^2}(u,x^1,x^2) f^2(u,x^1,x^2,x^3) \\ \vdots \\ \left(\prod_{k=1}^{q-2} \frac{\partial f^k}{\partial x^{k+1}}(u,x)\right) f^{q-1}(u,x) \end{pmatrix}$$

where $z^k \in \mathbb{R}^{n_1}$, $k = 1, \ldots, q$. According to assumptions (A1) and (A2), the map Φ is one to one. Let Φ^c denote its converse. Before deriving the dynamics of z, let us introduce the following notations :

- $\Lambda(u,x)$ is the diagonal matrix:

$$\Lambda(u,x) = diag\left(I_{n_1}, \frac{\partial f^1}{\partial x^2}(u,x), \frac{\partial f^1}{\partial x^2}(u,x)\frac{\partial f^2}{\partial x^3}(u,x), \ldots, \prod_{k=1}^{q-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x)\right) \quad (3)$$

Notice that according to Assumption (A1), $\Lambda(u,x)$ is left invertible. One shall denote by $\Lambda^+(u,x)$ its left inverse. Now, one can easily check that:

$$\Lambda(u,x)f(u,x) = Az + G(u,x) \text{ or equivalently}$$
$$f(u,x) = \Lambda^+(u,x)Az + \Lambda^+(u,x)G(u,x) \quad (4)$$

where the $n_1 q \times n_1 q$ square matrix A and the vector field $G(u,x) \in \mathbb{R}^{n_1 q}$ are respectively given by:

$$A = \begin{bmatrix} 0 & I_{n_1} & 0 & & 0 \\ \vdots & \ddots & I_{n_1} & \ddots & 0 \\ 0 & & \ddots & \ddots & I_{n_1} \\ 0 & \dots & & 0 & 0 \end{bmatrix} \quad (5)$$

$$G(u,x) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \left(\prod_{k=1}^{q-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x)\right) f^q(u,x) \end{pmatrix}$$

Proceeding as in (Hammouri and Farza, 2003; Farza *et al.*, 2004), one can show that the transformation Φ puts system (1) under the following form:

$$\begin{cases} \dot{z} = Az + \varphi(u,z) + \dfrac{\partial \Phi}{\partial x}(u,x)\bar{\varepsilon}(t) \\ y = Cz = z^1 \end{cases} \quad (6)$$

where $\varphi(u,z)$ has a triangular structure i.e.

$$\varphi(u,z) = \begin{pmatrix} \varphi^1(u,z^1) \\ \varphi^2(u,z^1,z^2) \\ \vdots \\ \varphi^k(u,z^1,\dots,z^k) \\ \vdots \\ \varphi^q(u,z) \end{pmatrix}$$

with $\varphi^k(u,z) \in \mathbb{R}^{n_1}$, $k = 1,\dots,q$ and

$$C = [I_{n_1}, 0_{n_1}, \dots, 0_{n_1}] \quad (7)$$

is $n_1 \times n_1 q$ matrix with 0_{n_1} denoting the $n_1 \times n_1$ null matrix.

3.2 Observers synthesis

As in the works related to the high gain observers synthesis (Bornard and Hammouri, 1991; Gauthier *et al.*, 1992; Farza *et al.*, 2004), one assumes that:
(A4) The functions $\Phi(u, \Phi^c(z))$ and $\varphi(u,z)$ are globally Lipschitz with respect to z uniformly in u.
Before giving our candidate observers, one introduces the following notations.
1) let Δ_θ be the block diagonal matrix defined by

$$\Delta_\theta = diag\left[I_{n_1}, \frac{1}{\theta}I_{n_1}, \dots, \frac{1}{\theta^{q-1}}I_{n_1}\right] \quad (8)$$

where $\theta > 0$ is a real number

2) Let S be the unique solution of the algebraic Lyapunov equation :

$$S + A^T S + SA - C^T C = 0 \quad (9)$$

where A and C are respectively given by equations (5) and (7). One can show that S is symmetric positive definite.

3) $\forall \xi = \begin{pmatrix} \xi^1 \\ \vdots \\ \xi^q \end{pmatrix} \in \mathbb{R}^{n_1 q}$ with $\xi^k \in \mathbb{R}^{n_1}$, $k = 1,\dots,q$, set $\bar{\xi} = \Delta_\theta \xi$ and let $K(\xi) \triangleq K(\xi^1) = \begin{pmatrix} K^1(\xi^1) \\ \vdots \\ K^q(\xi^1) \end{pmatrix} \in \mathbb{R}^{n_1 q}$ with $K^k(\xi^1) \in \mathbb{R}^{n_1}$, $k = 1,\dots,q$ be a vector of smooth functions satisfying:

$$\forall \xi \in \mathbb{R}^{n_1 q} : \bar{\xi}^T K(\xi^1) \geq \frac{1}{2}\xi^T C^T C \xi \quad (10)$$

$$\exists \sigma > 0; \forall \xi \in \mathbb{R}^{n_1 q} : \|K(\xi^1)\| \leq \sigma \|\xi^1\| \quad (11)$$

where the matrices Δ_θ and C are respectively given by (8) and (7).

A candidate observer for system (6) is:

$$\begin{aligned} \dot{\hat{z}}(t) = {} & A\hat{z} + \varphi(u,\hat{z}) - \theta\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1) \\ & - \frac{\partial \Phi}{\partial x}(u, \Phi^c(\hat{z})) \\ & \left(\Lambda^+(u,\Phi^c(\hat{z})) - \left(\frac{\partial \Phi}{\partial x}(u,\Phi^c(\hat{z}))\right)^+\right) \\ & \theta\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1) \end{aligned} \quad (12)$$

where $\hat{z} = \begin{bmatrix} \hat{z}^1 \\ \hat{z}^2 \\ \vdots \\ \hat{z}^q \end{bmatrix} \in \mathbb{R}^{n_1 q}$ with $\hat{z}^k \in \mathbb{R}^{n_1}$, $k = 1,\dots,q$; S, C and Δ_θ are respectively given by equations (9), (7) and (8); $\tilde{z} = \hat{z} - z$ where z is the unknown trajectory of system (6); $K(\tilde{z}) \in R^{n_1 q}$ satisfies conditions (10) and (11); u is the input of system (6) and $\theta > 0$ is a real number.
Indeed one states the following :
Theorem 1. *Assume that system (6) satisfies Assumptions (A1) to (A4). Then,*

$$\exists \theta_0 > 0;\ \forall \theta > \theta_0;\ \exists \lambda > 0;\ \exists \mu_\theta > 0;\ \exists M_\theta > 0;$$

$$\forall u \in U;\ \forall \hat{z}(0) \in R^{n_1 q};\ \textit{one has}\ :$$

$$\|\hat{z}(t) - z(t)\| \leq \lambda\theta^{q-1}e^{-\mu_\theta t}\|\hat{z}(0) - z(0)\| + M_\theta \delta$$

where $z(t)$ is the unknown trajectory of (6) associated to the input u, $\hat{z}(t)$ is any trajectory of system (12) associated to (u,y) and δ is the upper bound of $\|\varepsilon\|$. Moreover, one has $\lim_{\theta \to \infty} \mu_\theta = +\infty$ and $\lim_{\theta \to \infty} M_\theta = 0$.

Proof of Theorem 1: One has:

$$\begin{aligned} \dot{\tilde{z}} = {} & A\tilde{z} - \theta\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1) + \varphi(u,\hat{z}) - \varphi(u,z) \\ & - \frac{\partial \Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t) - \Gamma(u,\hat{z})\theta\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1) \end{aligned}$$

where $\Gamma(u,\hat{z}) =$
$\frac{\partial\Phi}{\partial x}(u,\Phi^c(\hat{z}))\left(\Lambda^+(u,\Phi^c(\hat{z})) - \left(\frac{\partial\Phi}{\partial x}(u,\Phi^c(\hat{z}))\right)^+\right)$.
Notice that $\Gamma(u,\hat{z})$ is a lower triangular matrix with zeros on its main diagonal. Moreover, using Assumption (A1) and (A4), one can easily deduce that $\Gamma(u,\hat{z})$ is bounded.
Now, one can easily check the following identities: $\theta\Delta_\theta^{-1}A\Delta_\theta = A$ and $C\Delta_\theta = C$. Set $\bar{z} = \Delta_\theta\tilde{z}$. One obtains:

$$\begin{aligned}\dot{\bar{z}} =& \theta A\bar{z} - \theta S^{-1}K(\tilde{z}^1) + \Delta_\theta\left(\varphi(u,\hat{z}) - \varphi(u,z)\right)\\ &- \Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t) - \theta\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1)\end{aligned}$$

Consider the quadratic function $V(\bar{z}) = \bar{z}^T S\bar{z}$, then

$$\begin{aligned}\dot{V} &= 2\bar{z}^T S\dot{\bar{z}}\\ &= 2\theta\bar{z}^T SA\bar{z} - 2\theta\bar{z}^T K(\tilde{z}^1)\\ &\quad +2\bar{z}^T S\Delta_\theta(\varphi(u,\hat{z}) - \varphi(u,z))\\ &\quad -2\bar{z}^T S\Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{z}^T S\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1)\\ &= -\theta V + \theta\bar{z}^T C^T C\bar{z} - 2\theta\bar{z}^T K(\tilde{z}^1)\\ &\quad +2\bar{z}^T S\Delta_\theta(\varphi(u,\hat{z}) - \varphi(u,z))\\ &\quad -2\bar{z}^T S\Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{z}^T S\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1)\end{aligned}$$

by equation (9).
Using (10), one obtains:

$$\begin{aligned}\dot{V} &= -\theta V + 2\theta\left(\frac{1}{2}\bar{z}^T C^T C\bar{z} - \bar{z}^T K(\tilde{z}^1)\right)\\ &\quad +2\bar{z}^T S\Delta_\theta(\varphi(u,\hat{z}) - \varphi(u,z))\\ &\quad -2\bar{z}^T S\Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{z}^T S\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1)\\ &\leq -\theta V + 2\bar{z}^T S\Delta_\theta(\varphi(u,\hat{z}) - \varphi(u,z))\\ &\quad -2\bar{z}^T S\Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{z}^T S\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}S^{-1}K(\tilde{z}^1) \qquad (13)\end{aligned}$$

Now, assume that $\theta \geq 1$, then, because of the triangular structure and the Lipschitz assumption on φ, one can show that :

$$\|\Delta_\theta\left(\varphi(u,\hat{z}) - \varphi(u,z)\right)\| \leq \zeta\|\bar{z}\| \qquad (14)$$

where ζ is a contant that does not depend on θ (see (Gauthier *et al.*, 1992)). Similarly, according to assumption (A1) and to the Lipschitz assumption on Φ (Assumption (A4)), $\Gamma(u,\hat{z})$ is bounded. Moreover, and since $\Gamma(u,\hat{z})$ is lower triangular with zeros on the main diagonal, one has:

$$\|\theta\Delta_\theta\Gamma(u,\hat{z})\Delta_\theta^{-1}\| \leq \gamma \ \text{ for } \theta \geq 1 \qquad (15)$$

where $\gamma > 0$ is a constant that does not depend on θ. Finally, according to the structure of ε and since $\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))$ is triangular, one can show that:

$$\|\Delta_\theta\frac{\partial\Phi}{\partial x}(u,\Phi^c(z))\bar{\varepsilon}(t)\| \leq \frac{\beta^{q-1}}{\theta^{q-1}}\delta \qquad (16)$$

where $\delta = \sup_{t\geq 0}\|\varepsilon(t)\|$ given in Assumption (A3) and β is given in (A1). Using inequalities (14), (15), (11) and (16) inequality (13) becomes:

$$\begin{aligned}\dot{V} &\leq -\theta V + 2\lambda_{max}(S)\|\bar{z}\|\left(\zeta\|\bar{z}\| + \gamma\sigma\|\tilde{z}^1\|\right)\\ &\quad + 2\lambda_{max}(S)\frac{\beta^{q-1}}{\theta^{q-1}}\delta\|\bar{z}\|\\ &\leq -(\theta - c_1)V + \frac{c_2}{\theta^{q-1}}\delta\sqrt{V}\end{aligned}$$

where $c_1 = 2\sigma^2(S)(\zeta + \gamma\sigma)$ and $c_2 = 2\beta^{q-1}\sigma(S)\sqrt{\lambda_{max}(S)}$ with $\lambda_{min}(S)$ and $\lambda_{max}(S)$ being respectively the smallest and the largest eigenvalues of S and $\sigma(S) = \sqrt{\frac{\lambda_{max}(S)}{\lambda_{min}(S)}}$.
Now taking $\theta_0 = max\{1, c_1\}$ and using the fact that for $\theta \geq 1$, $\|\bar{z}(t)\| \leq \|\tilde{z}(t)\| \leq \theta^{q-1}\|\bar{z}(t)\|$, one can show that for $\theta > \theta_0$, one has :

$$\begin{aligned}\|\tilde{z}(t)\| &\leq \theta^{q-1}\sigma(S)\exp\left[-\left(\frac{\theta - c_1}{2}\right)t\right]\|\tilde{z}(0)\|\\ &\quad + 2\beta^{q-1}\frac{\sigma^2(S)}{(\theta - c_1)}\delta\end{aligned}$$

It is easy to see that λ, μ_θ and M_θ needed by the theorem are: $\lambda = \sigma(S)$, $\mu_\theta = \frac{\theta - c_1}{2}$ and $M_\theta = 2\beta^{q-1}\frac{\sigma^2(S)}{(\theta - c_1)}$.This ends the proof.

3.3 Observers equations in the original coordinates

Proceeding as in (Farza *et al.*, 2004), one can show that observer (12) can be written in the original coordinates x as follows:

$$\dot{\hat{x}} = f(u,\hat{x}) - \theta\Lambda^+(u,\hat{x})\Delta_\theta^{-1}S^{-1}K(\tilde{x}^1) \qquad (17)$$

where S, C, Δ_θ and $\Lambda^+(u,x)$ are given above, $\hat{x} = \begin{bmatrix}\hat{x}^1\\ \hat{x}^2\\ \vdots\\ \hat{x}^q\end{bmatrix} \in \mathbb{R}^n$ with $\hat{x}^k \in \mathbb{R}^{n_k}$, $k = 1,\ldots,q$; u the input of system (1) and $\tilde{x} = \hat{x} - x$ where x is the unknown trajectory of system (1).

4. SOME PARTICULAR OBSERVERS

Some expressions of $K(\tilde{x}^1)$ that satisfying conditions (10) and (11) shall be given in this section and the so-obtained observers are discussed. These expressions will be given in the new coordinates z in order to easily check conditions (10) and (11) as well as in the original coordinates x in order to easily recognize the structure of the resulting observers.

4.1 High gain observers

Consider the following expression of $K(\tilde{\xi})$:

$$\begin{aligned} K_{HG}(\tilde{z}) &= C^T C\tilde{z} = C^T \tilde{z}^1 \\ &= C^T \tilde{x}^1 = C^T \bar{C}\tilde{x} \end{aligned} \tag{18}$$

One can easily check that expression (18) satisfies conditions (10) and (11). Replacing $K(\tilde{z})$ by expression (18) in (17) gives rise to a high gain observer (see e.g. (Gauthier ***et al.***, 1992; Hammouri and Farza, 2003; Farza ***et al.***, 2004)).

4.2 Sliding mode like observers

At first glance, the following vector seems to be a potential candidate for the expression of $K(\tilde{z})$:

$$\begin{aligned} K(\tilde{z}) &= kC^T C sign(\tilde{z}) = kC^T sign(\tilde{z}^1) \\ &= kC^T sign(\tilde{x}^1) = kC^T \bar{C} sign(\tilde{x}) \end{aligned} \tag{19}$$

where $k > 0$ is a real number and 'sign' is the usual sign function with $sign(\tilde{z}^1) = \begin{pmatrix} sign(\tilde{z}^1_1) \\ \vdots \\ sign(\tilde{z}^1_{n_1}) \end{pmatrix}$, $\tilde{z}^1_i \in \mathbb{R}$, $i = 1, \ldots, n_1$. Indeed, condition (10) is trivially satisfied by (19). Similarly, for bounded input bounded output systems, condition (11) is also satisfied for relatively high values of k. However, expression (19) cannot be used due the discontinuity of sign function. Indeed, such discontinuity makes the stability problem not well posed since the Lyapunov method used throughout the proof is not valid. In order to overcome these difficulties, one shall use continuous functions which have similar properties that those of the sign function. This approach is widely used when implementing sliding mode observers. Indeed, consider the following function:

The Tanh function:

$$\begin{aligned} K_{Tanh}(\tilde{z}) &= kC^T C Tanh(\tilde{z}) = kC^T Tanh(\tilde{z}^1) \\ &= kC^T Tanh(\tilde{x}^1) = kC^T \bar{C} Tanh(\tilde{x}) \end{aligned} \tag{20}$$

where $Tanh$ denotes the hyperbolic tangent function and $k > 0$ is a real number.

Similarly to the hyperbolic tangent function, one can easily check that the inverse tangent function, the inverse sinus function, etc., also constitute valid expressions for $K(\tilde{z})$. Besides, one can consider new valid expressions for $K(\tilde{z})$ by adding $K_{Tanh}(\tilde{z})$ to $K_{HG}(\tilde{z})$. This gives rise to a sliding mode observer similar to that used in (Filipescu ***et al.***, 2003).

5. EXAMPLE

Consider the following dynamical system:

$$\begin{cases} \dot{x}_1 = (a - x_1)x_3 + u_1 \\ \dot{x}_2 = x_1 x_3 - x_2 + u_2 \\ \dot{x}_3 = -k_1 x_3^3 + x_4(1 + k_2 x_4^2) \\ \dot{x}_4 = \varepsilon(t) \\ y = (x_1\ x_2)^T \end{cases} \tag{21}$$

where $x = (x_1\ x_2\ x_3\ x_4)^T \in \mathbb{R}^4$, $a > 0, k > 0$ are constant real parameters, $u = (u_1\ u_2)^T \in \mathbb{R}^2$ and $y = (x_1\ x_2)^T$ respectively denote the measured inputs and outputs, and ε stands for any unknown bounded function. It is easy to see that system (21) is under form (1) with $q = 3$ and $x^1 = (x_1\ x_2)^T$, $x^2 = x_3$, $x^3 = x_4$.

System (21) is similar to that considered in (Farza ***et al.***, 2004) and the authors constructed a set Ω which is positively invariant under the dynamics of (21). Using this fact, one can easily check Assumptions (A1) to (A4) and observers under form (17) can then be used to estimate the trajectories of system (21). One shall give in what follows two sets of results provided by a high gain observer (obtained by using expression (18) in (17) and and by a sliding mode like observer (obtained by using expression (20) in (17)).

In order to simulate practical situations and before being used by the observers, each of the measurements of x_1 and x_2 has been corrupted by a uniformly distributed random signal produced by SIMULINK with zero mean value and a standard deviation to 0.33. The true time evolutions of x_3 and x_4 (issued from model simulation) with their respective estimates provided by the high gain observer are compared in figure 1. Figure 2 shows the same results obtained with the sliding mode like observer. For simulation purposes, the time evolution of x_4 has been specified as a trapezoidal signal varying between 100 and 40, $u_1(t) = 100\cos(\pi t)$, $u_2(t) = 100\sin(\pi t)$, $a = 5$, $k_1 = 0.02$ and $k_2 = 10^{-4}$. The employed value of θ was 50 in both observer and the value of the parameter k was equal to 1 in the sliding mode observer. The initial conditions for the model and the observer are: $x_1(0) = \hat{x}_1(0) = 0$; $x_2(0) = \hat{x}_2(0) = 0$; $x_3(0) = 5$; $\hat{x}_3(0) = 1$; $\hat{x}_4(0) = 0$. Figures 1 and 2 show the good agreement between the estimated and simulated variables. Recall that the time evolution of the state x_4 considered for simulation purposes is ignored by the observers.

The obtained results clearly show similar behaviours of both observers. In fact, many other numerical simulations have been carried out and they do confirm such

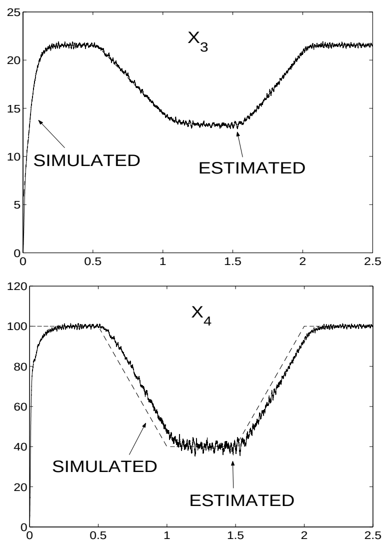

Fig. 1. Estimation results with high gain observer

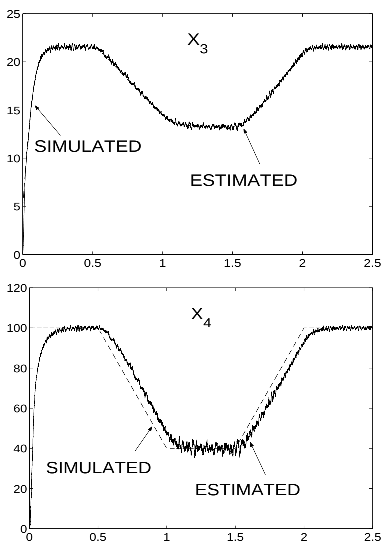

Fig. 2. Estimation results with sliding mode observer

a fact.

Conclusion: A set of observers has been designed for a class of nonlinear systems. The appealing features of the proposed observers are the easiness of their implementation and their ability to give rise to different observers having different structures. It has been shown that high gain observers and sliding mode like observers can be derived from the set of proposed observers.

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