NONLINEAR PATH FOLLOWING CONTROL OF FULLY ACTUATED MARINE VEHICLES WITH PARAMETER UNCERTAINTY

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Abstract: The paper addresses the problem of steering a fully actuated marine vehicle along a desired path. The methodology adopted for path following control deals explicitly with vehicle dynamics and plant parameter uncertainty. Furthermore, it avoids tight constraints on initial conditions that are present in other approaches reported in the literature. The main contribution of the paper is the development of an efficient two-stage procedure: first, a path following control law is designed based on the kinematics only; this is followed by the design of a robustly stabilizing controller for the vehicle dynamics. A coupling term in the final dynamic control law that emerges naturally out of the design procedure ensures stability of the resulting closed-loop system. Simulation results illustrate the performance of the control system proposed.

Keywords: Marine Vehicles, Nonlinear Control, Parameter Uncertainty, Lyapunov Stability, Backstepping, Differential Inclusions, Linear Matrix Inequalities.

1. INTRODUCTION

Fast paced progresses in marine robotics are steadily affording scientists advanced tools for ocean exploration and exploitation. However, much work remains to be done before marine robots can roam the oceans freely, acquiring scientific data at the temporal and spatial scales that are naturally imposed by the phenomena under study. To meet these goals, robots must be equipped with systems to steer them accurately and reliably in the harsh marine environment. For this reason, there has been over the last few years considerable interest in the development of advanced methods for motion control of marine vehicles, including surface and underwater robots. Namely, point stabilization, trajectory tracking, and path following control methodologies.

Point stabilization refers to the problem of steering a vehicle to a final target point, with a desired orientation. Trajectory tracking requires a vehicle to track a time-parameterized reference curve. Finally, path following control aims at forcing a vehicle to converge to and follow a desired spatial path, without any temporal specifications. The latter objective occurs for example when it is required that an autonomous surface vessel cover a certain area by performing a “lawn mowing” maneuver along desired tracks with great accuracy, at speeds determined by a scientific end-user. The underlying assumption in path following control is that the vehicle’s forward speed tracks a desired speed profile, while the controller acts on the vehicle’s orientation to drive it to the path. Typically, smoother convergence to a path is achieved when path following strategies are used instead of trajectory tracking control laws, and the control signals are less likely to be pushed to saturation.

The paper addresses the problem of steering a fully actuated marine vehicle along a desired path. For previous work in this field, the reader is referred to (Aicardi, *et al.*, 2001; Encarnação, *et al.*, 2000; Encarnação and Pascoal, 2000; Fossen, 2002; Lapierre, *et al.*, 2003) and the references therein. The new methodology adopted for path following control deals explicitly with vehicle dynamics and plant parameter uncertainty. Furthermore, it avoids tight constraints on initial conditions that are present in a number of path-following control strategies described in the literature. These issues have also been addressed in (Lapierre, *et al.*, 2003) for the case of an underactuated autonomous underwater vehicle by resorting to pure backstepping techniques. The main contribution of the present paper is the development of an efficient two-stage procedure for the design of dynamic path following controllers. The new methodology departs considerably from mainstream work reported in the literature and resembles the inner-outer loop designs that are pervasive in the aircraft industry. First, a path following control law is designed based on the kinematics only. The strategy adopted at this stage borrows from the non-singular path following control strategy for wheeled robots introduced in (Soetanto, *et al.*, 2003). The first step is then followed by the design of a robustly stabilizing controller for the vehicle dynamics. A coupling term in the final dynamic control law that emerges naturally out of the design procedure ensures stability of the resulting closed loop system. Controller design relies on Lyapunov theory, backstepping techniques, and the theory of differential inclusions. Simulation results illustrate the performance of the control system proposed.

The paper is organized as follows. Section 2 contains the theoretical results that underpin the new methodology proposed for path following control system design. Section 3 describes the application of the methodology developed to the control of a fully actuated marine vehicle with parameter uncertainty. It also describes illustrative results obtained in simulation. Finally, Section 4 contains the summary and discusses related problems that warrant further research.

2. MAIN THEORETICAL RESULT

We start by stating and proving the following theoretical results that will be later applied to the development of a path following controller for a generic tug boat.

Theorem. Assume the plant G to be controlled is given by

$$G = \begin{cases} \dot{x} = f(x) + g(x)y \\ \dot{y} = A(p)y + B(p)u \end{cases} \tag{1}$$

where the parameter p belongs to a compact set M and $B(p)$ is invertible for all $p \in \mathrm{M}$. Further assume that the following conditions hold true.

C.1. $\exists\, V(x) > 0$, $V(x) \to \infty$, $\|x\| \to \infty$ and $\phi(x, y)$, $\phi(0,0) = 0$ such that

$$\frac{\partial V(x)}{\partial x}(f(x) + g(x)\phi(x)) \le -b_1 \|x\|_2^2$$

with $\|\phi\| \le b_2 \|x\|_2 + b_3$, for some $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$.

C.2. $\forall p \in P$, $\exists K(p)$ and $P(p) > 0$ such that

$$F^T(p)P(p) + P(p)F(p) < 0 ,$$

where

$$F(p) := A(p) + B(p)K(p) . \tag{2}$$

C.3. $\exists K_z > 0$ such that $\forall p \in P$

$$\begin{bmatrix} K_z b_1 - b_3^2 I & \sqrt{\left(1+b_2^2\right)} P(p)F(p) \\ \sqrt{\left(1+b_2^2\right)} F^T(p)P(p) & I \end{bmatrix} > 0 .$$

If all aforementioned hold, then the control

$$u = Ky + B^{-1}\left(-\frac{1}{2}P^{-1}K_z z - \frac{1}{2}P^{-1}\left(\frac{\partial V(x)}{\partial x} g\right)^T + \dot{\phi} \right)$$

will render the origin of system G globally asymptotically stable.

Proof. Let $u = Ky + v$ and $z = y + \phi$.
Then

$$G = \begin{cases} \dot{x} = f(x) + g(x)\phi - g(x)z \\ \dot{z} = (A(p) + B(p)K(p))y + B(p)v - \dot{\phi} \end{cases}$$

Define $V_1 = V + z^T P z$ and compute its derivative

$$\begin{aligned} \dot{V}_1 = {} & \frac{\partial V(x)}{\partial x}(f(x) + g(x)\phi(x)) + \frac{\partial V(x)}{\partial x} g(x)z \\ & + z^T P(F(p)y + B(p)v - \dot{\phi}) \\ & + (F(p)y + B(p)v - \dot{\phi})^T Pz \end{aligned}$$

Let

$$v = B(p)^{-1}\left(-\frac{1}{2}P^{-1}K_z z - \frac{1}{2}P^{-1}\left(\frac{\partial V(x)}{\partial x} g\right)^T + \dot{\phi} \right).$$

Then,

$$\begin{aligned} & \dot{V}_1 = \frac{\partial V(x)}{\partial x}(f(x) + g(x)\phi(x)) - z^T K_z z + z^T PF(p)y \\ & + y^T F^T(p)Pz \le -b_1 \|x\|_2^2 - z^T K_z z + z^T PF(p)(y - \phi + \phi) \\ & + (y - \phi + \phi)^T F^T(p)Pz = -b_1 \|x\|_2^2 - z^T (K_z - PF(p) \\ & - F^T(p)P)z + z^T PF(p)\phi + \phi^T F^T(p)Pz \le -b_1 \|x\|_2^2 \\ & - \lambda_{\min}(K_z)\|z\|_2^2 + 2\|PF(p)\|\left(b_2\|x\|_2 + b_3\right)\|z\|_2 + b_3^2 - b_3^2 \\ & = -\begin{bmatrix} \|z\|_2 & \|x\|_2 & b_3 \end{bmatrix} \\ & * \begin{bmatrix} \lambda_{\min}(K_z) & \|PF(p)\| b_2 & \|PF(p)\| \\ \|PF(p)\| b_2 & b_1 & 0 \\ \|PF(p)\| & 0 & 1 \end{bmatrix} \begin{bmatrix} \|z\|_2 \\ \|x\|_2 \\ b_3 \end{bmatrix} + b_3^2 \end{aligned}$$

Therefore, $\forall p \in \mathrm{M}$

$$\dot{V}_1 < 0 \Leftarrow \lambda_{\min}(K_z)b_1 - \|PF(p)\|^2 b_2^2 - \|PF(p)\|^2 > b_3^2$$
$$\Leftarrow K_z b_1 - \left(1+b_2^2\right) PF(p)F^T(p)P > b_3^2 I$$

The proof follows by using the third assumption and taking Schur complements.

3. APPLICATION: PATH FOLLOWING FOR A SUPPLY VESSEL

In this section the results developed above are applied to the development of a path following control law for a supply vessel.

3.1 Vehicle Modeling: Dynamics.

The model of the vessel adopted in this paper can be found in (Fossen and Strand, 1999). Specifically its low frequency dynamics are written as

$$M\dot{v} + Dv = \tau\,, \quad \tau = B_u u\,,$$

where the state vector $v = [v_x, v_y, r]^T$ includes the surge, sway and yaw modes of the ship, the control vector τ determines forces and moments generated by the propulsion system, u is the control input vector, and B_u is a constant matrix that represents the actuator configuration (see Fossen and Strand, 1999). The mass matrix *M* and damping matrix *D* assume the following form

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix},$$

$$D = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & mv_{x0} - Y_r \\ 0 & -N_v & mx_G v_{x0} - N_r \end{bmatrix}.$$

Detailed explanations of each term in the expressions above can be found in (Fossen and Strand, 1999; Fossen, 1994). However, of special interest is the parameter v_{x0} which determines the surge speed of the vessel. Notice that other parameters in the matrices *M* and *D* (along with the explicit dependence of D_{23} and D_{33}) can depend on v_{x0}. In this paper we assume that $v_{x0} > 0$.

3.2 Path following: Problem Definition and System Equations.

The presentation in this section is inspired by the work in (Soetanto, *et al.*, 2003) to which the reader is referred for more details. With respect to Fig.1, let {I} denote an inertial frame and {F} a Serret-Frenet frame. Let *P* be an arbitrary point on the path to be followed and *Q* be the center of mass of the tug boat. Then *Q* can be resolved in {I} as $q_I = [x, y, 0]^T$ or in {F} as $q_F = [s_1, y_1, 0]^T$. Let ψ_c denote the angle between {I} and {F},

$$R = \begin{bmatrix} \cos\psi_c & -\sin\psi_c & 0 \\ \sin\psi_c & \cos\psi_c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the rotation matrix from {I} to {F}, and $r_c = \dot{\psi}_c$. Then $r_c = \kappa\dot{s}$, where s denotes the curvilinear abscissa of *P* along the path and κ denotes the path curvature. Let the vector *p* denote the position of point *P* in {I}. The rate of change of p with respect to {I} resolved in {F} can be written as

$$\left(\frac{d}{dt}p\right)_F = [\dot{s}, 0, 0]^T.$$

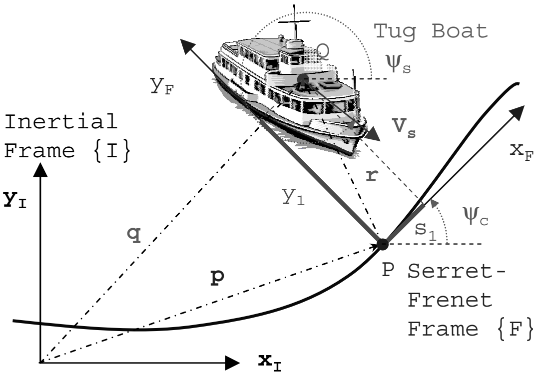

Fig. 1. Path following problem geometry.

On the other hand, the rate of change of q is

$$\left(\frac{d}{dt}q\right)_I = \left(\frac{d}{dt}p\right)_I + R^{-1}\left(\frac{d}{dt}\begin{bmatrix} s_1 \\ y_1 \\ 0 \end{bmatrix}\right)_F + R^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ r_c \end{bmatrix} \times \begin{bmatrix} s_1 \\ y_1 \\ 0 \end{bmatrix}\right).$$

and therefore

$$R\left(\frac{d}{dt}q\right)_I = \left(\frac{d}{dt}p\right)_F + \left(\frac{d}{dt}\begin{bmatrix} s_1 \\ y_1 \\ 0 \end{bmatrix}\right)_F + \left(\begin{bmatrix} 0 \\ 0 \\ r_c \end{bmatrix} \times \begin{bmatrix} s_1 \\ y_1 \\ 0 \end{bmatrix}\right).$$

Let

$$\left(\frac{d}{dt}q\right)_I = [\dot{x}, \dot{y}, 0]^T,$$

and

$$\left(\frac{d}{dt}[s_1, y_1, 0]^T\right)_F = [\dot{s}_1, \dot{y}_1, 0]^T.$$

Straightforward computations show that (Soetanto, *et al.*, 2003)

$$R\begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix}=\begin{bmatrix}\dot{s}_1+\dot{s}(1-\kappa y_1)\\ \dot{y}_1+\kappa\dot{s}s_1\\ 0\end{bmatrix}.$$

Denote by ψ_s the angle between {I} and {B} and by ψ_F - the angle between {B} and {F}. Let $r=\dot{\psi}_s$. Then, using simple algebra it can be shown that

$$\begin{cases}\dot{s}_1=-\dot{s}(1-\kappa y_1)+v_x\cos\psi_F-v_y\sin\psi_F\\ \dot{y}_1=-\kappa\dot{s}s_1+v_x\sin\psi_F+v_y\cos\psi_F\\ \dot{\psi}_F=r-\kappa\dot{s}\end{cases}$$

Given any arbitrary velocity profile for the vessel, the path following problem can now be reduced to driving $[s_1,y_1,\psi_F]^T$ to zero by rotating the vessel around its *z*-axis, i.e. by deriving an appropriate control law for *r*.

Following the presentation in Section 2 let the parameter $p:=v_{x0}$. The complete system of equations for the path following problem is given by

$$G_s=\begin{cases}\dot{x}=g_1(x)\dot{s}+g_2(x)v\\ \dot{v}=A(v_{x0})v+B(v_{x0})u\end{cases}\qquad(3)$$

where

$$x=\begin{bmatrix}s_1\\ y_1\\ \psi_F\end{bmatrix},\ g_1(x)=\begin{bmatrix}-(1-\kappa y_1)\\ -\kappa s_1\\ -\kappa\end{bmatrix},$$

$$g_2(x)=\begin{bmatrix}\cos\psi_F & -\sin\psi_F & 0\\ \sin\psi_F & \cos\psi_F & 0\\ 0 & 0 & 1\end{bmatrix},$$

$$A(v_{x0})=M^{-1}D(v_{x0}),\ B(v_{x0})=M^{-1}B_u.$$

Equations (3) are in the form of the equation (1) for the plant *G* used in the theorem of Section 2. Next we apply the results of this theorem to obtain a path following controller for the supply vessel.

3.3 Path Following: Controller Design.

First, we shall obtain a stabilizing controller ϕ for the kinematics. Consider the candidate Lyapunov function

$$V=\frac{1}{2}s_1^2+\frac{1}{2}y_1^2+\frac{1}{2}(\psi_F-\delta)^2,$$

where δ is defined below. Computing the time-derivative of *V* yields

$$\begin{aligned}\dot{V}&=s_1\dot{s}_1+y_1\dot{y}_1+(\psi_F-\delta)(\dot{\psi}_F-\dot{\delta})\\ &=s_1(-\dot{s}(1-\kappa y_1)+v_x\cos\psi_F-v_y\sin\psi_F)\\ &+y_1(-\kappa\dot{s}s_1+v_x\sin\psi_F+v_y\cos\psi_F)\\ &+(\psi_F-\delta)(r-\kappa\dot{s}-\dot{\delta})\\ &+y_1v_x\cos\delta+y_1v_y\sin\delta-y_1v_x\cos\delta-y_1v_y\sin\delta\end{aligned}$$

Let

$$\begin{cases}\dot{s}=K_1s_1+v_x\cos\psi_F-v_y\sin\psi_F\\ r=-K_2(\psi_F-\delta)+\dot{\delta}+\kappa\dot{s}-\dfrac{y_1}{\psi_F-\delta}\\ \quad *(v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta)\end{cases}.$$

Notice how the rate of progression of point P along the path is here viewed as an extra control input. Using the difference of sines and cosines formulae it is easy to show that

$$\frac{y_1}{\psi_F-\delta}(v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta)$$

is nonsingular. Therefore,

$$\begin{aligned}\dot{V}&=-K_1s_1^2-K_2(\psi_F-\delta)^2-y_1v_x\sin\delta-y_1v_y\cos\delta\\ &=-K_1s_1^2-K_2(\psi_F-\delta)^2-y_1V_s(\frac{v_x}{V_s}\sin\delta+\frac{v_y}{V_s}\cos\delta)\\ &=-K_1s_1^2-K_2(\psi_F-\delta)^2-y_1V_s\sin(\delta+\beta),\end{aligned}$$

where $V_s=\sqrt{v_x^2+v_y^2}$, $\beta=\sin^{-1}\frac{v_y}{V_s}$.

Let $\delta=\sin^{-1}(\theta_a\frac{y_1}{|y_1|+\varepsilon})-\beta,$ where $\theta_a>0$, $\varepsilon>0$.

Then

$$\dot{V}=-K_1s_1^2-K_2(\psi_F-\delta)^2-\frac{y_1^2}{|y_1|+\varepsilon}V_s<0.$$

Using the definitions introduced in the theorem in Section 2, $b_1=\left\|\left[K_1,K_2,\frac{u_{0\min}}{\varepsilon}\right]^T\right\|$ and

$$\phi:=\begin{bmatrix}K_1s_1+v_x\cos\psi_F-v_y\sin\psi_F\\ v_x\\ v_y\\ -K_2(\psi_F-\delta)+\dot{\delta}+\kappa(K_1s_1+v_x\cos\psi_F\\ -v_y\sin\psi_F)-\dfrac{y_1}{\psi_F-\delta}(v_x\sin\psi_F-v_x\sin\delta\\ +v_y\cos\psi_F-v_y\cos\delta)\end{bmatrix}.$$

To obtain b_2, and b_3, express ϕ as follows

$$\phi:=\Xi+\begin{bmatrix}v_x\cos\psi_F-v_y\sin\psi_F\\ v_x\\ v_y\\ \kappa(v_x\cos\psi_F-v_y\sin\psi_F)\end{bmatrix}+\begin{bmatrix}0\\0\\0\\ \dot{\delta}\end{bmatrix},$$

then

$$\|\phi\|\le\|\Xi\|+\left\|\begin{bmatrix}v_x\cos\psi_F-v_y\sin\psi_F\\ v_x\\ v_y\\ \kappa(v_x\cos\psi_F-v_y\sin\psi_F)\end{bmatrix}\right\|+\left\|\begin{bmatrix}0\\0\\0\\ \dot{\delta}\end{bmatrix}\right\|$$

(the expression for Ξ is given by

$$\Xi := \begin{bmatrix} K_1 s_1 \\ 0 \\ 0 \\ -K_2\left(\psi_F-\delta\right)+\kappa K_1 s_1-\dfrac{y_1}{\psi_F-\delta}(v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta) \end{bmatrix}).$$

Using simple algebra and the last two terms in the expression above it can be shown that $b_3=\sqrt{2u_{0\max}^2(2+\kappa^2)}+\dot{\delta}_{\max}$, where it was assumed that $\forall t$, $|\dot{\delta}|\le\dot{\delta}_{\max}$. Now, to obtain b_2 consider

$$\|\phi\| = \left\|\begin{bmatrix} K_1 s_1 \\ 0 \\ 0 \\ -K_2\left(\psi_F-\delta\right)+\kappa K_1 s_1-\dfrac{y_1}{\psi_F-\delta}(v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta) \end{bmatrix}\right\|$$

$$\le \left\|\begin{bmatrix} K_1 s_1 \\ 0 \\ 0 \\ -K_2\left(\psi_F-\delta\right)\end{bmatrix}\right\| + \left\|\begin{bmatrix} 0 \\ 0 \\ 0 \\ \kappa K_1 s_1\end{bmatrix}\right\| + \left\|\begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{y_1}{\psi_F-\delta}(v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta)\end{bmatrix}\right\|$$

$$= \left\|\begin{bmatrix} K_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -K_2\end{bmatrix}\begin{bmatrix} s_1 \\ y_1 \\ \psi_F-\delta\end{bmatrix}\right\| + \left\|\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \kappa K_1 & 0 & 0\end{bmatrix}\begin{bmatrix} s_1 \\ y_1 \\ \psi_F-\delta\end{bmatrix}\right\|$$

$$+ \left\|\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \dfrac{v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta}{\psi_F-\delta} & 0\end{bmatrix}\begin{bmatrix} s_1 \\ y_1 \\ \psi_F-\delta\end{bmatrix}\right\|$$

$$\le \left(\sqrt{K_1^2+K_2^2}+\kappa K_1+\left|\frac{v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta}{\psi_F-\delta}\right|\right)\left\|\begin{bmatrix} s_1 \\ y_1 \\ \psi_F-\delta\end{bmatrix}\right\|.$$

Finally, using basic trigonometry we obtain that

$$\left\|\frac{v_x\sin\psi_F-v_x\sin\delta+v_y\cos\psi_F-v_y\cos\delta}{\psi_F-\delta}\right\| = \left\|2V_T\cos\left(\beta+\frac{\psi_F+\delta}{2}\right)\frac{\sin\dfrac{\psi_F-\delta}{2}}{\psi_F-\delta}\right\| \le u_{0\max}.$$

Therefore, $b_2=\sqrt{K_1^2+K_2^2}+\kappa K_1+u_{0\max}$.

Next, a stabilizing controller $K(u_0)$ is determined using the theory of differential inclusions, see (Boyd, *et al.*, 1994; El Ghaoui and Niculescu, 2000 and the references therein). Suppose the surge speed of the vessel $v_{x0}\in[v_{x0\min},v_{x0\max}]$ .Then $A(v_{x0})\in Co(A(v_{x0\min}),A(v_{x0\max}))$. Suppose $\exists Y>0$ W_1 and W_2 such that

$$A(v_{x0\min})Y+BW_1+YA^T(v_{x0\min})+W_1^TB<0$$

and

$$A(v_{x0\max})Y+BW_2+YA^T(v_{x0\max})+W_2^TB<0$$

Let $W(v_{x0})=\dfrac{v_x-v_{x0\min}}{v_{x0\max}-v_{x0\min}}W_2+\dfrac{v_{x0\max}-v_x}{v_{x0\max}-v_{x0\min}}W_1$, $P=Y^{-1}$ and $K(v_{x0})=W(v_{x0})P$. Then $K(v_{x0})$ and $P>0$ satisfy the second assumption of the theorem of Section 2 and can now be used to obtain the positive definite matrix K_z.

Following the third assumption of this theorem and using notation (2), suppose $\exists K_z>0$ such that $\forall v_{x0}\in[v_{x0\min},v_{x0\max}]$

$$\begin{bmatrix} K_z b_1 - b_3^2 I & \sqrt{(1+b_2^2)} PF(v_{x0}) \\ \sqrt{(1+b_2^2)} F^T(v_{x0}) P & I \end{bmatrix} > 0 .$$

Then the control

$$u = Ky + B^{-1}\left(-\frac{1}{2}P^{-1}K_z z - \frac{1}{2}P^{-1}\left(\frac{\partial V(x)}{\partial x}g\right)^T + \dot{\phi}\right)$$

renders the origin of system (3) globally asymptotically stable.

3.4 Simulation Results.

This section illustrates the performance of the nonlinear controller obtained above in simulation with a dynamic model of a surface vessel. The simulation experiment required that the vessel approach and track a circle. Figure 2 includes the position plot. It illustrates the algorithm's performance when capturing a circle from a large initial offset. Clearly, the controller developed performs the task well.

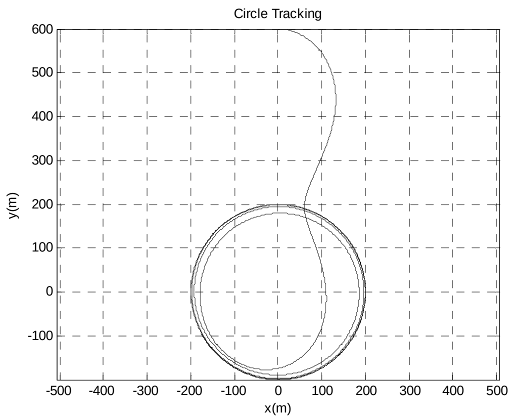

Figure 2. Circle tracking with large initial offset.

Figure 3 includes time histories of several key variables. In particular, it shows that the forward and lateral errors s_1 and y_1, respectively converge smoothly to zero in the vicinity of 150 *sec* (this is due to a large initial error). Furthermore, the function δ introduced to provide a desired shape to the steering command starts by commanding 90º-turn initially, then converges to zero when the vehicle captures the commanded path.

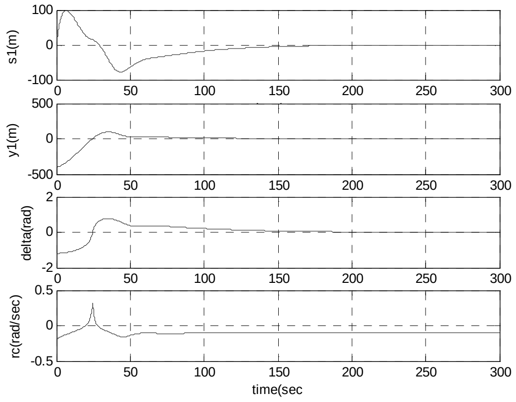

Figure 3. Time histories of s_1, y_1, δ, and r_c.

4. CONCLUSIONS

The paper introduced a new methodology for the design of path following controllers for fully actuated marine vehicles with parameter uncertainty. Its main contribution was the development of an efficient two-stage design procedure that resembles inner-outer loop designs that are commonly used in the aircraft industry. As such, it departed considerably from mainstream work reported in the literature. Simulation results illustrated the performance of the control system proposed. Future work will address the extension of the results to underactuated marine and underwater vehicles.

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