MULTIVARIABLE BOUNDARY CONTROL APPROACH BY INTERNAL MODEL, APPLIED TO IRRIGATION CANALS REGULATION

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Abstract: This paper deals with the regulation problem of a class of irrigation canals using the Saint-Venant partial differential equations (pde). The Internal Model Boundary Control (IMBC) approach is used and the multireaches case is considered (several reaches in cascade). Perturbation theory of exponential semigroup used for control synthesis is extended here to nonhomogeneous hyperbolic systems, as the multireaches regulation model is described by hyperbolic pde's. Experimental results (on the Valence experimental canal) are encouraging for an extension to the real case. Additionally, a multi-model approach was introduced to allow wider water level variations.

Keywords: Shallow water equation, irrigation canal, perturbation theory, multivariable boundary control.

1. INTRODUCTION

Open surface hydraulic systems were studied by different approaches (Georges, 2002; Malaterre, 2003) in modelling or control. The usual model is the Saint Venant equations also named shallow water equations with regard to the control. Two approaches are currently used: indirect approach in finite dimensional (the pde's are approximated) and the direct one in infinite dimension (methods and tools directly relate to pde's) (Russel, 2002; Touré, 2002).
This paper is located in the second approach, using directly partial differential equations for control synthesis (Pohjolainen, 1982; Sakawa, 1975; Touré, 1997).
The internal model boundary control is proposed for control synthesis for one or many reaches regulation. Internal model boundary control was introduced in (Touré, 1997) for dissipative parabolic and exponentially stable systems. It is extended here to the hyperbolic case. The spatial dependency of variables is taken into account. Conservation properties of semigroup stability give the control synthesis, using some perturbations theory results (Kato, 1966).
In the first section, the non linear model for a rectangular canal is given in order to define a linear regulation model around an equilibrium state. Regulation problem is then defined for a canal with reaches in cascade, where hydraulics constructions are gates or overflows.
The second part deals with the control synthesis. The boundary control model is well posed to set up the essential properties to be conserved, due to the structural perturbation of the closed loop. Associated to the particular structure of

the adapted internal model, the extended system is represented by a closed operator, generator of an exponentially stable C_0-semigroup. The internal control law is taken as a multivariable integral controller or a proportional integral one. Then, synthesis parameters are obtained by a direct application of the operators and semigroup perturbations theory results (Kato, 1966; Pohjolainen, 1982; Touré, 1997).
In the last part, experimental results are given in multireaches case for water level tracks around equilibrium states. Moreover, a multi-model approach is introduced to allow a wider range of real water level variations.

2. THE CANAL REGULATION PROBLEM: A BOUNDARY CONTROL SYSTEM

The class of open surface canals is considered. Considered cases are: one reach followed by an overflow (Fig. 1, e.g. p^{th}: terminal reach) and reaches in cascade (Fig. 1 reaches $p-2$, $p-1$). Considering a reach, e.g. i^{th} one, the following notations are used:

- $Q(x,t)$ denotes the water-flow,
- $Z(x,t)$ is the water level in the canal,
- $L(i)$ is the i^{th} reach length , to be controlled between the upstream $x = x_{up} = 0_i$ and the downstream $x = x_{do} = L(i)$.
- $U_i(t)$ is the opening of the $(i+1)^{th}$ gate, $U_0(t)$ is the first one.

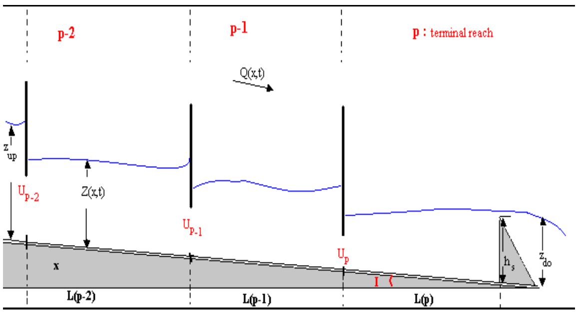

Fig. 1. Canal scheme: multireaches in cascade

Considering there are p reaches, then let

$$x \in \Omega = \cup_{i=1}^{p}]0_i, L(i)[\text{ and } \xi = \begin{pmatrix} Z \\ Q \end{pmatrix} \in X$$

$$X = \left[\prod_{i=1}^{p} L^2(0_i, L(i)) \times L^2(0_i, L(i)) \right]. \quad (1)$$

The control problem is the stabilization of the height and/or the water-flow, around an equilibrium behavior for each considered reach.

2.1 *Model of Saint-Venant*

The reaches are supposed to have a sufficient length $L(i)$ such that a uniform movement can be assumed in the lateral direction. The shallow water's non linear pde for a rectangular canal can be written as follows (Georges, 2002; Malaterre, 2003):

$$\partial_t Z = -\partial_x \frac{Q}{b} \quad (2)$$

$$\partial_t Q = -\partial_x \left(\frac{Q^2}{bz} + \frac{1}{2} g b Z^2\right) + gbZ(I - J) \quad (3)$$

$$Z(x,0) = Z_0(x), \qquad Q(x,0) = Q_0(x), \quad (4)$$

where b is the canal width, g the gravity constant, I the bottom slope, J the slope's rubbing expressed with the Manning-Strickler expression and R the hydraulic radius:

$$J = \frac{n^2 Q^2}{(bZ)^2 R^{4/3}}, \quad R = \frac{bZ}{b + 2Z}. \quad (5)$$

The boundary conditions are stated for each reach. For all, hydraulic constructions are supposed **submerged**. Coefficients and functions depend on considered reach and considered hydraulic construction. For example, let consider the i^{th} reach, $x_{up} = 0_i$ $x_{do} = L(i)$, with a upstream gate. Then upstream boundary equation is given by:

$$Q(0_i, t) = U_{i-1}(t) \Psi_1(Z(0_i, t)). \quad (6)$$

In the same way, at downstream, when there is a gate, downstream boundary equation is:

$$Q(L(i), t) = U_i(t) \Psi_3(Z(L(i), t)), \quad (7)$$

For a terminal reach (p^{th}), generally the hydraulics construction is an overflow, so the downstream boundary condition is the overflow equation:

$$Z(L(p), t) = \Psi_2(Q(L(p), t)). \quad (8)$$

Ψ_i, $1 \le i \le 3$, are given by:
$\Psi_1(Z) = K_{i-1}\sqrt{2g(z_{up} - Z)}$ with $Z < z_{up}$, $\Psi_3(Z) = K_i \sqrt{2g(Z - z_{do})}$ with $Z > z_{do}$, and $\Psi_2(Q) = (\frac{Q^2}{2gK_p^2})^{1/3} + h_s$, with $Z > h_s$,
where z_{up} and z_{do} are respectively the water levels before the upstream gate and after the downstream gate. K_i is the product of gate (or overflow) width and water-flow coefficient of the gate. The output to be controlled is the level at $x_{do} = L(i)$.

2.2 *A regulation model*

The regulation problem is the stabilization of the height and/or the water-flow around an equilibrium behavior $(z_e(x), q_e(x))$ for each considered

reach. So a linearized model with variable coefficients can be involved to describe the variations around this equilibrium behavior.
An equilibrium state of the system checks the following equations:

$$\partial_x z_e = gbz_e \frac{I + J_e + \frac{4}{3} J_e \frac{1}{1+2z_e/b}}{gbz_e - q_e^2/bz_e^2},$$
$$\partial_x q_e = 0 \tag{9}$$

Remark 1. The fluvial case is assumed, i.e.:

$$\mathbf{z_e} > \sqrt[3]{q_e^2/(gb^2)} = \mathbf{z_c}. \tag{10}$$

Note that q_e is constant but z_e is space dependent. Considering one equilibrium state for i^{th} reach, the linearized system around an equilibrium state is, $\xi = \left(Z\ Q\right)^t \in X_i = L^2(0_i, L(i)) \times L^2(0_i, L(i))$:

$$\begin{aligned}\partial_t \xi(t) &= (\partial_t z(t)\ \ \partial_t q(t))^t \\ &= A_1(x)\partial_x \xi(x) + A_2(x)\xi(x) \end{aligned} \tag{11}$$
$$\xi(x,0) = \xi_0(x) \tag{12}$$

Boundary limits are:

for an upstream gate:

$$\begin{aligned} &q(0_i,t) - u_{i-1,e}\partial_z \Psi_1(z_e(0_i))z(0_i,t) \\ &\quad = u_{i-1}(t)\Psi_1(z_e(0_i)) \end{aligned} \tag{13}$$

for a downstream overflow:

$$z(L(i),t) - \partial_q \Psi_2(q_e)q(L(i),t) = 0 \tag{14}$$

for a downstream gate:

$$\begin{aligned} &q(L(i),t) - u_{i,e}\partial_z \Psi_3(z_e(L(i)))z(L(i),t) \\ &\quad = u_i(t)\Psi_3(z_e(L(i))) \end{aligned} \tag{15}$$

where $u_{i-1,e}$, $u_{i,e}$ are respectively the i^{th} gate upstream and downstream equilibrium state opening. u_{i-1}, u_i are the opening variations at upstream and downstream. Moreover

$$A_{1,i}(x) = \begin{pmatrix} 0 & -a_{1,i}(x) \\ -a_{2,i}(x) & -a_{3,i}(x) \end{pmatrix}, \tag{16}$$
$$A_{2,i}(x) = \begin{pmatrix} 0 & 0 \\ a_{4,i}(x) & -a_{5,i}(x) \end{pmatrix}, \tag{17}$$

with $a_{1,i}(x) = 1/b$, $a_{5,i}(x) = \frac{2gbJ_{e,i}(x)z_{e,i}(x)}{q_{e,i}}$, $a_{2,i}(x) = gbz_{e,i}(x) - \frac{q_{e,i}^2}{bz_{e,i}^2(x)}$, $a_{3,i}(x) = \frac{2q_{e,i}}{bz_{e,i}(x)}$, $a_{4,i}(x) = gb(I + J_{e,i}(x) + \frac{\frac{4}{3}J_{e,i}(x)}{1+2z_{e,i}(x)/b})$.
Let F_i and G_i be the matrical writing of the boundary conditions (13)-(15).

The linearized system around an equilibrium state, different for each p reaches, is written as:

$$\partial_t \xi(t) = A_e(x)\partial_x \xi(x) + B_e(x)\xi(x) \tag{18}$$
$$\xi(x,0) = \xi_0(x) \tag{19}$$
$$F(\xi, u_e) = G(u(t)), \tag{20}$$

where $\xi = (z_1\ q_1\ z_2\ q_2 \ldots z_p\ q_p)^t \in X$ (eq. 1), F and G are the generalization of F_i, G_i, their coefficients are adapted according to the hydraulics construction used on each considered reach.

Operators $A_e(x)$ and $B_e(x)$ are the generalization of operators $A_{1,i}(x)$ (16) and $A_{2,i}(x)$ (17) respectively. Indeed:

$$A_e = \mathrm{diag}(A_{1,i})_{1\le i\le p} \tag{21}$$
$$\text{and} \quad B_e = \mathrm{diag}(A_{2,i})_{1\le i\le p}, \tag{22}$$

with $x \in [0, L(i)]$, $\forall\ 1 \le i \le p$ and $A_{1,i}(x)$ (16) is the representative matrix of the state $\xi_i(x) = (z_i(x)\ q_i(x))^t$ of i^{th} reach. The same is done for $B_e(x)$, $x \in [0, L]$.
Coefficients $a_{j,i}(x)$, $1 \le j \le 5$ depend on the equilibrium state $\xi_{e,i}(x) = (z_{e,i}(x)\ q_{e,i})^t$, $x \in [0, L(i)]$, $\forall\ 1 \le i \le p$.

The control problem is to find the variations of the control vector $u(t)$ such that the water levels at each downstream reach $x = x_{L(i)}$ (i.e. the output variables) track reference signals $r_i(t)$, different for each reach.
The reference signal $r_i(t)$ is chosen, for all cases, constant or no persistent (a step stable response of a non oscillatory system).

3. CONTROL SYNTHESIS

The system is first written as a classical boundary control system. Associated to the internal model structure, the closed loop system is described as an open loop perturbation. Control law parameters are tuned such that perturbations preserve the open loop system properties: closed operator and semigroup exponentially stable.

3.1 The abstract boundary control system, open loop system

The linearized boundary control model can be formulated as follows ($x_{up} = 0_i$ and $x_{do} = L(i)$ for i^{th} reach):

$$\partial_t \xi(t) = A_d(x)\xi(t), \quad x \in \Omega =]0_i, L(i)[, \quad t > 0 \tag{23}$$
$$F_b \xi(t) = B_b u(t), \text{ on } \Gamma = \partial\Omega, t > 0$$
$$\xi(x,0) = \xi_0(x) \tag{24}$$

where $A_d(x) = A_e(x)\partial_x + B_e(x)$.
Output variable y_i is measured at $x_i = L(i)$:

$$y_i(t) = C_i \xi(t), \quad t \ge 0, \ y(t) \in Y = \mathbb{R} \tag{25}$$

where C_i is a bounded operator (representation of the measurement with $\mathbf{1}_{x_i \pm \mu}(x) = \mathbf{1}_{[x_i-\mu, x_i+\mu]}(x)$ is the function such that equals 1 if $x \in [x_i - \mu, x_i + \mu]$, else 0, and $\mu > 0$):

$$C_i \xi = \begin{pmatrix} \frac{1}{2\mu}\int_{x_i-\mu}^{x_i+\mu} \mathbf{1}_{x_i \pm \mu} & 0 \end{pmatrix} \xi dx, \quad \mu > 0. \quad (26)$$

Output variable y is measured for all $x_j = L(j)$, $1 \leq j \leq p$, $\mathbf{y(t)} = \mathbf{C}\xi(\mathbf{t}) \in \mathbf{Y} = \mathbb{R}^{\mathbf{p}}$, $t \geq 0$ where C is a bounded operator (C_i get the type of (26)):

$$C\xi = (diag(C_i))_{1 \leq i \leq p} \xi dx, \quad \mu > 0.$$

For n gates, $u(t) \in \mathbb{R}^n$, $u \in C^{\alpha}([0, \infty], U)$.

The abstract boundary control system is obtained by change of variables and operators (Fattorini, 1968; Touré, 1997) and the system (23-24) becomes:

$$\begin{aligned} \dot{\varphi}(t) &= A\varphi(t) - D\,\dot{u}(t), \quad \varphi(t) \in D(A), \quad t > 0 \\ \varphi(0) &= \xi(0) - Du(0) \end{aligned} \quad (27)$$

where: $\varphi(t) = \xi(t) - Du(t) \quad \forall t \geq 0$.
D is a bounded operator from $U \to X$, such that:

$$Du \in D(A_d) \text{ and } F_b(Du(t)) = B_b u(t) \quad \forall u(t) \in U$$

and $Im(D) \subset Ker(A_d)$). So $D(A) = \{\varphi \in D(A_d) : F_b\varphi = 0\} = D(A_d) \cap Ker(F_b)$ and $A\varphi = A_d\varphi, \quad \forall \varphi \in D(A)$ on X.

From a previous work (Dos Santos, 2004), it is obvious that the open loop system is a well-posed and exponentially stable system, i.e.:

$$\varphi(t) = T_A(t)\varphi_0 - \int_0^t T_A(t-s) D\,\dot{u}(s) ds$$

with $\|T_A(t)\|_X < Me^{-wt}$, $M > 0$ $w > 0$ for all $t \geq 0$, under the fluvial behavior assumption (10).

The control objective can be now achieved by a simple control law employed in the IMBC control structure.

3.2 *The IMBC structure: closed loop*

The Internal Model Boundary Control (IMBC) structure is a particular case of the classical IMC structure since it contains an internal feedback on the model. It allows to get best performances from closed loop added.

Tracking model M_r and low pass filter model M_f are stable systems of finite dimension. For the regulation, a proportional integral feedback control is chosen for the control law:

$$\begin{aligned} u(t) &= \alpha_i \kappa_i \int \varepsilon(s) ds + \alpha_p \kappa_p \varepsilon(t) \\ &= \alpha_i \kappa_i \zeta(t) + \alpha_p \kappa_p\,\dot{\zeta}(t), \end{aligned} \quad (28)$$

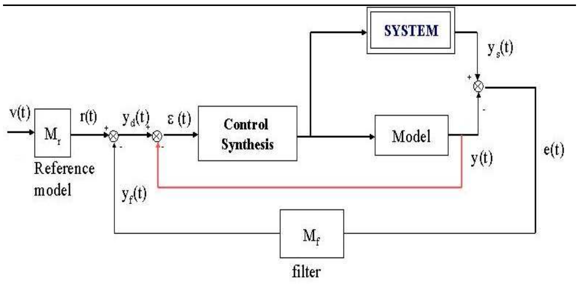

Fig. 2. IMBC structure

with $\dot{\zeta}(t) = \varepsilon(t)$.
Moreover, $\varepsilon(t) = y_d(t) - y(t)$ acts like an integrator compared to the "real" measured output, indeed:

$$\varepsilon(t) = r(t) - y(t) - y_f(t)$$

and

$$\begin{aligned} \lim_{t \to \infty} \varepsilon(t) &= \lim_{t \to \infty} [r(t) - y(t) - (y_s(t) - y(t))] \\ &= \lim_{t \to \infty} v(t) - y_s(t), \end{aligned} \quad (29)$$

as $r(t)$ and $e(t)$ are no persistent, i.e. their limits exist and are bounded.

Using previous relation, one get:

$$\zeta(t) = \int r(s) - y(s) ds$$

so $\dot{\zeta}(t) = r(t) - y(t) = r(t) - C\varphi(t) - CDu(t)$.

Then, the control law can be written as follows:

$$u(t) = \alpha_i \kappa_i \zeta(t) + \alpha_p \kappa_p\,\dot{\zeta}(t)$$

$(I - \alpha_p \kappa_p CD) u(t) = \alpha_i \kappa_i \zeta(t) + \alpha_p \kappa_p C\varphi(t) - \alpha_p \kappa_p r(t)$.
So, assuming that $(I - \alpha_p \kappa_p CD)$ is inversible, and that W is its left pseudo inverse, such that $W(I - \alpha_p \kappa_p CD) = I$, one get:

$$u(t) = \alpha_i W \kappa_i \zeta(t) + \alpha_p W \kappa_p C\varphi(t) - \alpha_p W \kappa_p r(t).$$

Let $x_a(t) = (\varphi(t) \;\; \zeta(t))^t$ be the new state space and $\tilde{\kappa_p} = \alpha_p \kappa_p$, $\tilde{\kappa_i} = \alpha_i \kappa_i$ then, the extended IMBC state space system is (from (27)):

$$\begin{cases} \dot{x_a}(t) = A(\alpha) x_a(t) + B(\alpha) r(t) + C(\alpha)\,\dot{r}(t) \\ x_a(0) = x_{a0} \end{cases} \quad (30)$$

where

$$A(\alpha) = \begin{pmatrix} (I - D\tilde{\kappa_p}C)A & 0 \\ (I + CDW\tilde{\kappa_p})C & 0 \end{pmatrix} + \begin{pmatrix} 0 & -D\tilde{\kappa_i}CDW\tilde{\kappa_i} \\ 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} -D\tilde{\kappa_i}C(I + DW\tilde{\kappa_p}C) & 0 \\ 0 & CDW\tilde{\kappa_i} \end{pmatrix}, \quad (31)$$

$$B(\alpha) = \begin{pmatrix} D\tilde{\kappa_i}(CDW\tilde{\kappa_p} + I) \\ -(CDW\tilde{\kappa_p} + I) \end{pmatrix}, \quad C(\alpha) = \begin{pmatrix} D\tilde{\kappa_p} \\ 0 \end{pmatrix}.$$

$A(\alpha)$ can be viewed as a bounded perturbation of A: $A(\alpha) = A_e + \alpha_i A_e^{(1)} + \alpha_i^2 A_e^{(2)}$, where $A_e^{(1)}$ and $A_e^{(2)}$ are bounded operators. Indeed, C, D and CD are bounded operators.

3.3 Stability results

Now the perturbation theory, from Kato's works (Kato, 1966), for control problem of infinite dimensional system (Pohjolainen, 1982; Pohjolainen, 1985; Touré, 1997) can be used.

Recall the proportional integral control

$$u(t) = \alpha_p \kappa_p (r(t) - y(t)) + \alpha_i \kappa_i \int_t (r(s) - y(s)) ds.$$

According to perturbation theory, the system stability is preserved if the following assumptions are checked:

$*$**1 There is a solution to the problem (27) if and only if rank(CD) = p** (Pohjolainen, 1982). The rank of CD ($rank(-CD) = p$), is checked by recurrence, with $p = n$ in the downstream overflow case and $p = n - 1$ in the other case so $p \leq n$ (Fig. 1).

$*$**2** $(\mathbf{I} - \alpha_\mathbf{p} \kappa_\mathbf{p} \mathbf{CD})$ **is inversible**.

$*$**3** According to theorem 3.6 of (Pohjolainen, 1982), a suitable selection for $\kappa_\mathbf{p}$ is: $\kappa_\mathbf{p} = [\mathbf{CD}]^{\ddagger}$ and **the series converges for sufficient small value of** $\alpha_\mathbf{p}$, i.e. ($\ddagger$ is the right pseudo inverse):

$$\mathbf{0} \leq \alpha_\mathbf{p} < (\sup_{\lambda \in \mathbf{\Gamma}} \mathbf{a} \|\mathbf{R}(\lambda; \mathbf{A})\| + \mathbf{b} \|\mathbf{A}\mathbf{R}(\lambda; \mathbf{A})\|)^{-\mathbf{1}}$$

where positive numbers $\mathbf{a}$ and $\mathbf{b}$ are selected so that $\|\mathbf{D}\kappa_\mathbf{p}\mathbf{C}\mathbf{x}\| \leq \mathbf{a}\|\mathbf{x}\| + \mathbf{b}\|\mathbf{A}\mathbf{x}\|$ for all $\mathbf{x} \in \mathbf{D}(\mathbf{A})$.

$*$**4 Stability of the series** $\sum_\mathbf{n} \alpha_\mathbf{i}^\mathbf{n} \mathbf{A}_\mathbf{e}^\mathbf{n}$ is given by following conditions with $\kappa_\mathbf{i} = -\theta(\mathbf{CD})^{\ddagger}$, $\mathbf{1} \geq \theta > \mathbf{0}$ (Touré, 1997):

$$\begin{cases} \mathbf{0} \leq \alpha_\mathbf{i} < \min_{\lambda \in \mathbf{\Gamma}} (\mathbf{a}\|\mathbf{R}(\lambda; \mathbf{A}_\mathbf{e})\| + \mathbf{1})^{-\mathbf{1}} \\ \mathbf{rg}(\mathbf{CDW}) = \mathbf{p} \\ \mathcal{R}\mathbf{e}(\sigma(\mathbf{CDW}\kappa_\mathbf{i})) < \mathbf{0} \end{cases} \quad (32)$$

Two cases can be studied similarly, the difference is that in first case the operator CD is inversible ($\kappa_p = [CD]^{-1}$), in the second it has a right pseudo inverse (CD is a $(n+1) \times n$ matrix, $\kappa_p = [CD]^{\ddagger}$). The former is the multireaches case with a downstream overflow, the latter one is the multireaches case with only gates.
It is supposed that $\kappa_i = -\theta[CD]^{\ddagger}$, $0 < \theta < 1$, and α_i checks the first condition of (32).
Results are similar in both cases.

Multireaches with a terminal overflow is the easier case. There are n reaches and $p = n$ gates. So matrix CD is a $n \times n$ inversible matrix with rank n. Moreover, κ_p is chosen to verify condition 2, i.e. $\kappa_p = [CD]^{-1}$. The coefficient α_p is chosen in order to check conditions 1, 2 and 3. Let k such that W is written as $W = k(I + \alpha_p \kappa_p CD)$, with $k = 1/(1 - \alpha_p^2)$. The checking of the three conditions leads to:

$$0 \leq \alpha_p < (\sup_{\lambda \in \Gamma} a \|R(\lambda; A)\|)^{-1} \quad (33)$$

with $a = \|D\kappa_p C\|$. Note that $D\kappa_p C : X \to X$.

4. EXPERIMENTAL RESULTS

Simulations were performed with Matlab and Simulink. They gave satisfactory results for a single reach (cf. (Dos Santos, 2003)) and for the multireaches case, too. Then, the proposed control law was implemented on the Valence (France) experimental canal. This pilote canal is an experimental process (length=8 m, width=0.1 m) with a rectangular basis, a variable slope and with three gates (three reaches and an overflow). Rubbing are weak and the fluvial hypothesis (10) is realized thanks to the variable slope.

4.1 Experimentations around an equilibrium state

Two reaches with three gates case is presented. Initial conditions of the both equilibrium states are the following: $Q_e = 1(dm^3.s^{-1})$

$$z_{e1}(0) = 1.21(dm), \; z_{e2}(0) = 1.01(dm).$$

Reference signals are described bellow for both reaches (Fig. 3):

$$r_{1,0} = 1.28(dm) \; r_{2,0} = 1.17(dm)$$

and

	$0 \leq t < 80(s)$:	$r_1(t) = r_{1,0}$,
FIRST	$80 \leq t < 330(s)$:	$r_1(t) = 120\% * r_{1,0}$
REACH	$330 \leq t < 470(s)$:	$r_1(t) = 90\% * r_{1,0}$
	$t \geq 470(s)$:	$r_1(t) = 110\% * r_{1,0}$
SECOND	$0 \leq t < 160(s)$:	$r_2(t) = r_{2,0}$
REACH	$160 \leq t < 320(s)$:	$r_2(t) = 70\% * r_{2,0}$
	$t \geq 320(s)$:	$r_2(t) = 83\% * r_{2,0}$

System oscillations are explained by the ratio between abscissas and ordinates. It only reproduces the wave phenomena and not instability.
All the experimental results show the suitability of this approach. Indeed, given an interval of $\pm 20\%$ around a given equilibrium state, results are still very satisfactory. However, if the variation asked is superior to $\pm 20\%$, the error between model and system increases dramatically.

4.2 Multi-model experimentation

Multi-model solution is proposed to solve this problem: different models are chosen to cover all

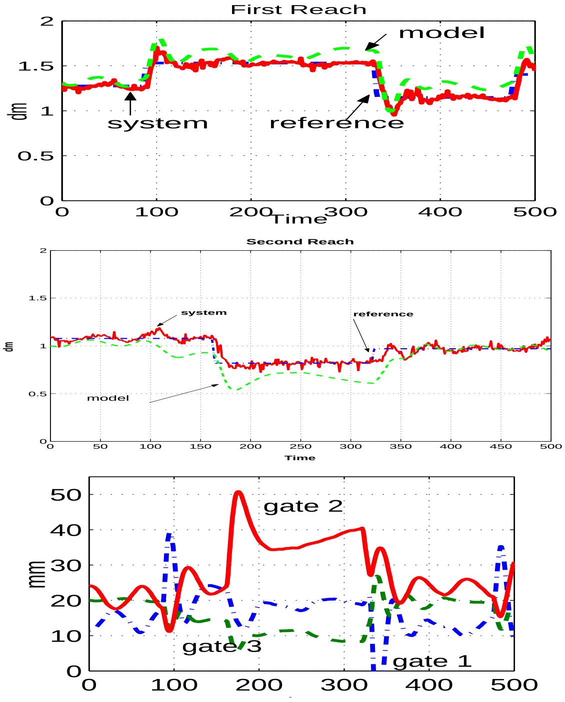

Fig. 3. Downstream levels in two reaches (dm) and opening of the three gates (mm)

water level variation domain, contained between z_{do} to z_{up} (cf. Figure 1). Then, during the control process, the model of each considered reach used by the IMBC structure depends on the real water level, all along the process. E.g. in Figure 4, considering the domain $[z_{do}, z_{up}]$ with a variation of $\pm 20\%$, three models were chosen. Other models should have been add to get even more precision, but one can observe that it is not useful. In the upper figure, the behavior is very satisfactory and the transitions between two models are very smooth, even if it goes through model 3 to model 1 (e.g. at time $t = 110(s)$). In the lower figure, the oscillations are about $1(cm)$ i.e. an error $< 10\%$.

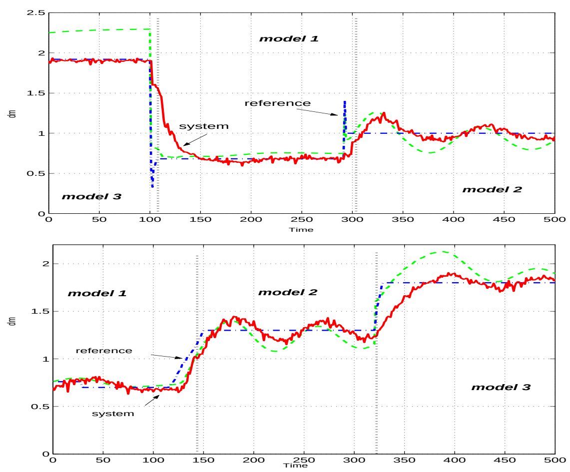

Fig. 4. Downstream levels (dm), varying between 0.5-2 (dm)

5. CONCLUSION

Internal Model Boundary Control seems to be suitable for the regulation of canal irrigation problems addressed in this paper. Moreover, the fact that the spatial evolution of the model parameters has been taken into account allows to consider canals in real situation. It also allows to use the multi-model approach more easily in order to take into account multi-level behavior control and multireaches control.

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