LOW ORDER CONTROL AUGMENTATION SYSTEM DESIGNS FOR THE HELICOPTER UH-60

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Abstract: This paper deals with the design of low order controllers for the tactical transport UH 60 Black Hawk helicopter. Two low order $\mathcal{H}_\infty$ loop shaping control design techniques, which are amendable to Linear matrix Inequalities, are presented and discussed. The first synthesis implements a *cone-complementarity* algorithm while the second method exploits sufficient conditions in the $\mathcal{H}_\infty$ problem leading to a simple, non-iterative control synthesis procedure. The validity of these control techniques are compared and demonstrated both in terms of computational efficiency and control performance achieved through a linearized model of the UH 60 helicopter.

Keywords: Reduced-order control, Linear Matrix Inequality Optimization, H_∞ Optimization, Linear Systems, Helicopter Control.

1. INTRODUCTION

The control of helicopters has attracted the attention of many researchers because it involves large variations in the system dynamics, strong inter-axis couplings and stringent performance requirements. Over the last fifteen years, robust linear control methods have been successfully applied to the design of robust linear controllers for helicopter augmentation systems (Yue and Postlethwaite, 1990). Among them, a design procedure which has been proven to be very successful both in simulation and in flight is the $\mathcal{H}_\infty$ control technique of McFarlane and Glover (McFarlane and Glover, 1992) (Walker and Postlethwaite, 1996), (Smerlas *et al.*, 1998), (Smerlas, 1999), (Postlethwaite *et al.*, 1999) with subsequent flight tests reported in (Walker *et al.*, 1999), (Smerlas *et al.*, 2001), (Postlethwaite *et al.*, 2004), (Prempain and Postlethwaite, 2004). However, most of this research focused on obtaining good performance about the hover. Clearly, a single linear controller cannot ensure performance across the entire flight envelope. Typically, a helicopter control system contains a family of linear controllers which have been designed at various points of the flight envelope. Then, this controller is implemented as a single control with parameters changing according to the scheduling variables (typically the longitudinal and the lateral velocities). A major difficulty with modern control techniques arises from the complexity of the controller. For a helicopter, depending on the control structure (one degree-of-of-freedom or two-degree-of-freedom controllers), on the complexity of the model and of the weighting functions, the order of each single linear controller can reach 15 to 30 states. High order controllers can be reduced by model reduction techniques, but it is preferable to obtain directly low order controllers with performance guarantees. The aim of this paper is to demonstrate that high

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performance, low order $\mathcal{H}_\infty$ helicopter controllers can be easily and efficiently computed using sufficient Linear Matrix Inequality (LMI) optimization techniques.

This paper summarizes a recent study into the design of a multivariable H_∞ performance

The paper is structured as follows. Section 2 presents the basic concepts of the control methods employed. Section 3 presents the H_∞ designs along with simulation results. Conclusions are given in section 4.

The notation used in this paper is fairly standard: $R^{m\times n}$ denotes the set of real $m \times n$ matrices, I_n is the $n \times n$ identity matrix, $A > 0$ means that A is symmetric and positive definite.

2. THEORETICAL BACKGROUND: STATIC $\mathcal{H}_\infty$ LOOP SHAPING CONTROL

2.1 LMI formulation

Without loss of generality, a static controller is considered. Let G_s be a strictly proper plant of order n having a stabilizable and detectable state-space realization:

$$G_s := \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \qquad (1)$$

with $A \in R^{n\times n}$,$B \in R^{n\times n_u}$, $C \in R^{n_y\times n}$. G_s can be considered the shaped plant in the Glover-McFarlane $\mathcal{H}_\infty$ loop shaping design procedure.

A minimal normalized left coprime factorization of $G_s = \tilde{M}^{-1}\tilde{N}$ is given by, (Zhou *et al.*, 1995), (Skogestad and Postlethwaite, 1997).

$$\left[\tilde{N}, \tilde{M}\right] = \left[\begin{array}{c|cc} A+LC & B & L \\ \hline C & D & I \end{array}\right]. \qquad (2)$$

where $L = -ZC^T$ and the matrix Z is the unique symmetric positive semi-definite solution to the algebraic Riccati equation

$$AZ + ZA^T - ZC^TCZ + BB^T = 0 \qquad (3)$$

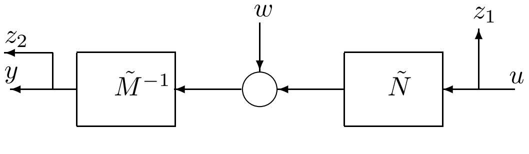

Fig. 1. Open loop Glover-McFarlane $\mathcal{H}_\infty$ loop shaping interconnection

Theorem 1. Let $L = -YC^T$ where $Y \geq 0$ is the stabilizing solution to

$$AY + YA^T - YC^TCY + BB^T = 0. \qquad (4)$$

There exists a static loop shaping controller K such that

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I + G_sK)^{-1}\tilde{M}^{-1} \right\|_\infty < \gamma \qquad (5)$$

if $\gamma > 1$ and if and only if there exist two positive definite matrix R and S solving the inequalities

$$S(A+LC) + (A+LC)^TS - \gamma CC^T < 0 \qquad (6)$$

$$\begin{pmatrix} AR + RA^T - \gamma BB^T & RC^T & -L \\ CR & -\gamma I_p & I_p \\ -L^T & I_p & -\gamma I_p \end{pmatrix} < 0 \qquad (7)$$

$$\mathcal{K}(R,S) := \begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0 \qquad (8)$$

and $rank(\mathcal{K}(R,S)) = n$.

Proof. See (Prempain and Postlethwaite, 2004).

It is well-known that in general the minimization of a rank constraint is hard to solve. Various heuristics have been developed to handle problems of this type. One simple heuristic, applicable when the matrix is symmetric positive semi-definite, is to minimize its trace in place of its rank. We have the following result:

Theorem 2. There exists a stabilizing static-output feedback controller if and only if the global minimum of the following optimization problem

$$\min Trace(RS) \qquad (9)$$

subject to (6), (7) and (8) is equal to n.

Proof. See e.g. (El Ghaoui *et al.*, 1997).

2.2 A cone complementarity algorithm

To solve the optimization problem (9), a linear approximation of $trace(XS)$ takes the form

$$\phi_{lin}(R,S) = constant + trace(S_0R + R_0S) \qquad (10)$$

From (10) the following iterative algorithm (El Ghaoui *et al.*, 1997) is:

(1) Find a feasible point S_0, R_0. If there are non exit, set $k = 1$.
(2) Solve the LMI problem
    minimize $O_k := trace(S_kR_{k-1} + R_kS_{k-1})$
    subject to (6), (7) and (8).
(3) Exit if $\|O_k - O_{k-1}\| < \epsilon$ where ϵ is given positive number. Otherwise, set $k = k+1$ and go to step 2.

El Ghaoui *et al.* (1997) have shown that such an algorithm converges and finds, at every step k, a controller of order that is less or equal to $n - \max(n_u, n_y)$.

2.3 Sufficient LMI conditions

Corollary 1. (Prempain and Postlethwaite, 2004). There exists a static loop shaping controller K such that

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I + G_s K)^{-1} \tilde{M}^{-1} \right\|_\infty < \gamma \quad (11)$$

if $\gamma > 1$ and if there exists a positive definite matrix R solving the inequalities

$$(A + LC)R + R(A + LC)^T < 0 \quad (12)$$

$$\begin{pmatrix} AR + RA^T - \gamma BB^T & RC^T & -L \\ CR & -\gamma I_p & I_p \\ -L^T & I_p & -\gamma I_p \end{pmatrix} < 0 \quad (13)$$

2.4 Controller reconstruction

For both methods, the controller K can be reconstructed using the analytic formulae given in (Iwasaki and Skelton, 1994). The final feedback controller K_{ST} is then obtained using the output feedback controller K with the shaping functions W_1 and W_2 such that $K_{ST} = W_1 K W_2$.

3. PLANT DESCRIPTION AND CONTROL OBJECTIVES

A linear the state-space model of the UH-60 has been obtained from a 20-state nonlinear flight mechanic model developed by the Flight Science & Technology department of the University of Liverpool. The linearized model used in this paper corresponds to the hover situation. The controlled outputs are θ, ϕ, r, and the measured outputs are θ, ϕ, r, p, q see table 1. θ, ϕ, ψ are the Euler angles, defining the orientation of the body axes relative to the earth and p, q, r are the angular velocities about the x-, y-, z-axes fixed to the fuselage. Note that the yaw attitude (ψ) is not directly controlled because in the case of a complete turn about the z-axis, the yaw attitude jumps from 2π to 0 producing a discontinuity in the measured signal. For the ease of control, both from pilot and control design view points, it is preferable to control directly the yaw rate which is approximatively equals to $\dot{\psi}$ when ϕ and θ have small values.

3.1 Low order $\mathcal{H}_\infty$ helicopter controller design

In this section, we demonstrate the design techniques above on a 12th order residualized model of the UH-60. In the sequel, the 12th order residualized model of the UH-60 is denoted G. For the hover operating point the weights were chosen as

$$W_1 = diag(8\frac{s+8}{s}, 8\frac{s+4}{s}, 9\frac{s+12}{s}) \quad (14)$$

$$W_2 = diag(2, 1.5, 1, 0.7, 0.7) \quad (15)$$

The weighting function were adjusted to meet the Level 1 handling quality requirement of the ADS-33 norm (anonymous, 1994). The nominal plant G and the shaping functions W_1 and W_2 are combined to form the shaped plant $G_s = W_2 G W_1$. The synthesis procedures described in section 2 were implemented using the LMI solvers (Gahinet *et al.*, 1995) and SeDuMi (Sturm, 2001). The SeDuMi solver was used to implement the cone complementary linearization algorithm while the sufficient conditions of corollary 1 were implemented with the LMI toolbox of Gahinet *et al.* (1995).

3.2 Static $\mathcal{H}_\infty$ Controller

The static version of the Glover and McFarlane design procedure described in this paper led to a final controller of order 3, for which the closed-loop attenuation is of 3.66. Figure 2 shows the singular value plot of the complementarity sensitivity function. This plot indicates a control bandwidth of about 6 rad/s. Good robustness is expected (no overshoot, low bandwidth). Figure 3 shows acceptable closed-loop time responses for this controller. However, note the presence of static couplings, which may be not desirable.

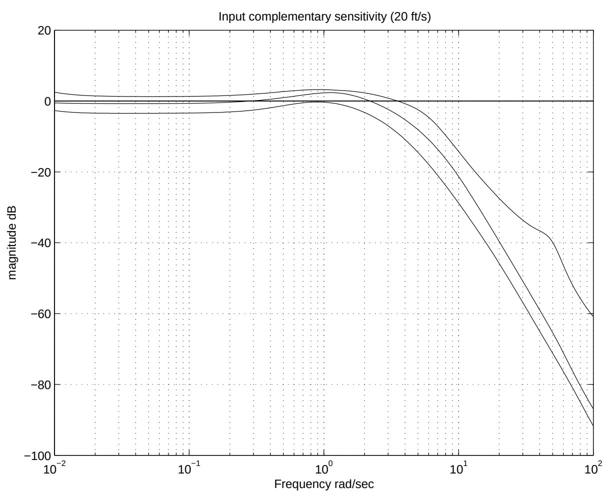

Fig. 2. Singular value plot of the complementarity sensitivity function

3.3 The cone complementarity algorithm

The right plot of figure 4 shows the eigenvalues of $RS - I$ versus the iterations. This plot suggests that a reduced order controller can be reconstructed. In this case, the routine *klmi* was able to reconstruct a controller of order 4 leading to a final controller of order 7 for which the closed-loop attenuation is $\gamma = 3.8$. The responses obtained with

Table 1. Plant inputs and measured outputs description

Measured outputs (y)	Description	Input (u)	Description
ϕ	roll attitude	θ_{1c}	lateral cyclic actuator
θ	pitch attitude	θ_{1s}	longitudinal cyclic actuator
r	yaw rate	θ_{0T}	tail rotor collective
p	roll rate		
q	pitch rate		

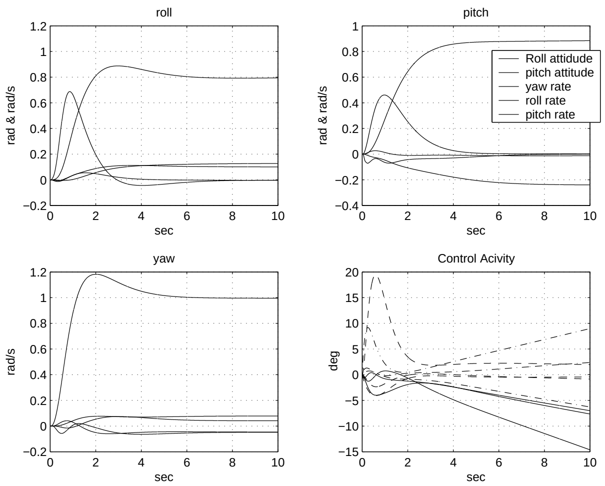

Fig. 3. Closed-loop time responses

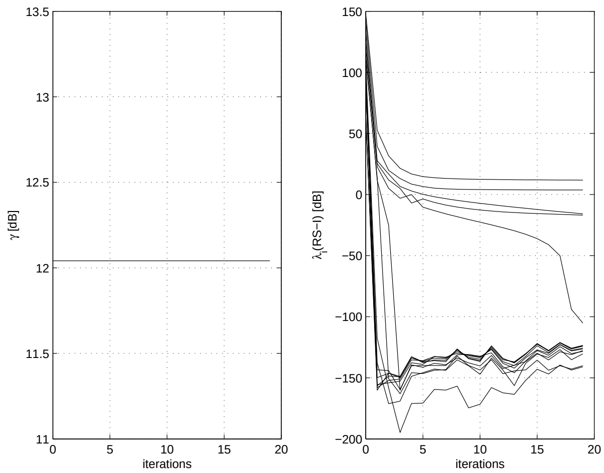

Fig. 4. Cone Complementary algorithm: left γ, right $\lambda_i(RS - I_n)$

cone complementary algorithm are very close to those obtained with the static controller approach and are therefore not reproduced here.

4. CONCLUSIONS

This paper has presented two autopilot designs for the helicopter UH60. A Glover-MacFarlane H_∞ type of design was conducted on a linearized model of the helicopter at the hover situation. The performance of the low order regulators obtained were found to be acceptable and comparable to the performance obtained with a full order controller. It is worth noting that the controller obtained with our non iterative approach has only 3 states while the controller obtained by the cone complementary algorithm possesses 7 states. Also, it is worth mentioning that the two methods do not require the same amount of computer work. The cone complementary algorithm involves $n(n+1)+1$ decision variables at each iteration. In contrast, our synthesis method based on the sufficient LMI conditions involves only $n(n+1)/2+1$ decision variables and has the advantage to be not iterative. Therefore, the method is computationally much more attractive than the cone complementarity algorithm for solving static $\mathcal{H}_\infty$ loop shaping control problems.

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