CONTROLLER EVALUATION AND REDESIGN USING RELAY EXPERIMENTS

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Abstract: In this paper it is presented a procedure for closed loop controller redesign using a relay based experiment. The present closed loop is evaluated in relation to a symmetrical optimum design. The loop gain is estimated at a few frequency points using an excitation derived from a relay experiment. The controller is redesigned to yield some stability margin using a controller procedure derived from the symmetrical optimum. Simulation examples illustrate the properties of the design scheme.

Keywords: PID controller, controller tuning, relay systems, symmetrical optimum design.

1. INTRODUCTION

Techniques for identification and controller redesign using closed-loop data are very attractive to industrial applications. The closed-loop identification doesn't cause stops in system operation unlike open-loop identification. Other reasons which can be listed are demands on safety in process operation, unstable processes and restrictions in production. Its has also been argued that in closed loop it is possible to obtain representative restricted complexity process models which can be used to redesign controllers such as PI and PID (Hof and Schrama, 1995). This is justified as the dynamics exhibited by the plant with the old controller is relevant to the new controller design.

On the other side, there exists several PID controller design procedures that do not use models, but the information of a few process transfer function frequency points. The earliest one is the Ziegler-Nichols frequency design technique (Ziegler and Nichols, 1942) from which a several technique have been derived. Usually, the information is obtained from relay experiments, which have proven to be very useful for process identification and on-line controller tuning (Åström and Hägglund, 1995).

When redesigning a controller, specially when using little information from the process transfer function, it is important to evaluate the robustness properties of the existing loop and to redesign the controller leaving some safety margins for the case of model errors. One common approach is to evaluate the gain and phase margins and use the information to redesign the controller as the one presented in (de Arruda and Barros, 2003*a*). In the present paper the controller evaluation and redesign is based on the symmetrical optimum technique (Kessler, 1958). This technique has advantages as robustness aspects (phase margin, gain margin, sensivity, neglected dynamics), desired closed loop characteristics and cover a large domain of current, real applications. It received some attention in recent years as it takes into account the existence of fast dynamics lumped in one pole. The pole is used as a limiting factor for the crossover frequency, thus resulting in a closed loop with some stability margin. The symmetrical optimum and a relay experiment have been recently used in (Voda and Landau, 1995) with good results, but in the context of open loop experiment.

In this paper a technique for controller evaluation and redesign based on the symmetrical optimum controller design technique is presented. A relay test applied to the closed loop is used to define a periodic excitation reference signal. The closed loop frequency response at a few points is estimated and the loop gain at those frequencies is computed. The loop gain frequency response variation between the estimated frequency points is analyzed to determine the presence of unmodelled dynamics in a frequency interval. The new crossover frequency is defined based on this information and the controller is redesigned. Here, the presentation is limited to the PI controller case, however the procedure can be easily extended to the PID controller case. Simulation examples are used to illustrate the proposed closed loop evaluation and closed loop design technique.

2. THE PROBLEM STATEMENT

Consider the closed loop shown in Fig. 1. The process transfer function is given by $G(s)$ while the controller is given by $C(s)$. The closed loop transfer function from the reference signal $r(t)$ to the process output $y(t)$ is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{L(s)}{1+L(s)} \qquad (1)$$

where $L(s) = G(s)C(s)$ is the *Loop Gain Transfer Function.*

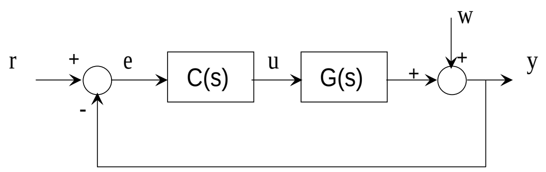

Fig. 1. The Closed Loop.

2.1 The Symmetrical Optimum Design

The symmetrical optimum design technique assumes a simple model with two poles, at zero and at $\omega_\Sigma = -1/T_\Sigma$ given as

$$G_{SO}(s) = \frac{K}{s(s+1/T_\Sigma)}.$$

The model assumes a slow dominating dynamics captured by an integrator and a pole at ω_Σ representing all fast dynamics, including time-delays.

The PI controller, given by

$$C_{SO}(s) = \frac{K_p(s+1/T_i)}{s},$$

is designed using the following equations

$$K_p = \frac{2\omega_g}{K}, \quad T_i = \frac{2}{\omega_g}, \quad \omega_g = \frac{1}{2T_\Sigma}.$$

The resulting loop gain transfer function is

$$L_{SO}(s) = \frac{\omega_g(2s+\omega_g)}{s^2(s+2\omega_g)}.$$

The estimated pole is located at frequency $2\omega_g$ while the controller zero is added at frequency $\omega_g/2$. The resulting loop gain has a 20db/decade asymptotic decay in the frequency range $[\omega_g/2, 2\omega_g]$, with the gain crossover (unity loop gain) frequency ω_g. These characteristics result in good robustness properties (see (Voda and Landau, 1995)).

The problem statement is: Given a closed loop system, evaluate how the closed loop compares to a symmetrical optimum design. If it is too far, redesign the controller to approximately and safely match the symmetrical optimum specification. This is made comparing the present closed loop gain crossover frequency to half of the estimated transfer function pole.

3. THE RELAY BASED ESTIMATION

In this paper it is not desired to estimate a plant model, instead only a few closed loop frequency points are evaluated. The frequencies are the gain crossover frequency and its second and forth harmonics. The excitation is derived after a relay experiment as described below.

3.1 The Relay Experiment

A basic procedure for the estimation of a general frequency point of a given transfer function using a relay feedback is presented in (de Arruda and Barros, 2003*b*). The feedback structure applied for loop transfer function estimation is presented in Fig. 2. The conditions of the limit cycle operation are defined by the following proposition.

Proposition 1. Consider the closed loop system shown in Fig (2). Assume that for a stable closed loop $T(s)$ and a real positive number r, the transfer function

$$F(s) = \frac{2}{r}\frac{T(s)}{T(s)\left(\frac{1-r}{r}\right)+1} - 1 \qquad (2)$$

is also stable. Then if a limit cycle is present it oscillates at a frequency ω_o such that

$$|L(j\omega_o)| \approx r\ .$$

Proof. See (de Arruda and Barros, 2003*b*). ■

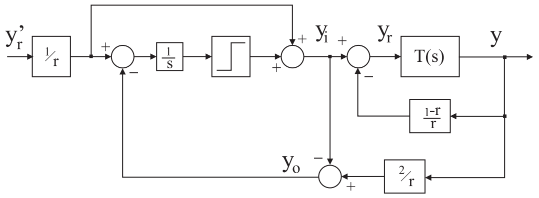

Fig. 2. Relay Closed Loop Experiment for Loop Transfer Function Estimation.

Selecting $r = 1$, the current gain crossover frequency ω_g can be estimated. This estimate is denoted $\hat{\omega}_g$. In this case the scheme reduces to the one presented in (Schei, 1992).

In order to estimate the loop gain at additional frequencies $2\omega_g$ and $4\omega_g$, an excitation signal is composed with three square waves with the above frequencies weighted by $0.4d$, $0.3d$ and $0.3d$ respectively, where d is the desired maximum amplitude. This frequencies define the vector $W = \{\omega_g, \quad 2\omega_g, \quad 4\omega_g\}$. One such example is shown in Fig.3.

The loop gain transfer function at the chosen frequencies is estimated using the DFT on the reference y_r and output signals y, computing the closed loop gain T_i and then recovering the loop gain (L_i) using the loop equations

$$L_i(j\omega) = \frac{T_i(j\omega)}{1 - T_i(j\omega)}.$$

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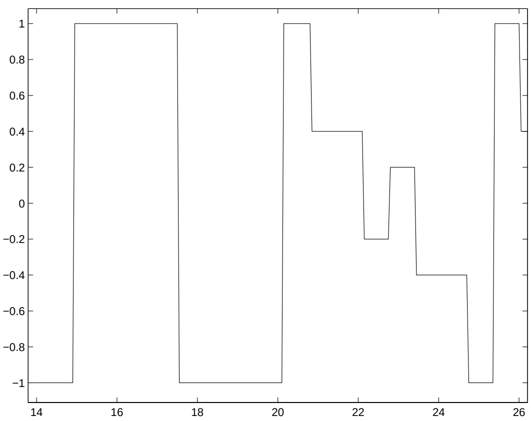

Fig. 3. The experiment. First a relay test is made then the derived excitation is applied at the setpoint.

3.2 The Fast Pole Estimation and Controller Evaluation

The loop gain frequency response at the estimated frequency points is analyzed to determine the presence of the fast pole within the frequency intervals defined by W. From the estimated loop gain at those frequencies, it is possible to evaluate in which interval the unmodelled pole ω_Σ is located using the following procedure.

In the symmetrical optimum design, between the crossover frequency ω_g and $2\omega_g$ the loop gain shown asymptotically decay as a pure first order system, that is, 20db/decade or 6db/octave. The evaluation procedure is made taking into account the contribution of a first order pole.

The true gain decay for a first order real pole includes a discount of 3db at the pole location and of 1db one octave higher. Using this information, the contribution of a real pole to the magnitude decay in the intervals $[\omega_g, 2\omega_g]$ and $[2\omega_g, 4\omega_g]$ can be specified as:

Table 1. Contribution of the pole to the magnitude decay within the intervals .

Pole	$[\omega_g, 2\omega_g]$	$[2\omega_g, 4\omega_g]$
ω_g	10 db	11 db
$2\omega_g$	8 db	10 db
$4\omega_g$	7 db	8 db

Based on the above data, two tests were defined: the first analyzing the decay in the interval $[\omega_g, 2\omega_g]$, and the second in the interval $[2\omega_g, 4\omega_g]$. The first test uses the following rule:

Table 2. Magnitude decay of the interval $[\omega_g, 2\omega_g]$ related with the estimated pole location.

Decay Range	Estimated Pole Location ($\hat{\omega}_\Sigma$)
≥ 11	$\hat{\omega}_g/2$
[10,11)	$\hat{\omega}_g$
[8,10)	$2\hat{\omega}_g$
[6,8)	$4\hat{\omega}_g$
< 6	$8\hat{\omega}_g$

The second test uses

Table 3. Magnitude decay of the interval $[2\omega_g, 4\omega_g]$ related with the estimated pole location.

Decay Range	Estimated Pole Location ($\hat{\omega}_\Sigma$)
≥ 12	$\hat{\omega}_g/2$
[11,12)	$\hat{\omega}_g$
[10,11)	$2\hat{\omega}_g$
[8,10)	$4\hat{\omega}_g$
< 8	$8\hat{\omega}_g$

The estimated pole location $\hat{\omega}_\Sigma$ is chosen in a conservative way as the smallest estimated pole location obtained from the two tests.

It should be remarked that the technique can be extended to evaluate the phase behavior, which would allow one to estimate the contribution of time delays, not considered in this paper.

In this paper the estimated pole is expressed as

$$\hat{\omega}_\Sigma = 2\alpha\hat{\omega}_g$$

with parameter α indicating the estimated pole relative to the current crossover frequency ω_g.

Then α can be calculated and be used in the controller redesign procedure next.

4. THE CONTROLLER REDESIGN PROCEDURE

The new crossover frequency is defined as $\omega'_g = \hat{\omega}_{\Sigma}/2 = \alpha\hat{\omega}_g$. The new controller zero is set at

$$\omega_z = \omega'_g/2.$$

It should be noted that the pole estimation in intervals results a more conservative new controller than if the model is estimated as the symmetrical optimum is used in the design because of the pole location chosen as explained before. It should be clear that the obtained controller is not a true symmetrical optimum design.

In order to avoid the estimation of the process gain, the new controller is designed as a modification of the old controller. The process model is assumed to be

$$G_{SO}(s) = \frac{K}{s\,(s + 2\alpha\omega_g)}.$$

The old PI controller is given by

$$C_1(s) = \frac{K_p^1\left(s + 1/T_i^1\right)}{s}$$

and the new PI controller

$$C_{SO2}(s) = \frac{K_p^2\,(s + \alpha\omega_g/2)}{s}.$$

The gain K_p^2 can be computed noticing that from the experiment

$$|G_{SO}(s)\,C_S(s)||_{s=j\omega_g} = 1$$

and for the design

$$|G_{SO}(s)\,C_{SO2}(s)||_{s=j\alpha\omega_g} = 1.$$

Equating the two equations, cancelling the process gain K and solving for K_p^2 it yields

$$K_p^2 = 2K_p^1\alpha^2\frac{\sqrt{\left(\frac{2\alpha}{T_i^1} - \omega_g\right)^2 + \left(\frac{1}{T_i^1} - 2\alpha\omega_g\right)^2}}{\omega_g\,(4\alpha^2+1)}.$$

5. SIMULATION EXAMPLES

In this section three simulation examples are shown which illustrate the use of the technique.

5.1 Example 1

The process is given by

$$G(s) = \frac{1}{(10s+1)(s+1)}$$

and the initial controller is

$$C_1(s) = \frac{18.1246\,(s + 1/2.1272)}{s}.$$

With the estimates shown in table (4)

Table 4. Actual and estimated loop gain transfer function points.

	$\lvert L(j\varpi)\rvert(db)$	$\lvert\hat{L}(j\omega)\rvert(db)$
ω_1	0.1950	0.3112
ω_2	-10.6955	-10.6955
ω_4	-22.3471	-22.3471

one computes $\alpha = 0.5$ and the new controller is

$$C_{SO2}(s) = \frac{3.9149\,(s + 1/3.3104)}{s}.$$

The step responses and the loop gains for both controllers are shown in Figures 4 and 5.

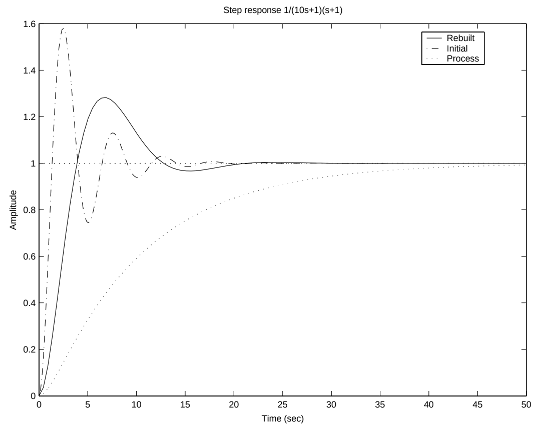

Fig. 4. Step Responses $\frac{1}{(10s+1)(s+1)}$.

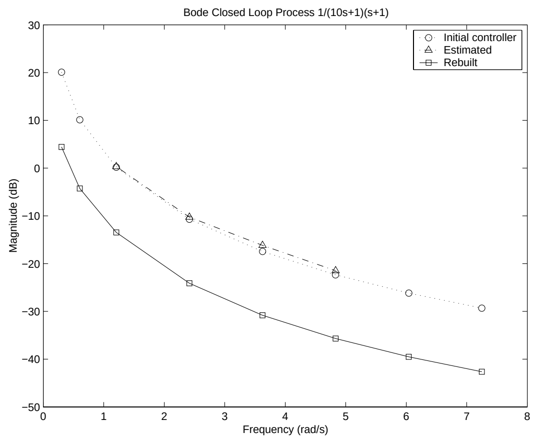

Fig. 5. Loop Gain Bode Diagrams $\frac{1}{(10s+1)(s+1)}$.

Besides, the process does not contain an integrator and the symmetrical optimum be formulated for plants containing integrators, it is clear that the robustness of the method had guaranteed a better result to the step response test.

5.2 *Example 2*

The process now is given by

$$G(s) = \frac{1}{(10s+1)(s+1)^2}$$

and the initial controller

$$C_1(s) = \frac{8.5491\,(s+1/4.8119)}{s}.$$

With the estimates shown in table (5)

Table 5. Actual and estimated loop gain transfer function points.

Redesign	$\|L(j\varpi)\|(db)$	$\|\hat{L}(j\omega)\|(db)$
ω_1	-0.0166	0.1531
ω_2	-11.6819	-11.3767
ω_4	-26.8685	-26.4242

one computes $\alpha = 0.5$ and the new controller is

$$C_{SO2}(s) = \frac{2.0328\,(s+1/6.3025)}{s}.$$

The step responses and the loop gains for both controllers are shown in Figures 6 and 7.

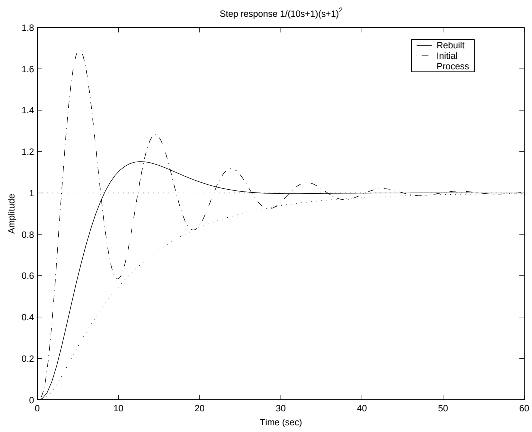

Fig. 6. Step Responses $\frac{1}{(10s+1)(s+1)^2}$

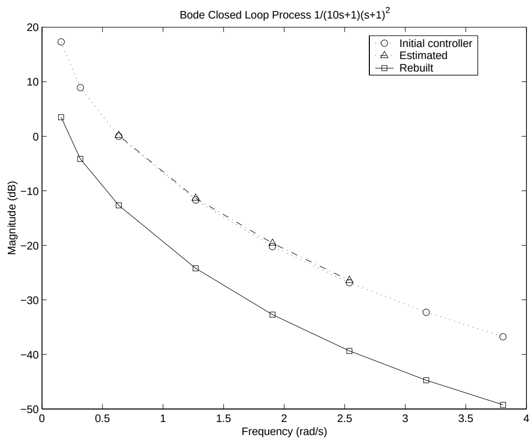

Fig. 7. Loop Gain Bode Diagrams $\frac{1}{(10s+1)(s+1)^2}$

5.3 *Example 3*

The process is given by

$$G(s) = \frac{1}{(s+1)^4}$$

and the initial controller

$$C_1(s) = \frac{2.8955\,(s+1/4.6840)}{s}.$$

With the estimates shown in table (6)

Table 6. Actual and estimated loop gain transfer function points for the initial system.

Redesign 1	$\|L(j\varpi)\|(db)$	$\|\hat{L}(j\omega)\|(db)$
ω_1	0.0701	0.2826
ω_2	-14.2674	-10.4078
ω_4	–34.6733	-25.6860

one computes $\alpha = 0.5$ and the new controller is

$$C_{SO2}(s) = \frac{0.7663\,(s+1/4.7110)}{s}.$$

The closed loop is too slow as it can be noted in Figure 8, so the procedure is repeated for this new loop. With the estimates shown in table 7

Table 7. Actual and estimated loop gain transfer function points for the redesigned system.

Redesign 2	$\|L(j\varpi)\|(db)$	$\|\hat{L}(j\omega)\|(db)$
ω_1	2.4020	2.4050
ω_2	-1.7640	-1.7281
ω_4	-6.6553	-6.4807

now $\alpha = 2.0$ and the new controller is

$$C_{SO2}(s) = \frac{2.0007\,(s+1/7.0028)}{s}.$$

The step responses and the loop gains for the three controllers are shown in Figures 8 and 9.

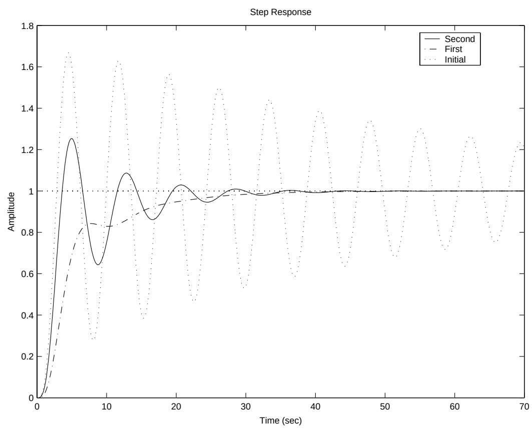

Fig. 8. Step Responses $\frac{1}{(s+1)^4}$.

From the results it can be seen that the technique have improved the stability of the close loop system in both experiments.

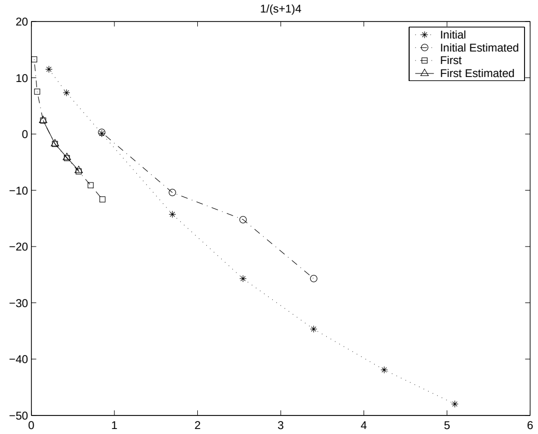

Fig. 9. Loop Gain Bode Diagrams $\frac{1}{(s+1)^4}$

6. CONCLUSIONS

In this paper a controller evaluation and redesign technique was presented. The closed loop is evaluated by estimating the loop gain response at the crossover frequency and the two first even harmonic frequencies. Then, using the loop gain magnitude decay, the closed loop is evaluated in a symmetrical optimum sense. The controller is redesigned on a design procedure based on the symmetrical optimum design, using an approximate and conservative evaluation of the fast pole. Simulation examples illustrate the use of the technique.

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