HIGH ORDER SLIDING MODE CONTROLLERS AND DIFFERENTIATORS FOR A SYNCHRONOUS GENERATOR WITH EXCITER DYNAMICS

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Abstract: The problem of angular velocity of rotor and voltage control in synchronous generator machines is considered. As a robust solution of this problem the block control technique with sliding mode is proposed. The physical implementations of the proposed control law for the real synchronous machines results in chattering due to the presence of exciter unmodelled dynamics. In order to reduce the chattering effect we design high order sliding mode controllers and exact sliding mode differentiators, ensuring the robustness of the closed-loop system.

Keywords: nonlinear system, sliding mode control, synchronous machine, robust control.

1. INTRODUCTION

Synchronous machines have a different time-scale nonlinear dynamics. The typical model of synchronous generators is the sixth order model in which the 60Hz. electromagnetic stator fast dynamics is neglected and only the slow mechanical and rotor flux dynamics are considered, see for example (Kundur, 1964; Sauer, 1998). The principal control objectives in synchronous machines are to maintain the required values of the synchronous speed and generator voltage. So, it is necessary to design the feedback controllers for synchronous generators for robust stabilization of both: frequency and voltage magnitude. Robustness implies operation with adequate stability margins and admissible performance level in spite of plant parameters variations and in the presence of external disturbances. The generator control strategies are commonly based on linearized dynamics equations and consequently only local stability for a specific operation point is achieved. Recently, to overcome the limitation of linear control, attention has been focused on implementation of modern control technique, e.g., an adaptive linear control (Son and Park, 2000), passivity-based approach (Ortega *et al*., 1998; Galaz *et al*., 2001), intelligent control such as fuzzy logic (Hiyama *et al*., 1997; Lown *et al*., 1997) and neural networks (Hsu and Chen, 1991), control based on direct Lyapunov method (Pai, 1989), feedback linearization (FL) technique (Gao *et al*., 1992; Bourlés *et al*., 1997), and control based on adaptive FL (Jain *et al*., 1994; Lahdhiri and Alouani, 1998). All of the mentioned controllers provide larger stability margins with respect to traditional ones. But these control schemes are sensitive with respect to uncertainties presented in the plant model and do not take into account practical limitation on the magnitude of the excitation control input. To solve this problem (Loukianov *et al*., 2004) proposed sliding mode observer-based controller using block control technique in order to obtain the control objectives, ensuring the robustness issue even in presence of disturbances. The block control technique is applied to design a nonlinear sliding surface in such way that the sliding-mode dynamics is described by desired linear system. However, when the exciter dynamics is considered, some problems appear. In a continuous control scheme, see for example (Sauer, 1998), the effects of fast actuator unmodelled dynamics rapidly decay if they are stable and faster than the slow motions of the system, and the omission of actuator dynamics is possible. On the other hand, first order sliding mode controllers loose robustness in the presence of unmodeled dynamics of relative degree two or more, converting sliding mode in fast switching oscillations. Such oscillations are named “chattering”, see for example (Bondarev, 1985; Utkin *et al.*, 1999; Fridman, 2002) and reference therein. The chattering comes out as low control accuracy, vibrations in mechanical parts and undesirable heat losses in electric power circuits.

To counteract the effects of chattering, this paper proposes the implementation of High Order Sliding Modes (HOSM) algorithms (Levant, 2003a, b). Taking into account that complete relative degree of synchronous machine plus exciter with respect to a sliding variable is three, then usage of Third Order Sliding Mode (TOSM) becomes reasonable. The implementation of TOSM needs the calculation of sliding function and their first and second time derivatives. In the case of the synchronous generators, direct calculations of the derivatives of sliding variables results in a computationally expensive control algorithm. For those reason we use HOSM exact differentiators (Levant, 2003a).

Section II presents the synchronous generator sixth order model. In section III we present design of a conventional sliding mode controller for the synchronous machine. The block controllable form of the machine is used to define the sliding surface. Section IV presents the structure of voltage exciter and gives a brief description of the causes of chattering in discontinuous control systems. In section V, Third Order Sliding Mode (TOSM) algorithm is used to compensate the effects of the unmodelled dynamics of the exciter. To estimate first and second time derivatives of the sliding variable the robust HOSM differentiators are used. Section VI presents the simulations results. Section VII gives some conclusions.

2. SYNCHRONOUS GENERATOR MODEL

2.1 Basic Equations

The mathematical models for the synchronous generator are based on the mechanical and electric equilibrium equations; see (Park, 1929; Rankin, 1945; Kundur, 1964). The mechanical equilibrium equations for a synchronous generator are given by

$$\frac{d\delta}{dt} = \omega - \omega_b \tag{1}$$

$$\frac{d\omega}{dt} = \frac{\omega_b}{2H}\left(T_m - T_e\right) \tag{2}$$

where δ is the power angle (rad.), ω is the angular velocity (rad./sec.), ω_b is the synchronous angular velocity (rad./sec.), H is the inertia constant (sec.), T_m is the mechanical torque (p.u.), and T_e is the electromechanical torque (p.u.). On the other hand, the electric equilibrium equations affected by the Park transformations (Park, 1929), are expressed as

$$V = R\,i + \omega\,G\varphi + \frac{d\varphi}{d\bar{t}} \tag{3}$$

$$\varphi = L\,i \tag{4}$$

where $\bar{t} = \omega_b t$, t is the time in seconds, $\bar{t}$ is the time in p.u.,

$V = \left[v_d, v_q, v_f, 0, 0, 0\right]^T$,

$\varphi = \left[\varphi_d, \varphi_q, \varphi_f, \varphi_g, \varphi_{kd}, \varphi_{kq}\right]^T$,

$i = \left[i_d, i_q, i_f, i_g, i_{kd}, i_{kq}\right]^T$, V means voltage, i means current, φ means flux linkage, r means resistance, L means inductance, and the subscripts mean: s stator, d direct axis circuit, q quadrature axis circuit, f field excitation circuit, g quadrature field circuit, kd direct axis damper, kq quadrature axis damper, md direct magnetizing, mq quadrature magnetizing,

$$R = \begin{bmatrix} -r_s & & & & & \\ & -r_s & & & 0 & \\ & & r_f & & & \\ & & & r_g & & \\ & 0 & & & r_{kd} & \\ & & & & & r_{kq} \end{bmatrix} \quad G = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} -L_d & 0 & L_{md} & 0 & L_{md} & 0 \\ 0 & -L_q & 0 & L_{mq} & 0 & L_{mq} \\ -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix}.$$

The equation for the electromechanical torque is

$$T_e = \varphi_d i_q - \varphi_q i_d\,. \tag{5}$$

2.2 External network

The synchronous machine is considered to be connected to an infinite bus through a transmission line, see Figure 1. The equilibrium equation between the synchronous machine and the external network is

$$\boldsymbol{V_{gen}} = L_{ext}\,\frac{d\,\boldsymbol{i_s}}{dt} + \boldsymbol{G_{ext}}(\omega)\,\boldsymbol{i_s} + V^{\infty}\boldsymbol{Y} \tag{6}$$

where

$\boldsymbol{V_{gen}} = \begin{bmatrix} v_d \\ v_q \end{bmatrix}$, $\boldsymbol{i_s} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$, $\boldsymbol{G_{ext}}(\omega) = \begin{bmatrix} R_{ext} & -\omega L_{ext} \\ \omega L_{ext} & R_{ext} \end{bmatrix}$,

$\boldsymbol{Y} = \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}$, L_{ext} is the line inductance and R_{ext} is the line resistance, V^{∞} is the infinite bus voltage settled in $1\angle 0^{\circ}$.

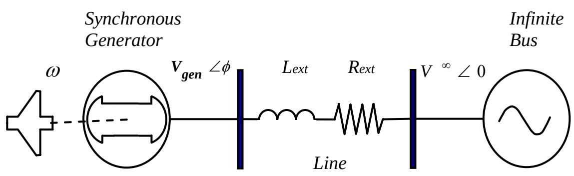

Figure 1. Single machine with infinite bus.

2.3 Linear magnetic model

From (1)-(6), we obtain the sixth order synchronous generator model. A more detailed development of the modeling, can be viewed in several power system literature, e.g. (Krause, 1986; Kundur, 1964; Sauer, 1998) and others.

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} F_1(\mathbf{x_1},\mathbf{x_2},T_m,i_d,i_q) \\ F_2(\mathbf{x_1},\mathbf{x_2},i_d,i_q) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} e_{fd} \quad (7)$$

$$F_1 = \begin{bmatrix} x_2 - \omega_s \\ d_m T_m - \left[a_{21}x_3 i_q + a_{22}x_4 i_d + a_{23}x_5 i_q + a_{24}x_6 i_d + a_{25} i_d i_q\right] \\ a_{41}x_3 + a_{42}x_4 + a_{43}x_5 + a_{44}x_6 + a_{45}\sin x_1 + a_{46}\cos x_1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} b_{11}x_4 + b_{12}x_6 + b_{13}i_q \\ b_{21}x_3 + b_{22}x_5 + b_{23}i_d \\ b_{31}x_4 + b_{32}x_6 + b_{33}i_q \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix},$$

where $\mathbf{x_1} = (x_1, x_2, x_3)^T$, $\mathbf{x_2} = (x_4, x_5, x_6)^T$, $x_1 = \delta$, $x_2 = \omega$, $x_3 = \varphi_f$, $x_4 = \varphi_g$, $x_5 = \varphi_{kd}$, $x_6 = \varphi_{kq}$, e_{fd} is the voltage injected to the machine from the voltage exciter. Neglecting the fast stator dynamics, the stator currents can be presented as continuous functions of the slow variables and voltage, that is, $i_d = h_1(\mathbf{x_1}, \mathbf{x_2}, v_d)$ and $i_q = h_2(\mathbf{x_1}, \mathbf{x_2}, v_q)$ .The coefficients of (7) depend on the plant parameters and are settled in the appendix.

3. BLOCK CONTROL AND FIRST ORDER SLIDING MODE

The principal advantage of sliding mode control is robustness in the presence of external and internal perturbations. To design a sliding controller we use the block controllable form for synchronous generator model (Loukianov, 1998). In subsection 3.1 the first order sliding mode controller is presented and in subsection 3.2 design of a sliding controller based on the block control approach is implemented for the synchronous generator model presented in section II.

3.1 First order sliding mode (FOSM)

Consider a nonlinear system of the form

$$\frac{dx}{dt} = f(x,u) \quad (8)$$

where $x \in R^n$, $u \in R$, f is smooth function of their arguments. The general sliding mode procedure is the following. First, let us design a nonlinear sliding surface in the state space of (8) in the form

$$s(x) = 0\,,\ s \in R$$

such that a unique solution of algebraic equation

$$\frac{d\,s}{dt} = Gf(x,\ u) = 0\,,\ G = \left\{\frac{d\,s}{dx}\right\}$$

with respect to the equivalent control, $u_{eq}(x)$ (Utkin, 1992), do exists, and the sliding mode equation (SME)

$$\frac{d\,x}{dt} = f\left(x, u_{eq}(x)\right) \quad (9)$$

$$s(x) = 0 \quad (10)$$

has the desired properties. Second, taking into account the system to be controlled, let us choose a discontinuous control

$$u(x) = \begin{cases} u^+(x) & if \quad s(x) > 0, \quad \left|u^+(x)\right| \le u_0 \\ u^-(x) & if \quad s(x) < 0, \quad \left|u^-(x)\right| \le u_0 \end{cases} \quad (11)$$

that makes the sliding surface (10) to be attractive.

3.2 First order sliding mode control for the synchronous machine.

To satisfy the control objective, rotor angle and velocity stability, the sliding surface will be designed using the block control technique (Loukianov, 1998). We define the control error as

$$\varsigma_2 = x_2 - \omega_b \quad (12)$$

The time derivative of (12) along (1), takes the form:

$$\dot{\varsigma}_2 = f_2(\mathbf{x_1}, \mathbf{x_2}, T_m) + b_2(\mathbf{x_1}, \mathbf{x_2})x_3 \quad (13)$$

where

$$f_2 = d_m T_m - \left(a_{22}x_4 h_1(\cdot) + a_{23}x_5 h_2(\cdot) + a_{24}x_6 h_1(\cdot) + a_{25}h_1(\cdot)h_2(\cdot)\right)$$

$b_2 = a_{21}h_2(\cdot)$, and $b_2(t)$ is a positive function of the time. So, choosing

$$x_3 = -b_2(\mathbf{x_1},\mathbf{x_2})^{-1}\left[f_2(\mathbf{x_1},\mathbf{x_2},T_m) + k_0\varsigma_2 - s_\omega\right] \quad k_0 > 0 \quad (14)$$

the right hand side of (13) becomes to be $-k_0\varsigma_2$. Then using (14), the switching surface can be defined as

$$s_\omega = b_2(\mathbf{x_1}, \mathbf{x_2})x_3 + f_2(\mathbf{x_1}, \mathbf{x_2}, T_m) + k_0(x_2 - \omega_b) = 0. \quad (15)$$

The projection motion on the subspace s_ω can be derived using (14) and (15) of the form

$$\dot{s}_\omega = f_s(\mathbf{x_1}, \mathbf{x_2}, T_m) + b_s(\mathbf{x_1}, \mathbf{x_2})e_{fd} \quad (16)$$

where f_s is a bounded function, $b_s(\cdot) = b_3 b_2(\cdot)$ and $b_s(t)$ is a positive function of the time. It is easy to see from (16) that the relative degree of the generator model with respect to sliding variable s_ω is one.

4. EXCITER DYNAMICS AND CHATTERING

For systems with first order sliding mode controllers, the presence of fast actuators of relative degree two or more caused the chattering. That is why it is necessary to design the controllers suppressing the chattering even in the presence of actuators.

The synchronous generator actuator is a rotating rectifier exciter with static voltage regulator, see Figure 2. Its model has the form (Sauer, 1998):

$$T_E \frac{de_{fd}}{dt} = -\left(K_E + S_E\right)e_{fd} + V_R \quad (17)$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_f - \frac{K_A K_F}{T_F} e_{fd} + K_A(V_{ref} - V_t) \quad (18)$$

$$T_F \frac{dR_f}{dt} = -R_f + \frac{K_F}{T_F} e_{fd} \quad (19)$$

where V_R is the exciter input, R_f is the rated feedback stabilizing transformer, T_E and K_E are the rotating exciter time constant and gain; T_F and K_F are the stabilizing transformer time constant and gain; T_A and K_A are the amplifier time constant and gain; and S_E is a saturation function.

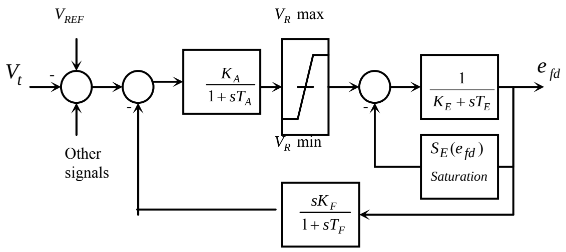

Figure 2. Rotating rectifier exciter with static voltage regulator.

We see that the relative degree of unmodelled exciter dynamics with respect to their output e_{fd} is two.

5. HIGH ORDER SLIDING MODE CONTROLLER

5.1 High order sliding mode (HOSM)

Consider a smooth dynamics system (8) with a smooth output function, $s(x) \in R$, and let the system be closed by some possibly-dynamical discontinuous feedback. The control task is to keep the output $s(x(t)) \equiv 0$. Then, provided that successive total time derivatives $s, \dot{s}, \ldots, s^{(r-1)}$ are continuous functions of the closed system state space variables, and the *r-sliding point set*

$$s = \dot{s} = \ddot{s} = \ldots = s^{(r-1)} = 0 \quad (20)$$

is non-empty and consists locally of Filippov trajectories, the motion on set (20) is called *r-sliding mode* (*rth-order sliding mode*) (Levant, 2003a).

HOSM presents even better robust performance than traditional first order sliding mode. HOSM dynamics converge toward the origin of surface coordinates in finite time always that the order of the sliding controller is equal or bigger than the sum of a relative degree of the plant and the actuator (Bondarev, 1985).

5.2 Third order sliding mode controllers (TOSM)

In our case of study the synchronous generator taking in the account the exciter dynamics, the complete plant model has relative degree three with respect to the designed output $s_0(\mathbf{x_1}, \mathbf{x_2})$ (15). Using (12) and (15) the plant dynamics is represented as

$$\dot{\delta} = \varsigma_2, \quad \dot{\varsigma}_2 = -k_0 \varsigma_2 + s_\omega \quad (21)$$

$$\dot{s}_\omega = s_1, \dot{s}_1 = s_2, \dot{s}_2 = \bar{f}_s(\mathbf{x_1}, \mathbf{x_2}, T_m) + \bar{b}_s(\mathbf{x_1}, \mathbf{x_2})u \quad (22)$$

where $\bar{f}_s$ is a bounded function, $\bar{b}_s(\cdot) = b_s(\cdot)/T_E T_A$.

So, to avoid the chattering we use the following third order sliding mode controller:

$$u = -\alpha\, sign\left(\ddot{s}_\omega + 2\left(|\dot{s}_\omega|^3 + |s_\omega|^2\right)^{1/6} sign\left(\dot{s}_\omega + |s_\omega|^{2/3} sign\, s_\omega\right)\right) \quad (23)$$

In (Levant, 2003a) it was show that if $\alpha_0 > 0$ then the state of the closed-loop system converges to the manifold $s_\omega = 0$, $s_1 = 0$, $s_2 = 0$ in a finite time. Then the sliding mode motion on this manifold is governed by the second order linear system (21)

$$\dot{\delta} = \varsigma_2, \quad \dot{\varsigma}_2 = -k_0 \varsigma_2$$

with desired eigenvalue $-k_0$.

To implement the proposed control (23) it is necessary to estimate variables s_1 and s_2

5.3 Exact robust differentiator

To estimate the derivatives s_1 and s_2 without its direct calculations, we will use the 2nd-order exact robust differentiator of the form (Levant, 2003a)

$$\dot{z}_0 = v_0 = -\lambda_0 |z_0 - s_\omega|^{2/3} sign(z_0 - s_\omega) + z_1$$

$$\dot{z}_1 = v_1 = -\lambda_1 |z_1 - v_0|^{1/2} sign(z_1 - v_0) + z_2$$

$$\dot{z}_2 = -\lambda_2\, sign(z_2 - v_1)$$

where z_0, z_1 and z_2 are the estimate of s_ω, s_1 and s_2, respectively, $\lambda_i > 0$, $i = 0,1,2$. Under condition $\lambda_0 > \lambda_1 > \lambda_2$ the third order sliding mode motion on will be established in a finite time. The obtained estimates, $z_1 = s_1 = \dot{s}_\omega$ and $z_2 = s_2 = \ddot{s}_\omega$ are then used in the controller (23).

6. SIMULATION RESULTS

The proposed control algorithms were tested on the sixth order model of synchronous generator connected through a transmission line to the infinite bus. The operation points of the machine were selected near the nominal values of reactive power and active power in order to give critical situations. The unmodeled dynamics of actuator was added to probe robustness of control structure and show the chattering effects.

6.1 Standard parameter of the machine

The parameters of the synchronous machine were obtained from data plate and test of the real generator. The parameters of the synchronous machine and external network in p.u. are:

$T'_{do} = 8.0$ sec. $T'_{qo} = 1.0$sec. $T''_{do} = 0.03$sec. $T''_{qo} = 0.07$sec. $L_d = 1.81$, $L'_d = 0.3$, $L''_d = 0.23$, $L''_q = 0.25$, $L_q = 1.76$, $L'_q = 0.6$, $L_{ext} = 0.1$, $R_{ext} = 0.001$. It allows us to obtain the parameters of model (7). The exciter parameter are: $K_A = 400$, $T_A = 0.02$,

$K_F = 0.03$, $T_F = 1$, $K_E = 1$, $T_E = 0.8$, $V_{r\max} = 7.25$, $V_{r\min} = -7.25$, $S_E \max = 0.86$. The steady state of the machine is calculated as $x_1(0) = 0.756535$, $x_2(0) = 376.9911$, $x_3(0) = 1.46803$, $x_4(0) = -0.63729$, $x_5(0) = 0.867576$, $x_6(0) = -0.637297$

6.2 Controllers

The controller used the gains $u_0 = 0.2$, $k_0 = 10$. Fort the TOSM controller, $\alpha = 0.12$. Parameters for the sliding differentiator were selected as, $\lambda_0 = 125$, $\lambda_1 = 115$, $\lambda_2 = 100$.

6.3 Results

Simulations of a 0.15 sec. short-circuit in the transformer terminals were used to evaluate the proposed sliding controllers. Figures show that in spite of the strong disturbance the controlled states hastily reach a steady state condition, exhibiting the stability of the closed loop system. Figure 3-6 depict that FOSM achieve the control goals even when the unmodelled dynamics of actuator are added, but chattering is presented. TOSM controller reduces the chattering. In Figure 7 is depicted the voltage generator with no change with FOSM or TOSM.

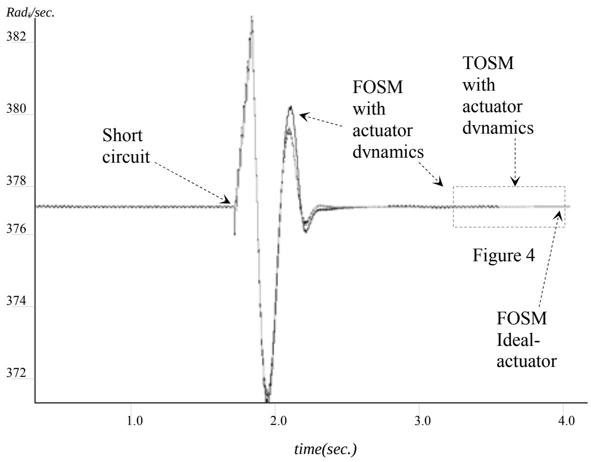

Figure 3. Angular velocity FOSM and TOSM.

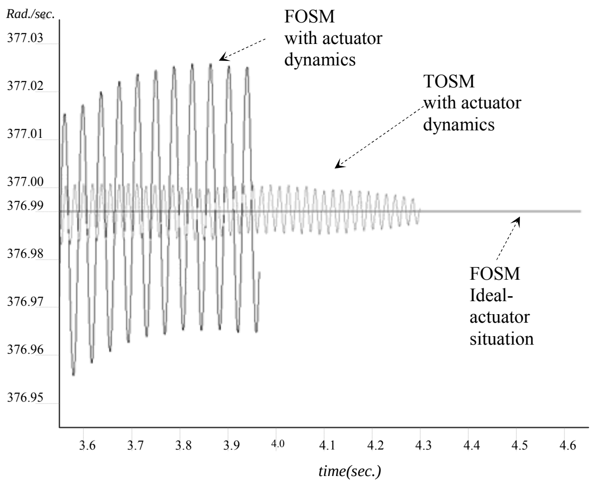

Figure 4. Angular velocity (zoom Figure 3).

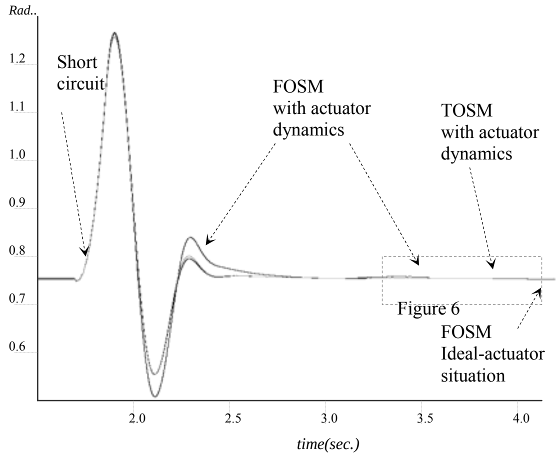

Figure 5. Power angle FOSM and TOSM.

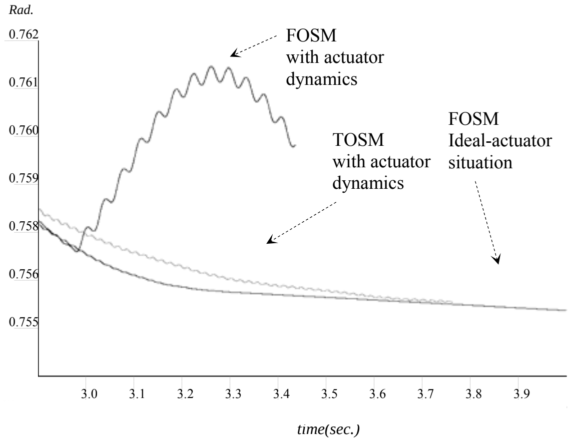

Figure 6. Power angle (Zoom Figure 5).

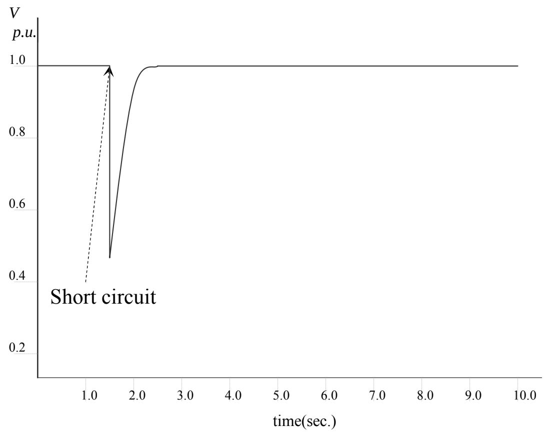

Figure 7. Generator voltage (FOSM and TOSM).

CONCLUSION

In this paper we have illustrated how the properties of block control technique can be combined with the robustness and simplicity of sliding mode. This combine controller design in the regulation of speed and voltage of a synchronous generator was used. However the presence of exciter dynamics causes chattering in SM control systems. It is shown that the usage of TOSM reduces the effects of chattering in the presence of exciter.

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APPENDIX

$X_j = \omega_b L_j$, $\ell_s, \ell_f, \ell_g, \ell_{kd}, \ell_{kq}$ are leakage inductances. L_{md} the d-axis mutual inductance, L_{mq} the q-axis mutual inductance, $L_d = L_{md} + \ell_s$, $L_q = L_{mq} + \ell_s$, $L_f = L_{md} + \ell_f$, $L_g = L_{mq} + \ell_g$,

$$L_d' = L_d - \frac{L_{md}^2}{L_f}, \qquad L_q' = L_q - \frac{L_{mq}^2}{L_g},$$

$$L_d'' = L_d' - \frac{\left(L_{md}'\right)^2}{L_{kd}'}, a_{21} = \frac{X_d'' - \ell_s}{X_d' - \ell_s}, L_q'' = L_q' - \frac{\left(L_{mq}'\right)^2}{L_{kq}'},$$

$$L_{kd}' = L_d' - \ell_s + \ell_{kd},\ L_{kq}' = L_q' - \ell_s + \ell_{kq},$$

$$a_{22} = \frac{X_q'' - \ell_s}{X_q' - \ell_s},\ a_{29} = \frac{1}{X_q''},\ a_{25} = X_q'' - X_d''$$

$$a_{28} = \frac{1}{X_d''},\ a_{23} = -\frac{X_d'' - X_d'}{X_d' - \ell_s},\ a_{24} = -\frac{X_q'' - X_q'}{X_q' - \ell_s},$$

$$a_{32} = \frac{1}{T_{do}'}\left[\frac{\left(X_d - X_d'\right)\left(X_d'' - X_d'\right)}{\left(X_d' - \ell_s\right)^2}\right], b_{33} = -\frac{1}{T_{qo}''}\left(X_q' - \ell_s\right)$$

$$a_{31} = \frac{1}{T_{do}'}\left[-1 - \frac{\left(X_d - X_d'\right)\left(X_d' - X_d''\right)}{\left(X_d' - \ell_s\right)^2}\right],\ b_3 = 1,$$

$$a_{33} = \frac{1}{T_{do}'}\left[-\left(X_d - X_d'\right) - \frac{\left(X_d - X_d'\right)\left(X_d'' - X_d'\right)}{\left(X_d' - \ell_s\right)}\right],$$

$$b_{11} = \frac{1}{T_{qo}'}\left[-1 - \frac{\left(X_q - X_q'\right)\left(X_q'' - X_q'\right)}{\left(X_q' - \ell_s\right)^2}\right], b_{21} = \frac{1}{T_{do}''},$$

$$b_{12} = \frac{1}{T_{qo}'}\left[-\frac{\left(X_q - X_q'\right)\left(X_q'' - X_q'\right)}{\left(X_q' - \ell_s\right)^2}\right],\ b_{22} = -\frac{1}{T_{do}''},$$

$$b_{13} = \frac{1}{T_{qo}'}\left[-\left(X_q - X_q'\right) - \frac{\left(X_q - X_q'\right)\left(X_q'' - X_q'\right)}{\left(X_q' - \ell_s\right)}\right],$$

$$b_{23} = -\frac{1}{T_{do}''}\left(X_d' - \ell_s\right), b_{31} = -\frac{1}{T_{qo}''},\ b_{32} = -\frac{1}{T_{qo}''},$$

$T_{do}', T_{do}'', T_{qo}', T_{qo}''$ are the d-axis or q-axis, transient, or subtransien time constants.