PRIMAL AND DUAL APPROACHES TO DISTRIBUTED CROSS-LAYER OPTIMIZATION ⋆

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Abstract: Several approaches for cross-layer design, e.g., coordinating the traditionally separated layers in wireless networks, have been proposed. However, protocols that are close to achieving the performance bounds are still lacking. We propose three distributed algorithms for joint congestion control and resource allocation in networks with variable capacities subject to a global resource constraint. Examples include spectrum assignment in wireless networks and wavelength allocation in optical networks. For scalability, we impose the additional constraint that nodes can only negotiate and exchange resources with their neighbors. The proposed algorithms consist of two complementary approaches based on decomposition techniques, in which congestion control and resource allocations are performed on different time-scales. Two of the algorithms can be shown to converge without network delays. *Copyright © 2005 IFAC*

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1. INTRODUCTION

In a desire to enhance performance of wireless networks, there has been a strong recent interest in methods for coordinating the traditionally separated networking layers. A range of optimization methods have been devised for evaluating the potential performance benefits of cross-layer designs, *e.g.*, (Johansson and Xiao, 2003) and the references therein, but there is still a shortage of protocols that come close to achieving these performance bounds. In a step towards these goals, we extend the approaches for analysis and design of congestion control for the fixed Internet, *e.g.*, (Low and Lapsley, 1999; Kelly *et al.*, 1998), to a class of networks with variable link capacities. We study how decomposition techniques can be used to devise protocols that jointly optimize end-to-end rates and resource allocation to maximize total network utility. The complexity of this problem depends on the structure of the resource constraints and the relationship between resource allocation and resulting capacities. When resource constraints are local to nodes, *e.g.*, a total power constraint at each node, it is relatively easy to derive a distributed solution. In this paper we consider the situation with a global resource constraint. Examples include time-slot or spectrum assignments in wireless networks and wavelength allocations in optical networks.

The key contributions of this paper are to demonstrate that the cross-layer optimization problem with a global resource constraint admits a dis-

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tributed solution, and to propose two complementary approaches for achieving the global optimum. The first approach is based on a dual decomposition technique. This method uses the classical source rate and congestion price updates of optimization flow control (Low and Lapsley, 1999) together with a resource management scheme (Ho *et al.*, 1980) where links exchange resources with their neighbors only. We demonstrate that this scheme converges to the optimum. However, our dual approach has the disadvantage that resource allocations have to be done at a fast time-scale, and that the resource allocation has to be performed to optimality. To complement the dual approach, we propose a distributed solution inspired by primal decomposition techniques. Contrary to the dual approach, this scheme relies on standard optimization flow control on the fast time-scale, while incremental resource re-allocations are performed on a slow time-scale. Using this approach we have proposed two different algorithms, a simple algorithm solving a relaxed version of the problem and a projection algorithm solving the full problem. The latter algorithm can be shown to converge to the global optimum.

2. PROBLEM STATEMENT AND NOTATION

We consider a network with L directed links shared by P sources. To each source, we associate an increasing and strictly concave function $u_p(s_p)$ which measures the utility source p has of sending at rate s_p. We assume that data is routed along fixed paths, represented by a routing matrix $R = [r_{lp}]$ with entries $r_{lp} = 1$ if source p uses link l and 0 otherwise. The aggregate communication rate on each link l is limited by its capacity $c_l(\varphi_l)$. We assume that the capacities are monotone increasing, concave functions, twice differentiable, and that there is a network-wide resource budget, *i.e.*, that $\sum_{l=1}^{L} \varphi_l \leq \varphi_{\text{tot}}$ and $\varphi_l \geq 0$. We would like to find the combination of source rates and resource allocation that maximizes the total network utility. Introducing the vectors $s = \begin{pmatrix} s_1 & \cdots & s_P \end{pmatrix}$, $\varphi = \begin{pmatrix} \varphi_1 & \cdots & \varphi_L \end{pmatrix}$, and $c(\varphi) = \begin{pmatrix} c_1(\varphi_1) & \cdots & c_L(\varphi_L) \end{pmatrix}$, this amounts to solving the following convex optimization problem

$$\begin{array}{ll} \text{maximize} & \sum_{p=1}^{P} u_p(s_p) \\ \text{subject to} & Rs \preceq c(\varphi), \qquad s_{\min} \preceq s \\ & \sum_{l=1}^{L} \varphi_l \leq \varphi_{\text{tot}}, \quad 0 \preceq \varphi \end{array} \tag{1}$$

in the variables s and φ. Our developments require two more technical assumptions: there should exist a resource allocation $\tilde{\varphi}$ such that $Rs_{\min} \prec c(\tilde{\varphi})$, and the routing matrix, R, should be such that all sources have data to send, and all links are at least used by one source. The first assumption simply states that the network should be able to support all sources when they use their minimum transmission rates; in a practical network, this will be taken care of by an admission control policy. As we will demonstrate in Proposition 1, the problem (1) is equivalent to

$$\begin{array}{ll} \text{maximize} & \sum_{p=1}^{P} u_p(s_p) \\ \text{subject to} & Rs \preceq c(\varphi), \qquad s_{\min} \preceq s \\ & \sum_{l=1}^{L} \varphi_l = \varphi_{\text{tot}}, \quad \varphi_{\min} \preceq \varphi \end{array} \tag{2}$$

which is the formulation we will use from now on.

Our problem formulation is rather general and different types of networks can be modelled in this way. For example, in wireless networks with Gaussian broadcast channels, the classical Shannon capacity formula gives that

$$c_l = W_l \log\left(1 + \frac{P_l}{\sigma_l W_l}\right) \tag{3}$$

where the adjustable parameters are W_l, the assigned bandwidth, and P_l, the power used in the link. Another case corresponds to distributed time sharing in a system operating under TDMA, for which the capacity is $c_l(\varphi_l) = \varphi_l c_{\text{tgt}}$

3. RELATED WORK

The optimization problem (2) is convex and is readily solved using centralized optimization techniques (*e.g.*, (Xiao *et al.*, 2004)). In this paper, we study how decomposition techniques can be used to devise distributed protocols to coordinate sources, routers and transmitters towards the optimal network operation. Although the use of decomposition techniques for devising distributed policies has a long history in economics (see, *e.g.*, (Arrow and Hurwicz, 1960)), the application to data networks is more recent Kelly *et al.* (1998) and Low and Lapsley (1999). Extensions to wireless networks can be found in, *e.g.*, (Xiao *et al.*, 2004; Neely *et al.*, 2003; Chiang, 2004).

Our solutions will make use of two distributed optimization algorithms from the literature: optimization flow control (Low and Lapsley, 1999) and center-free resource allocation (Ho *et al.*, 1980; Xiao and Boyd, 2003). Optimization flow control solves the problem of distributed utility maximization in networks with fixed link capacities, and is believed to be a relatively faithful abstraction of the operation of TCP/AQM in the fixed Internet. The center-free resource allocation algorithm considers distributed optimization under nearest neighbor communication constraints. These algorithms will now be briefly discussed.

3.1 Optimization flow control

Low and Lapsley (1999) address the problem of adjusting the communication rates between nodes in a network under fixed link capacity constraints. In our notation, the problem is

$$\begin{array}{ll} \text{maximize} & \sum_{p=1}^{P} u_p(s_p) \\ \text{subject to} & Rs \preceq c, \quad s_{\min} \preceq s \end{array} \tag{4}$$

The Lagrange dual function of this problem is

$$g(\lambda) = \max_{s_{\min} \preceq s} \left\{ \sum_{p=1}^{P} u_p(s_p) - \lambda^T(Rs - c) \right\} \tag{5}$$

$$= \max_{s_{\min} \preceq s} \left\{ \sum_{p=1}^{P} u_p(s_p) - s_p \sum_{l=1}^{L} r_{lp}\lambda_l \right\} + \lambda^T c \tag{6}$$

In the dual formulation, each link is associated with a "congestion price", λ_l (a Lagrange multiplier). A key observation is that sources can compute their optimal rate individually, based on the total congestion price $\sum_{l=1}^{L} r_{lp}\lambda_l$, using the source rate algorithm

$$s_p = \arg \max_{s_{\min} \leq s} u_p(z) - z \textstyle\sum_{l=1}^{L} r_{lp}\lambda_l \tag{7}$$

To solve the dual problem,

$$\begin{array}{ll} \text{minimize} & g(\lambda) \\ \text{subject to} & \lambda \succeq 0 \end{array}$$

one can use the projected gradient method

$$\lambda_l(t+1) = [\lambda_l(t) - \gamma(c_l - \textstyle\sum_{p=1}^{P} r_{lp}s_p)]^+$$

where $[\cdot]^+$ is the projection on the positive orthant and γ is the step length. Thus links can update their congestion prices individually, based only on knowledge of the local excess capacity.

3.2 Center-free resource allocation algorithms

The algorithm by Ho *et al.* (1980), Xiao and Boyd (2003) solves the resource allocation problem

$$\begin{array}{ll} \text{minimize} & \sum_{l=1}^{L} f_l(\varphi_l) \\ \text{subject to} & \sum_{l=1}^{L} \varphi_l = \varphi_{\text{tot}} \end{array} \tag{8}$$

under the assumptions that f_l are convex, twice continuously differentiable, with the second derivative bounded below, $m_i \leq f_l''(\varphi_l) \leq n_i$, with $m_i > 0$, n_i known. The optimality conditions are $\mathbf{1}^T\varphi^\star = \varphi_{\text{tot}}$ and $\nabla f(\varphi^\star) = \beta^\star \mathbf{1}$, with $\mathbf{1} = \begin{pmatrix} 1 & \ldots & 1 \end{pmatrix}^T \in \mathbb{R}^L$. Using the update rule

$$\varphi(t+1) = \varphi(t) - W\nabla f(\varphi(t)) \tag{9}$$

with $\mathbf{1}^T W = 0$, the new allocation $\varphi(t+1)$ will always be feasible. The limitation that links should only be allowed to communicate and exchange resources with its neighbors translates into a sparsity constraint on W. Ho *et al.* (1980) have shown that the distributed algorithm converges to the optimal solution if W is chosen to satisfy

$$\begin{array}{l} W \text{ is irreducible} \\ W_{ij} \leq 0 \\ W = W^T, W\mathbf{1} = 0 \\ \sum_{j \in \mathcal{N}(i)} |W_{ij}| < 1/m_{max},\ i = 1, ..., P \end{array} \tag{10}$$

where $\mathcal{N}(i)$ is the set of neighboring links to i. A simple way to satisfy these conditions is to use the *Metropolis* weights (Xiao and Boyd, 2003)

$$\begin{array}{l} W_{ij} = -\min\left\{ \dfrac{1}{|\mathcal{N}(i)|m_i}, \dfrac{1}{|\mathcal{N}(j)|m_j} \right\} + \epsilon, j \in \mathcal{N}(i) \\ W_{ii} = -\sum_{j \in \mathcal{N}(i)} W_{ij} \\ W_{ij} = 0, \text{ otherwise} \end{array}$$

where ϵ is a small positive constant. Ho *et al.* (1980) have extended the approach to handle non-negativity constraints on resources, $\varphi_l \geq 0$. This is accomplished by identifying the φ:s that will be zero at optimality and by finding a starting point that fulfills certain conditions. Details is provided in Ho *et al.* (1980), and it is always possible to do this initialization procedure.

We will also need to maximize a sum of concave functions, more specifically $\sum_l \lambda_l c_l(\varphi_l)$. This is done analogously, by identifying f_l with $-\lambda_l c_l(\varphi_l)$.

4. DISTRIBUTED OPTIMIZING PROTOCOLS BASED ON DECOMPOSITION

Inspired by the philosophy of Low and Lapsley (1999) to view network components as distributed processors solving the network wide utility maximization problem, we are now ready to look for similar solutions to (2). The main challenge is to coordinate source-rate selections with the resource allocations under network-wide resource constraint. We will develop two complementary approaches, based on primal and dual decomposition techniques. Note that primal and dual in this paper have their mathematical programming meaning, and they do not indicate wether the congestion control algorithms are running at the sources or on the links. While the dual approach is similar to the classical approach for fixed networks, the use of primal decomposition techniques appears to be novel in this context.

To get some insight into the structure of the optimal solution, we apply the Karush-Kuhn-Tucker (KKT) conditions to the problem (1).

Proposition 1. Under the assumptions made in Section 2, the optimal solution to (1) satisfies

$$
\begin{aligned}
&Rs^\star = c(\varphi^\star), \quad s_{\min} \preceq s^\star \\
&\sum_{l=1}^{L} \varphi_l^\star = \varphi_{\text{tot}}, \quad \varphi_{\min} \preceq \varphi^\star \\
&\lambda_l^\star c_l'(\varphi_l^\star) = \mu^\star \quad \forall l
\end{aligned}
$$

where $\varphi_{\min} = \min_l c_l^{-1}(s_{\min}) > 0$.

Proof. Omitted. □

It is interesting to see that all links become bottlenecks and that all resources are used. This is not the case for networks with fixed capacities. Also note that $\lambda_l^\star c_l'(\varphi_l^\star)$ will be equal for all links.

4.1 Dual decomposition

Introducing Lagrange multipliers λ_l, $l = 1, \dots, L$ for the capacity constraints in (2), we form the partial Lagrangian

$$
L(s, \varphi, \lambda) = \left\{ \sum_p u_p(s_p) - \lambda^T (Rs - c(\varphi)) \;\middle|\; \begin{array}{c} s_{\min} \preceq s \\ \sum_l \varphi_l = \varphi_{\text{tot}} \\ \varphi_{\min} \preceq \varphi \end{array} \right\}
$$

and the associated dual function

$$
g(\lambda) = \sup_{s,\varphi} \{L(s, \varphi, \lambda)\} = \underbrace{\sup_{s_{min} \preceq s} \left\{ \sum_p u_p(s_p) - \lambda^T Rs \right\}}_{\text{Network}} + \underbrace{\sup_{\substack{\sum_l \varphi_l = \varphi_{\text{tot}} \\ \varphi_{\min} \preceq \varphi}} \lambda^T c(\varphi)}_{\text{Resource allocation}}
$$

Thus, the dual function decomposes into a network subproblem and a resource allocation subproblem. The network subproblem is identical to the source algorithm in optimization flow control, while the second subproblem can be dealt with using the center-free algorithm. The corresponding Lagrange dual problem is

$$
\begin{aligned}
&\text{minimize} \quad g(\lambda) \\
&\text{subject to } \lambda \succeq 0
\end{aligned}
$$

If we assume that the link capacities are strictly concave, then the partial Lagrangian is strictly concave in (s, φ) and the dual function is differentiable (Bertsekas, 1999, Proposition 6.1.1) with

$$
\nabla g(\lambda) = c(\varphi^\star(\lambda)) - Rs^\star(\lambda)
$$

The dual variables can be updated using a projected gradient method

$$
\lambda_l(t+1) = [\lambda_l(t) - \gamma(c_l(\varphi_l^\star(t)) - \sum_{p=1}^{P} r_{lp} s_p^\star(t))]^+ \tag{11}
$$

This update can be carried out locally by links based on their current excess capacities. Convergence of this scheme follows similarly to the proof in Low and Lapsley (1999).

The dual algorithm:

While (not optimal) {
- Find φ by solving the resource allocation problem using the method in Section 3.2
- Find s by solving the network subproblem using the source algorithm (7)
- Use these optimal values to compute the gradient and update λ with (11) }

Note that the optimal resource allocation and source rates can be found in parallel, but the optimal solutions to both subproblems are found before the dual variables are updated. From a practical perspective, this approach has the disadvantage that resource allocations have to be done at a fast time-scale and that the resource allocation algorithm (at least in the most basic analysis) has to be executed to optimality before the dual variables can be updated.

4.2 Primal decomposition

To complement the dual approach, we will now develop a distributed solution in which optimization flow control is carried out on a fast time-scale, while resource re-allocations are performed on a slower time-scale. To this end, we re-write (2) as

$$
\begin{aligned}
&\text{maximize} \quad \nu(\varphi) \\
&\text{subject to} \sum_{l=1}^{L} \varphi_l = \varphi_{\text{tot}}, \quad \varphi_{\min} \preceq \varphi
\end{aligned} \tag{12}
$$

where we have introduced

$$
\nu(\varphi) = \sup_{s_{\min} \preceq s} \left\{ \sum_p u_p(s_p) | Rs \preceq c(\varphi) \right\} \tag{13}
$$

Note that $\nu(\varphi)$ is simply the optimal network utility that can be achieved by optimization flow control under resource allocation φ. Consequently, to evaluate $\nu(\varphi)$ we can simply fix the resource allocation and execute the distributed algorithm presented in Section 3.1.

Before attempting to solve the problem (12), we will establish some basic properties of $\nu(\varphi)$.

Proposition 2. $\nu(\varphi)$ is concave and a supgradient, $h(\varphi)$, of $\nu(\varphi)$ at φ is given by

$$
h(\varphi) = \begin{pmatrix} \lambda_1^\star c_1'(\varphi_1) & \cdots & \lambda_L^\star c_L'(\varphi_L) \end{pmatrix}
$$

where λ_l are optimal Lagrange multipliers for the capacity constraints in (13).

Proof By strong duality,

$$
\begin{aligned}
\nu(\varphi) &= \inf_{\lambda \succeq 0} \sup_{s_{\min} \preceq s} \sum_{p=1}^{P} (u_p(s_p) - s_p q_p) + \sum_{l=1}^{L} \lambda_l c_l(\varphi_l) \\
&= \inf_{\lambda \succeq 0} \tilde{g}(s(\lambda)) + \sum_{l=1}^{L} \lambda_l c_l(\varphi_l)
\end{aligned}
$$

with $q_p = \sum_{l=1}^{L} r_{lp}\lambda_l$. Thus, since $\nu(\varphi)$ is the pointwise infimum of concave functions, it is concave.

Let $\lambda^\star$ be the optimal Lagrange multipliers for a resource allocation vector φ. For any other resource allocation $\tilde{\varphi}$, it holds that

$$\nu(\tilde{\varphi}) \le \sup_{s_{\min}\preceq s}\left\{\sum_p u_p(s_p) - s_p q_p^\star + \sum_{l=1}^{L}\lambda_l^\star c_l(\tilde{\varphi}_l)\right\}$$
$$\le \nu(\varphi) + \sum_{l=1}^{L}\lambda_l^\star\{c_l(\varphi_l) + c_l'(\varphi_l)(\tilde{\varphi}_l - \varphi_l) - c(\varphi_l)\}$$
$$= \nu(\varphi) + \sum_{l=1}^{L}\lambda_l^\star c_l'(\varphi_l)(\tilde{\varphi}_l - \varphi_l)$$

with $q_p^\star = \sum_{l=1}^{L} r_{lp}\lambda_l^\star$. This, by the definition of a supgradient, concludes the proof. □

4.2.1. Relaxed primal algorithm To get a simple solution the problem can be relaxed to only demand $\sum \varphi = \varphi_{\text{tot}}$. Starting at any primal feasible point φ satisfying $\sum_l \varphi_l = \varphi_{\text{tot}}$, we propose to use the distributed resource update law

$$\varphi(t+1) = \varphi(t) - \alpha W h(\varphi(t)) \qquad (14)$$

where W is chosen in the same way as in Section 3.2 and α is a steplength parameter. Although $\nu(\varphi)$ is not immediately separable, the gradient can be evaluated locally by links, and the updates involve resource exchanges between neighboring links. Although this algorithm perform consistently well in simulations, we have at this point not been able to establish theoretical convergence.

The relaxed primal method:

While (not optimal) {
- Find the optimal s by solving the optimization flow problem using the method in Section 3.1
- Use these optimal values to compute the supgradient and update φ with (14) }

4.2.2. Primal algorithm Since a supgradient of ν is available, it is natural to use a projected supgradient algorithm

$$\varphi_{t+1} = [\varphi_t + \alpha h(\varphi(t))]^+ \qquad (15)$$

with diminishing stepsize, α. Here $[\cdot]^+$ denotes distributed projection, *i.e.*, solves the following projection problem in a distributed fashion

$$\begin{aligned} &\text{minimize} \quad ||\varphi - \varphi^0||_2^2 \\ &\text{subject to} \sum_{l=1}^{L}\varphi_l = \varphi_{\text{tot}}, \quad \varphi_{\min} \preceq \varphi \end{aligned} \qquad (16)$$

The projection problem has a separable objective function and it can be solved with the center-free algorithm. The primal algorithm can be shown to converge in the limit.

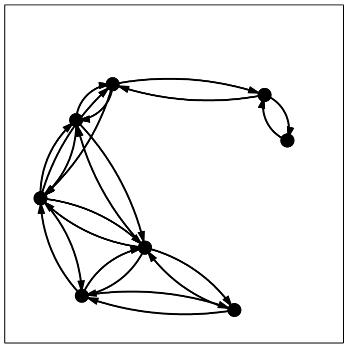

Fig. 1. The wireless network. The arcs denote directed wireless links, the dots denote nodes.

The primal method:

While (not optimal) {
- Find the optimal s by solving the optimization flow problem using the method in Section 3.1
- Use these optimal values to compute the supgradient
- Execute distributed projection and use this to update φ with (15) }

To summarize: The primal methods rely on solving the optimization flow problem on a fast time-scale and performing incremental updates of the resource allocation in an ascent direction of the total network utility on a slower time-scale. The source rate and link price updates are carried out in a distributed way, similarly to optimization flow control, while the resource updates are based on resource exchanges between neighboring links.

5. EXAMPLE

We now demonstrate the algorithms on the sample network shown in Figure 1. The network has been generated by placing 8 nodes randomly in a unit square, and introducing direct links between all nodes which are within a distance d of each other. The value of d has been chosen to be as small as possible while guaranteeing that the network is strongly connected. The link capacities are

$$c_l(\varphi_l) = \varphi_l \log\left(1 + \frac{\gamma_l}{\varphi_l}\right) \qquad (17)$$

where $\gamma_l = \frac{1}{d_l^2}$ and d_l is distance between the communicating nodes. This is a special case of (3). The resource limits $\varphi_{\min}$ and φ_{tot} are set to 0.1 and 10 respectively. The utility functions are $u_p(s_p) = \log(s_p)$, which corresponds to proportional fairness (Kelly *et al.*, 1998). The minimum source rate, $s_{\min}$, was set to 10^{-6}. The example problem was solved with the dual, relaxed primal, and primal methods. The step lengths were approximately tuned to optimize the convergence rate for the three algorithms. The norm of the differences between the current and optimal values

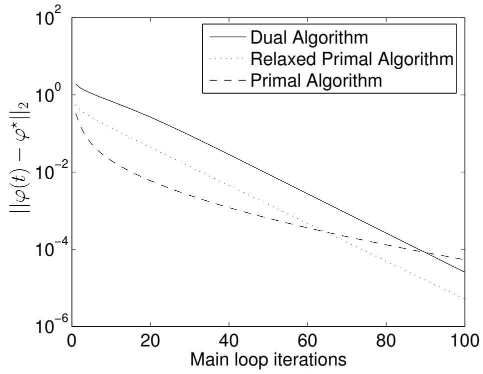

Fig. 2. The norm of the resource allocation minus the optimal resource allocation versus main loop iterations.

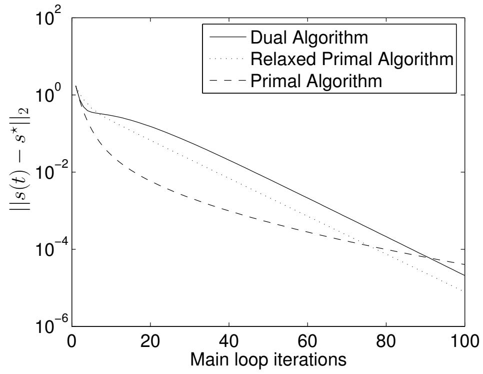

Fig. 3. The norm of the source rates minus the optimal source rates versus main loop iterations.

the decision variables versus number of main loop iterations are shown in Figure 2 for φ and Figure 3 for s. All methods converge in the limit, as predicted by theory for the dual method and the primal method. The dual method seems to be relatively sensitive with respect to the initial point, and can exhibit significantly worse performance than shown in the figures. The primal method shows rapid initial convergence, but slows down in the end.

6. CONCLUSIONS

In this paper we have considered distributed utility maximization of a wireless network under variable capacity constraints. Based on decomposition techniques from mathematical programming, we have proposed a primal and a dual approach, which rely on the solution of utility network maximization under fixed capacity constraints and center-free resource allocation. The dual approach yields an algorithm that solves the full problem using a gradient projection method. The primal approach yielded two algorithms, the relaxed primal algorithm that solves a relaxation of the original problem using a subgradient method, and the primal algorithm that solves the full problem using a projected subgradient method.

We are currently investigating the convergence properties of the relaxed primal algorithm. A natural next step is to complement the equilibrium analysis presented in this paper with an analysis of the dynamical properties of the proposed protocols.

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