AN ELECTROMECHANICAL SIGMA-DELTA MODULATOR FOR MEMS GYROSCOPE

Byung Su Chang*, Jang Gyu Lee*, Taesam Kang **, and Woon-Tahk Sung*

** School of Electrical Engineering and Computer Science,*
Seoul National University, Seoul, 151-744, Korea
*** Department of Aerospace Engineering,*
Konkuk University, 143-701, Seoul, Korea

Abstract: This paper presents a design and analysis of electromechanical sigma-delta modulator for MEMS gyroscope, which enables us to control the proof mass and to obtain an exact digital output without additional A/D conversion. The system structure and the circuit realization of the sigma-delta modulation are simpler than those of the analog sensing and feedback circuit. Based on the electrical sigma-delta modulator theory, a compensator is designed to improve the closed loop performance of the sensor. With the designed compensator, a closed loop performance of the gyroscope is enhanced to have wider bandwidth, larger dynamic range and direct digital output compared with those of an analog open loop system.

Keywords: Gyroscope, MEMS, Sigma-delta modulator, Inertial sensor, INS

1. INTRODUCTION

The inertial sensors are basic elements that compose Inertial Navigation System (INS). The gyroscope is basic inertial sensor, which can measure external angular rate. The MEMS gyroscope is an inertial angular rate sensor fabricated using MEMS technology. It has many good properties: it is small, light, and low power consuming. Moreover, it can be mass-produced inexpensively. Therefore, it has many applications in various fields.

The gyroscope is a high-Q system, which results in small bandwidth and has theoretically nonlinear characteristics. When external angular rate is applied to the MEMS gyroscope, the proof mass vibrating at resonant frequency is forced to vibrate in orthogonal direction due to the Coriolis force. The angular rate can be estimated by measuring the amplitude of the orthogonal oscillation. However, in this open loop case, the dynamic range of gyroscope is limited. Furthermore, the nonlinearity becomes larger as the amplitude of the orthogonal oscillation becomes larger (Sung, *et al.*, 2004). To overcome these disadvantages, a closed loop controller named rebalance loop can be used. A rebalance loop is a kind of feedback controller that makes the orthogonal oscillation be small. The magnitude of the feedback control is proportional to the Coriolis force due to the angular rate input. Therefore, the control input is used for the measurement of external angular rate input. Furthermore, the bandwidth of the gyroscope can be made large by using suitable compensator.

Most of feedback control technique is analog control. Analog feedback has good performance and high sensitivity. However, the implementation of the high performance A/D converter, which is necessary for the navigation systems, is very difficult. Therefore, sigma-delta modulation technique, which has been widely used in high-resolution A/D and D/A converters (Norsworthy, *et al.*, 1997), has been studied to realize high-performance MEMS sensor systems because it gives precise digital output without A/D converter. It is especially attractive in sensor applications where the signals are relatively narrow-band and a large over-sampling ratio can be easily achieved.

Electromechanical sigma-delta feedback, however, is different from electrical sigma-delta modulator. Electrical sigma-delta modulator consists of pure electric signals, but electromechanical sigma-delta feedback is a very complicated mixed signal feedback system. It has building blocks in multiple domains, such as mechanical and electrical, continuous-time and discrete-time, analog and digital, as well as linear and nonlinear. Mapping such an electromechanical sigma-delta feedback into pure electrical sigma-delta modulator will be helpful in understanding and designing. After system mapping, the close loop system is well designed using the established theory of conventional sigma-delta modulator.

In this paper, model equations of MEMS gyroscope are obtained and an electromechanical sigma-delta modulator is designed via electrical modulator theory. A simulation using MATLAB® Simulink® is performed to verify the performance of the closed loop rebalance loop with designed modulator. The results show the designed rebalance loop using modulator improves bandwidth as well as it gives digital output signal, directly.

2. MODEL EQUATION OF MEMS GYROSCOPE

MEMS gyroscope is generally composed of a mass and beams that is designed to oscillate. The mass readily oscillates in two orthogonal directions. A simple model of a vibrating gyroscope is shown in Fig. 1.

The governing equations of the vibratory gyroscope can be expressed using equations (1)-(6). To use Coriolis force, the proof mass is driven at its resonance frequency. When the proof mass is driven by the electrostatic force or $F_{driving}(t)$ along the x-axis (driving axis), the equation of the dynamic motion of the mass can be simply expressed as a second order mass-damper-spring system.

$$M_x\ddot{x}(t)+C_x\dot{x}(t)+K_x x(t)=F_{driving}(t) \quad (1)$$

Where M_x is the mass of the moving plate, C_x, the damping coefficient of the air-damper and K_x is the spring constant along the driving axis. Then the transfer function from the driving force to the displacement of the plate along x-axis is given by

$$G_{drive}(s)=\frac{X(s)}{F(s)}=\frac{1/M_x}{s^2+\frac{C_x}{M_x}s+\frac{K_x}{M_x}} \quad (2)$$

If the angular rate input $\Omega_z(t)$ is applied along z-axis (input axis), Coriolis force is induced, which is given by (3).

$$F_{coriolis}(t)=2M_x\dot{x}(t)\Omega_z(t) \quad (3)$$

Then, the equation of the dynamic motion along y-axis (sensing axis) is obtained by the same manner in x-axis case. The dynamic equation and the transfer function of the sensing mode can be expressed as (4) and (5), respectively.

$$M_y\ddot{y}(t)+C_y\dot{y}(t)+K_y y(t)=F_{coriolis}(t) \quad (4)$$

$$G_{sense}(s)=\frac{Y(s)}{F_{coriolis}(s)}=\frac{1/M_y}{s^2+\frac{C_y}{M_y}s+\frac{K_y}{M_y}} \quad (5)$$

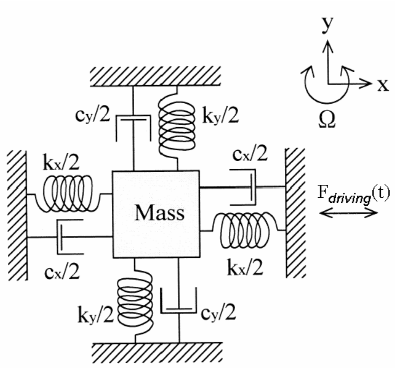


Fig. 1. Simple model of MEMS vibrating gyroscope

3. SIGMA-DELTA MODULATOR

3.1 Electrical sigma-delta modulator theory

Sigma-delta modulation technique was first introduced by Inose and Yasuda (Inose and Yasuda, 1963) and it became more popular for using as a high resolution A/D and D/A converter. Since this system contains a delta modulator and an integrator, they are named sigma-delta modulator, where 'sigma' denoted the summation performed by integrator. The sigma-delta modulator consists of a coarse (low resolution) quantizer preceded by a filter, both embedded into a negative feedback loop. It uses oversampling and quantization error shaping to achieve high accuracy. In other words, it trades speed for resolution, and analog circuit accuracy for simple digital circuit.

To rigorously analyze a sigma-delta modulator in the frequency domain is a difficult task due to the presence of the nonlinear quantizer. If quantization error is uncorrelated with the input signal and is uniformly distributed in quantizer output band, and is an independent identically distributed (iid.) sequence, then this analysis can be simplified as the input independent additive white noise approximation for the quantization error (Norsworthy, *et al.*, 1997; Kiss, 2002). Therefore, the deterministic nonlinear system is replaced by a linear stochastic system. The linearized model of the sigma-delta modulator is presented in Fig. 2. Then, transfer function from input $U(z)$ and quantization error $E(z)$ to output $V(z)$ are expressed as (6)

$$V(z)=\left(U(z)-V(z)\right)H(z)+E(z)$$
$$V(z)=\frac{H(z)}{1+H(z)}U(z)+\frac{1}{1+H(z)}E(z) \quad (6)$$

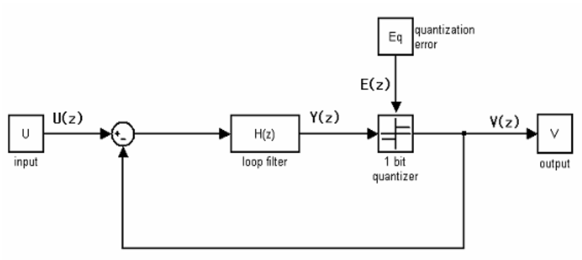


Fig. 2. Linearized model of sigma-delta modulator

From (6), it results that the sigma-delta modulator process is independently separated into the signal and the noise components. Therefore, its signal transfer function, *STF(z)* and noise transfer function, *NTF(z)* can be defined as follows.

$$STF(z) = \left.\frac{V(z)}{U(z)}\right|_{E(z)=0} = \frac{H(z)}{1+H(z)}$$

$$NTF(z) = \left.\frac{V(z)}{E(z)}\right|_{U(z)=0} = \frac{1}{1+H(z)} \qquad (7)$$

Also, one can write the output signal *V(z)* as the combination of the input signal, *U(z)* and the quantization noise signal, *Q(z)*. Each of them is filtered by the corresponding transfer function,

$$V(z) = STF(z)U(z) + NTF(z)Q(z). \qquad (8)$$

If one chooses a low-pass loop filter *H(z)*, which has large magnitude over the low frequencies of interest signal band (also called "signal band" or "in band" or "baseband"), and small magnitude (large attenuation) over high frequencies (also called "out of band"), then the magnitude of the signal transfer function |*STF(z)*| will be approximated into unity over the signal band, hence it will not distort the signal. However, the magnitude of the noise transfer function, |*NTF(z)*| will be approximated into zero over the same band, hence the quantization noise will be reduced significantly. By doing so, the signal-band spectral composition of the analog input and digital output signals will be linearly related, but outside the signal band, the spectral compositions of them will differ substantially. An example of frequency response of sigma-delta modulator is shown Fig. 3.

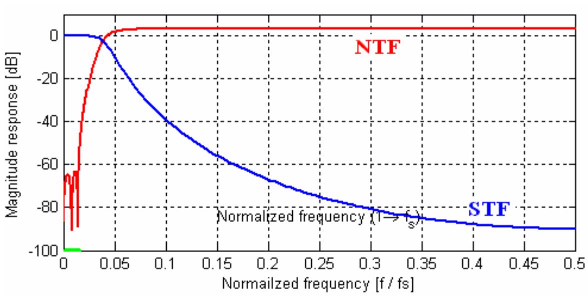


Fig. 3. STF and NTF of conventional sigma-delta modulator

3.2 Electromechanical force feedback by sigma-delta modulator

The block diagram of the sigma-delta feedback loop is shown in Fig. 4. The mechanical transducer is embedded in the loop. In addition, a compensator for improving performance is added in the loop. This modulator is represented in multiple domains, different from the electrical case. Position sensing part converts the displacement to the voltage signal, and actuator in feedback loop converts the voltage signal to the feedback force.

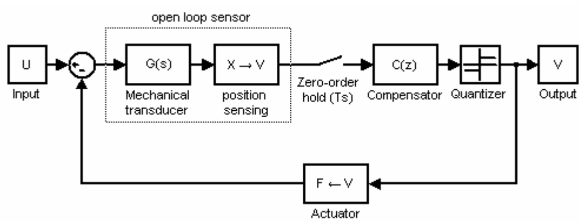


Fig. 4. Block diagram of electromechanical sigma-delta modulator

For easy analysis, mapping of electromechanical sigma-delta modulator into a pure electrical system will be helpful in understanding and designing closed loop gyroscope system. The mechanical transducer, which is embedded in the loop, has 2nd order transfer function. That is represented as (5). Since sigma-delta modulator is discrete system, it is necessary to convert continuous transfer function into discrete one. Assume that sampling period is Ts and sampling model is zero-order-hold model (Oppenheim and Schafer, 1999), then the discrete transfer function of mechanical transducer is represented as (9).

$$\begin{aligned} G(z) &= \frac{z-1}{z}\cdot Z\left\{\frac{1}{s}\cdot G(s)\right\} \\ &= \frac{1}{K_y}\cdot\frac{Az+B}{z^2-2e^{-aT_s}\left(\cos bT_s\right)+e^{-2aT_s}} \end{aligned}$$

$$\begin{cases} a = \dfrac{C_y}{2M_y}, \quad b = \sqrt{\dfrac{K_y}{M_y}-a^2} \\ A = 1-e^{-aT_s}\cos bT_s - \dfrac{a}{b}e^{-aT_s}\sin bT_s \\ B = e^{-2aT_s} + \dfrac{a}{b}e^{-aT_s}\sin bT_s - e^{-aT_s}\cos bT_s \end{cases} \qquad (9)$$

Because bandwidth of position sensing part and actuator are much higher than mechanical structure, the position sensing part and the actuator are modelled as a DC gain block, K_s and K_f respectively. Then, a loop filter of electromechanical sigma-delta modulator is expressed as (10).

$$G(z)\cdot K_s\cdot C(z) \qquad (10)$$

The linearized model of the electromechanical sigma-delta modulator is presented in Fig. 5. Therefore, the NTF and STF are calculated as (11), (12).

$$NTF(z) = \frac{1}{1+K_sK_fG(z)C(z)} \qquad (11)$$

$$STF(z) = \frac{K_s G(z) C(z)}{1 + K_s K_f G(z) C(z)} \qquad (12)$$

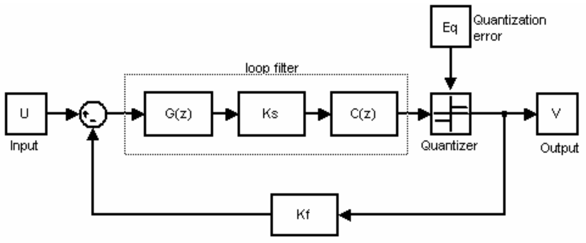


Fig. 5. Linearized model of electromechanical sigma-delta modulator

4. DESIGN OF ELECTROMECHANICAL SIGMA-DELTA MODULATOR

Now, a compensator block is designed to improve the resolution of the closed loop system. First of all, an interested signal band should be determined. In MEMS gyroscope system, driving signal is sinusoidal wave whose frequency is a resonance frequency of the mass-damper-spring system. Then the external angular rate signal is modulated by the driving signal. Since the bandwidth of conventional gyroscope is about 100Hz, the interested signal band is around the resonance frequency with additional ±100Hz.

4.1 Bandpass sigma-delta modulator theory

In the case of ordinary sigma-delta modulator, the interested signal band is a low frequency region. Therefore, the loop filter of the modulator has a frequency response characteristic like a low pass filter. In this case, the interested signal band is around the resonance frequency. Therefore, the loop filter of this case has a bandpass property.

Bandpass sigma-delta modulators operate in much the same manner as conventional (low pass) modulator. The design of bandpass converters has much in common with low pass modulator design (The magnitude of the signal transfer function $|STF(z)|$ will be approximated into unity over the signal band, the magnitude of the noise transfer function $|NTF(z)|$ will be approximated into zero over the same band). An example of frequency response of bandpass sigma-delta modulator is shown in Fig. 6.

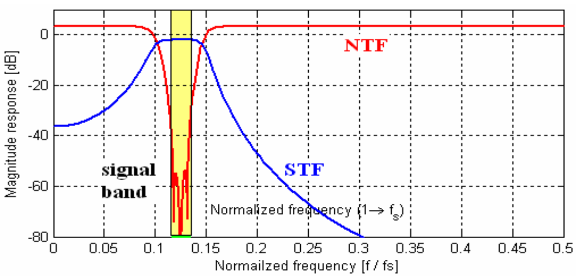


Fig. 6. STF and NTF of bandpass sigma-delta modulator

4.2 Design of compensator

First, NTF synthesize process is accomplished. To reduce quantization noise around the resonance frequency, NTF is designed as band reject filter via a generalized filter approximator/optimizer (Jantzi, *et al.*, 1994). The optimizer adjusts the poles and zeros of NTF such that its magnitude of frequency response closely matches the measures of ideal value. In this paper, the 6^{th} order NTF is designed. Fig. 7 represents Bode plot of designed NTF. It shows that the designed NTF rejects the quantization noise in signal band.

Then, compensator transfer function *C(z)* is calculated by (11). Table 1 shows the parameters of the MEMS gyroscope. The Bode plot of designed compensator is shown in Fig. 8. After obtaining the compensator, STF is calculated by (12). The bode plot of closed loop system and open loop system. From Fig. 9, the displacement of the mass along sensing axis is regulated well. Moreover, Fig. 10 shows that the closed loop system has wider bandwidth and lower phase distortion then the open loop case.

Table 1 Parameters of MEMS gyroscope

Parameter	Value
Quality factor	150
Resonant frequency	19540.7 rad/sec
Mass	9.1595×10^{-9} Kg
Spring constant K_y	3.475 N/m
Damping coef. C_y	1.1894×10^{-6} N/m/sec
K_s	1.5×10^{8} V/m
K_f	2.5×10^{-9} N/V

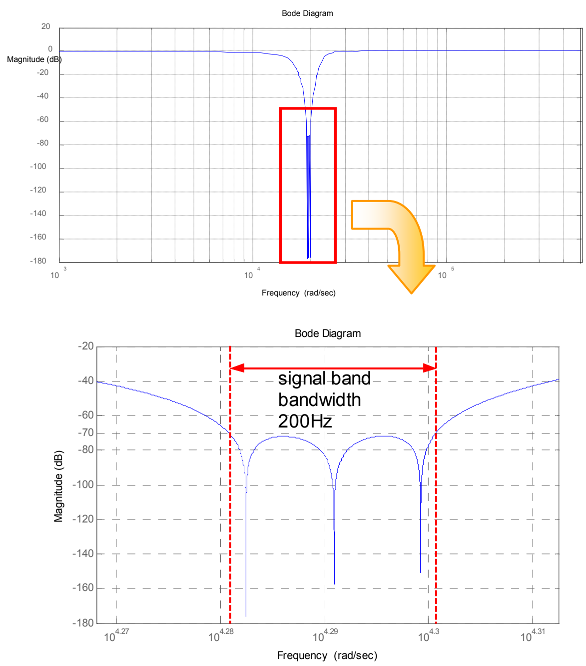


Fig. 7. Bode plot of designed NTF

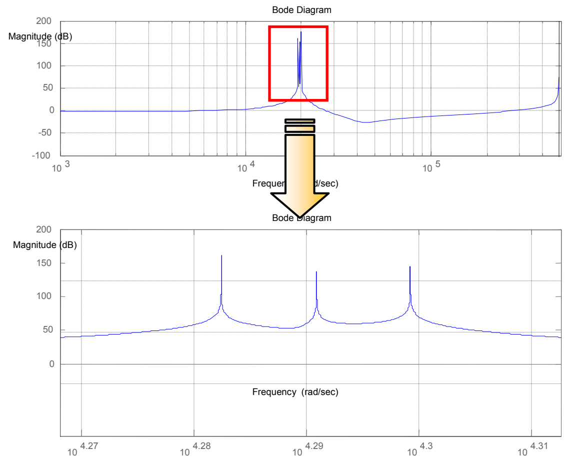


Fig. 8. Bode plot of designed compensator

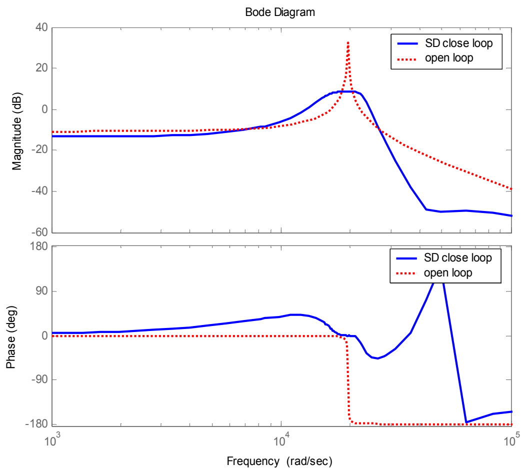
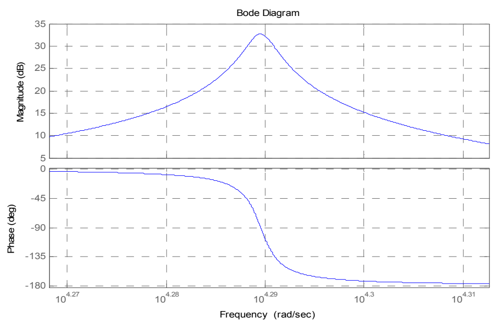
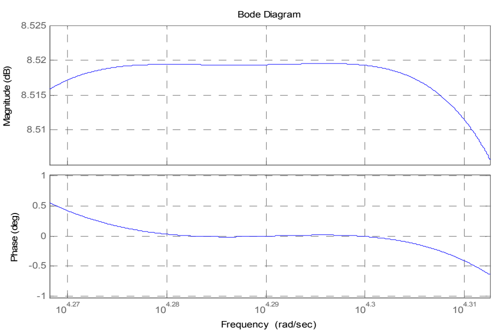


Fig. 9. Bode plot of open loop and closed loop system



(a) Open loop



(b) Closed loop

Fig. 10. Bode plot of open loop and closed loop system: magnified view around the signal band

5. DECIMATION FILTER AND DEMODULATOR

5.1 Decimation filter

The designed sigma-delta modulator is only half of the whole gyroscope system. Because the modulator output signal is not external angular rate signal but oversampled 1-bit digital pulse signal, the signal is digitally converted to the desired form through a digital filtering and resampling process called decimation.

To reconstruct the digitized signal with high SNR, the out-of-band quantization noise must be rejected. If modulator design process is accomplished well, the quantization noise in the signal band is minimized. It is then left to decimator to filter out the unwanted noise in out-of-band region so that it is not aliased into the signal band by the decimation process.

Let us then discuss traditional decimator design method such as sinc^K filter. This method is very attractive for implementation because it doesn't require multiplier, but adders and registers only (Hoegenauer, 1981; Norsworthy, *et al.*, 1997). The transfer function for sinc^K decimation filter has the general form as (13),

$$H_{deci} = \left(\frac{1 - z^{-R}}{1 - z^{-1}} \right)^K \qquad (13)$$

where R is decimation factor which represents resampling rate to remove all signal energy above π / R frequency range. K is the order of the sinc^K filter. In order to prevent aliasing, K should be at least 1 plus the order of the modulator.

The most efficient implementation is formulated by cascading K stages of accumulators operation at the high sampling rate, followed by K stages of cascaded differentiators operating at the low sampling rate (Dijkstra, *et al.*, 1988). Fig. 11 shows a block diagram of the 3rd order sinc^K filter.

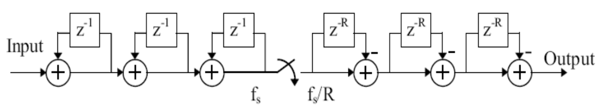


Fig. 11. Block diagram of the 3rd order sinc^K decimation filter

5.2 Demodulator

After the decimation process, we obtain sensing axis displacement signal, which is converted to digital form. Since the displacement along sensing axis due to the Coriolis acceleration is modulated by the driving signal, the demodulation process is essentially needed to extract the external angular rate signal. Because the modulated signal is converted to digital form, demodulator can be implemented in software. In addition, analog electronic parts such as analog multiplier and phase shifter are not needed.

6. SIMULATION RESULT

Using the designed system and parameters of the gyroscope, a simulation is performed by MATLAB® Simulink®. Fig. 12 shows displacement output of mass when the gyroscope operates in open loop. That is the form, whose input angular rates are modulated sinusoidal driving signal.

Fig. 13 represents displacement output when the gyroscope operates with electromechanical sigma-delta feedback. First plot shows output with compensator and second plot shows output without compensator. From the result, it is confirmed the compensator much enhances the performance of the sigma-delta feedback loop.

Fig. 14 shows that the output of whole gyroscope system when 10 deg/sec step input is applied. From this result, decimation filter and demodulator operate as desired.

7. CONCLUSION

In this study, an electromechanical sigma-delta modulator for MEMS gyroscope is proposed, which enables us to control the proof mass and to obtain a digital output directly. Based on electrical sigma-delta modulator theory, designed compensator improves closed loop performance such as larger bandwidth and digital output comparing with the results of analog open-loop system. Moreover the dynamic range of gyroscope is improved, since displacement along sensing axis is well regulated.

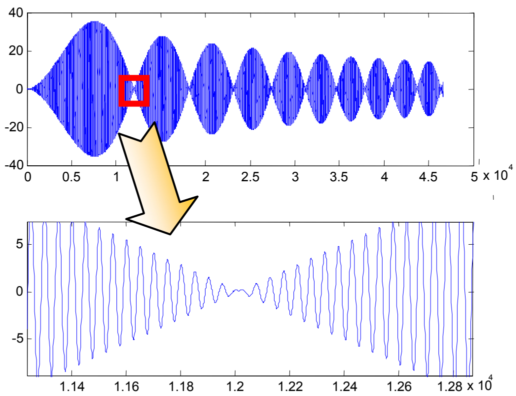


Fig. 12. Displacement of the mass in open loop system

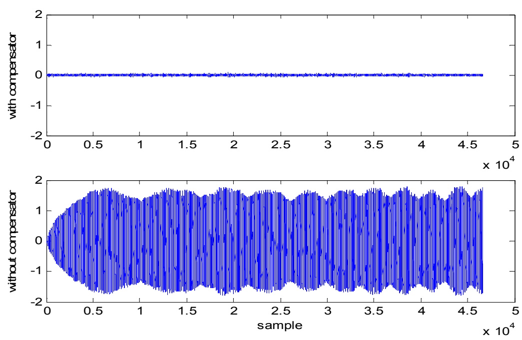


Fig. 13. Displacement of the mass when the sigma-delta closed loop gyroscope operates with and without compensator

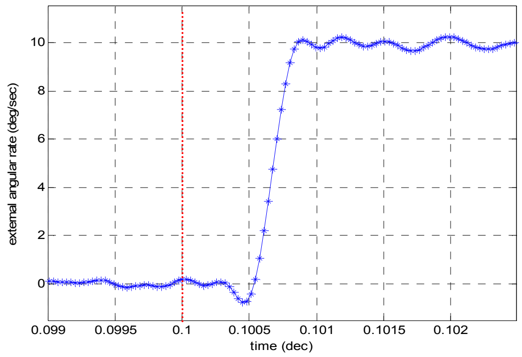


Fig. 14. Output of whole gyroscope system when 10 deg/sec step input is applied at 0.1 sec

ACKNOWLEDGEMENTS

This work has been supported by the BK21 SNU-KU Research Division for Information Technology, Seoul National University, Automation and Systems Research Institute (ASRI) in Seoul National University, the Korea Research Foundation Grant (KRF - 2004 - 0055 - B00047), and Aerospace Technology Development Project.

REFERENCES

Dijkstra, E., Nys, O., Piguet, C. and Degrauwe, M. (1998). On the use of modulo arithmetic comb Filters in sigma delta modulators, *IEEE proc. ICASSP'88*, 2001-2004.

Hoegenauer, E. G. (1981). An economical class of digital filters for decimation and interpolation, *IEEE Transactions on Acoustics, Speech and Signal Processing*, **vol. ASSP-29**, 155-162.

Inose, H. and Yasuda, Y. (1963). A unity bit coding method by negative feedback, *Proc. IEEE*, **vol. 51**, 1524-1535.

Jantzi, S., Ouslis, C. and Sedra, A.S. (1994). The design of transfer functions for delta-sigma converters, *Proc. 1994 IEEE Int. Symp. Circuit Syst.*, **vol.5**, 433-435.

Jiang, X. and Bhave, S. A. (2002). ΣΔ Capacitive interface for a vertically-driven X&Y-axis rate gyroscope, *in 28th European Solid-State Circuit Conference*, 639-642.

Kiss, P. (2002) *Design guide of high-order delta-sigma modulators - an empirical study*, Technical memorandum #30003544-020415-01, Agere Systems, New Jersey: Murray Hill.

Norsworthy, S., Scherier, R. and Temes, G. C. (1997). *Delta-Sigma Data Converters theory, design, and simulation*, IEEE press, New York.

Oppenheim, A. V. and Schafer, R. W. (1999). *Discrete-time signal processing*, Prentice Hall, New Jersey.

Sung, W.T., Lee, J.G., Song, J.W. and Kang, T. (2004). H_∞ Controller design of MEMS gyroscope and its performance test. *Proc. IEEE PLANS 2004*, 63-69.