

STATE ESTIMATION IN COORDINATED CONTROL WITH A NON-STANDARD INFORMATION ARCHITECTURE

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Abstract: An optimal estimation problem is studied with a non-classical information architecture in the vehicle formation, coordinated control context. This information architecture prohibits the direct application of Kalman filter approaches. This paper shows that such a suboptimal feasible estimator is attainable by recasting it as a linear matrix inequality convex optimization problem. A simple example is given and several interesting design issues are then discussed. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Coordinated control is an approach to manipulating a large scale system, which consists of many autonomous but communicating subsystems, to fulfil certain tasks. An example is the control of a fleet of vehicles to maintain their formation. Typically the size of the entire system prohibits a global solution because the collection of the global state information and the computation of a global control law are overly demanding. Furthermore the localization of the sensors and actuators on individual subsystems militates against a global solution because of limited communication capacity. Therefore, in this coordinated control context, each subsystem runs a localized control law based on some limited information of the whole system. There are two central design issues; the local control law and the information architecture, the latter of which decides the amount and the accuracy of the available information for a subsystem.

Recent papers studying decentralized and distributed control with communication constraints have tended to concentrate on the best achievable performance

(Tatikonda and Mitter, 2004) or the minimal permissible communication bandwidth (Nair *et al.*, 2004). The difficulty of optimal control in a decentralized problem with non-traditional information structure was demonstrated by Witsenhausen (Witsenhausen, 1968) in his famous example, which is still the subject of research (Lee *et al.*, 2001). Here we adopt a different approach focused on feasible design rather than optimality.

The tools developed in (Yan and Bitmead, 2003; Yan and Bitmead, 2005) rely on constrained full-state Model Predictive Control (MPC) married with state estimation based on Kalman filtering to formulate a coordinated vehicle control problem. Interactions between vehicles are incorporated via a no-collision constraint being included into the MPC problems solved locally at each vehicle. Each vehicle estimates the states of its neighboring vehicles and the state estimate covariance is used in tightening these constraints, through the requirement to maintain a small probability of collision. Thus typically large state estimate covariance leads to more conservative constraints.

With these design principles for coordinated control in mind, we need to develop a systematic approach to the construction of state estimators for other vehicles' states using communicated measurements and for the computation of the resultant state estimate covariances. The contribution of this paper is to formulate and solve this problem with a non-traditional information structure when a priori information is shared about all the local control laws. This solution is pre-computed globally to determine the bandwidth assignment of the communication capacity in links between the individual vehicles in the coordinated formation.

This paper studies the question of how a vehicle achieves the best local estimation of its neighbors from its localized information. A simple, but generalizable, two-vehicle scenario is considered. The local control laws are assumed to be a linear feedback of both state vectors (actually the local state estimates). This kind of mixed feedback control is usual in the coordinated control context, although it does not fully capture constrained model predictive control. In a classical information architecture, each vehicle needs the measurements and the control values of the others to calculate a shared common state estimate. This paper studies a non-classical type. The control values cannot be transmitted, instead the control laws (i.e. feedback gains) are known at every site. Each vehicle runs a Kalman filter to estimate its own state and needs to design an observer for the states of other vehicles. The key issues are the calculation of observer gains and covariance matrices of the observer errors. The covariance measures the quality of the observer estimate, may be used to recast the constraints in MPC, and is important for accomplishing the system performance requirement as in (Yan and Bitmead, 2003).

The observers take a similar form to a Kalman filter except for the exact control values and the observer gain matrices. As only control laws are given, the observers use the feedback of the best estimate at hand. Because of the mixed structure of the controllers, the observer errors depend on each other as well as the Kalman filter errors. As a result, the covariance matrix calculation of each observer error is much more complicated than that of a Kalman filter. Our method is to write down an augmented error system including the two Kalman filter errors and the two observer errors. The covariance matrix of the augmented system has a clean form and linear matrix inequality techniques can help to find a related optimal observer gain that minimizes a function of the estimation error covariance while taking care of the stability requirement in steady state. Details are written in Section 2 following by a scalar example in Section 3.

2. PROBLEM FORMULATION

This section is devoted to a simple two-vehicle formation problem. Our focus is to formulate a reasonable state estimator according to the non-classical information structure, which does not allow transmitting the control actions but instead assumes the state feedback control laws are known to all parties a priori.

The dynamics and the measurement of the vehicles can be described as follows:

Vehicle 1

$$x_{k+1}^1 = A_1 x_k^1 + B_1 u_k^1 + w_k^1, \quad (1)$$

$$y_{1,k}^1 = C_{11} x_k^1 + v_{1,k}^1, \quad (2)$$

$$y_{1,k}^2 = C_{12} x_k^2 + v_{1,k}^2. \quad (3)$$

Vehicle 2

$$x_{k+1}^2 = A_2 x_k^2 + B_2 u_k^2 + w_k^2, \quad (4)$$

$$y_{2,k}^1 = C_{21} x_k^1 + v_{2,k}^1, \quad (5)$$

$$y_{2,k}^2 = C_{22} x_k^2 + v_{2,k}^2. \quad (6)$$

Where x stands for the state, y for the measurement, w for the process noise with $w_k^i \sim N(0, Q_i)$ and v for measurement noise with $v_{j,k}^i \sim N(0, R_{ji})$. There are superscripts and subscripts throughout this paper with superscripts meaning 'of' and subscripts meaning 'at'; for example, $y_{2,k}^1$ is the measurement of vehicle 1 taken by (at) vehicle 2.

There are four state estimators based on different sets of measurements. The state estimators using y_1^1 and y_2^2 are standard Kalman filters,

Kalman Filter 1@1:

$$\begin{aligned} \hat{x}_{1,k+1|k}^1 &= A_1 \hat{x}_{1,k|k}^1 + B_1 u_k^1, \\ \hat{x}_{1,k+1|k+1}^1 &= \hat{x}_{1,k+1|k}^1 + M_{11} (y_{1,k+1}^1 - C_{11} \hat{x}_{1,k+1|k}^1), \\ M_{11} &= A_1 \Sigma_{1,k|k}^1 C_{11}^T (C_{11} \Sigma_{1,k|k}^1 C_{11}^T + R_{11})^{-1}, \\ \Sigma_{1,k+1|k+1}^1 &= A_1 \Sigma_{1,k|k}^1 A_1^T - M_{11} C_{11} \Sigma_{1,k|k}^1 A_1^T + Q_1. \end{aligned} \quad (7)$$

Kalman Filter 2@2:

$$\begin{aligned} \hat{x}_{2,k+1|k}^2 &= A_2 \hat{x}_{1,k|k}^1 + B_2 u_k^2, \\ \hat{x}_{2,k+1|k+1}^2 &= \hat{x}_{2,k+1|k}^2 + M_{22} (y_{2,k+1}^2 - C_{22} \hat{x}_{2,k+1|k}^2), \\ M_{22} &= A_2 \Sigma_{2,k|k}^2 C_{22}^T (C_{22} \Sigma_{2,k|k}^2 C_{22}^T + R_{22})^{-1}, \\ \Sigma_{2,k+1|k+1}^2 &= A_2 \Sigma_{2,k|k}^2 A_2^T - M_{22} C_{22} \Sigma_{2,k|k}^2 A_2^T + Q_2. \end{aligned} \quad (8)$$

The (cross) estimators based on y_2^1 and y_1^2 are more interesting and the major difficulty is the lack of the knowledge about the control between the two vehicles. Nevertheless, these two estimators take an observer form similar to a Kalman filter,

Estimator 1@2:

$$\begin{aligned}\hat{x}_{2,k+1|k}^1 &= A_1 \hat{x}_{2,k|k}^1 + B_1 \bar{u}_k^1, \\ \hat{x}_{2,k+1|k+1}^1 &= \hat{x}_{2,k+1|k}^1 + M_{21}(y_{2,k+1}^1 - C_{21} \hat{x}_{2,k+1|k}^1).\end{aligned}\quad (9)$$

Estimator 2@1:

$$\begin{aligned}\hat{x}_{1,k+1|k}^2 &= A_2 \hat{x}_{1,k|k}^2 + B_2 \bar{u}_k^2, \\ \hat{x}_{1,k+1|k+1}^2 &= \hat{x}_{1,k+1|k}^2 + M_{12}(y_{1,k+1}^2 - C_{12} \hat{x}_{1,k+1|k}^2).\end{aligned}\quad (10)$$

Remarks:

- The controls applied in these two estimators \bar{u}^1 and \bar{u}^2 are different from the real values of u^1 and u^2 . Suitable choices of \bar{u}^1 and \bar{u}^2 should be decided according to the off line knowledge about u^1 and u^2 and the associated control laws.
- The observer gain matrices M_{21} and M_{12} are unknown at the moment. It will be shown soon that the gains relate to the state error covariance matrices, i.e. the performance measure, of these two cross estimators. Hence the task of this paper is seeking design values of M_{21} and M_{12} .
- The state estimate covariance has an effect on control performance. Studying the covariances off line will enable us to manage them (and hence the control performance) via adjusting/designing the proper information architecture (e.g. adjusting R_{ij} which is directly tied to the assigned communication channel capacity).

The key issue for *Estimator 1@2* and *Estimator 2@1* to work is the knowledge of the control. In this paper, each vehicle will apply a state (estimate) feedback control law involving both its own and the other's state estimate. At this stage, we assume this control to be linear and, since coordination is involved, to include all local state estimates.

$$\begin{aligned}u_k^1 &= K_{11} \hat{x}_{1,k|k}^1 + K_{12} \hat{x}_{1,k|k}^2 + l_1, \\ u_k^2 &= K_{21} \hat{x}_{2,k|k}^1 + K_{22} \hat{x}_{2,k|k}^2 + l_2.\end{aligned}$$

The control gains K_{ij} and the additive constant vectors l_i are known to both of the vehicles. Then it is reasonable to apply

$$\begin{aligned}\bar{u}_k^1 &= K_{11} \hat{x}_{2,k|k}^1 + K_{12} \hat{x}_{2,k|k}^2 + l_1, \\ \bar{u}_k^2 &= K_{21} \hat{x}_{1,k|k}^1 + K_{22} \hat{x}_{1,k|k}^2 + l_2.\end{aligned}$$

Usually \bar{u}^i and u^i are not the same. Hence the 'cross estimation' will have a covariance larger than that of the Kalman filter, which should be minimized or at least bounded as per requirement.

To derive the covariance matrix of the estimates, an expression for the estimate errors should be derived first. As usual define $\tilde{x}_j^i = x^i - \hat{x}_j^i$, then

Filtering Error Equations:

$$\tilde{X}_{k+1} = \bar{M}_r \bar{A} \tilde{X}_k + \bar{M}_r \bar{B} w_k - \bar{M} v_{k+1}, \quad (11)$$

where

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{1,\cdot}^1 \\ \tilde{x}_{2,\cdot}^1 \\ \tilde{x}_{1,\cdot}^2 \\ \tilde{x}_{2,\cdot}^2 \end{bmatrix}, \quad w_k = \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix},$$

$$v_{k+1} = \begin{bmatrix} v_{1,k+1}^1 \\ v_{2,k+1}^1 \\ v_{1,k+1}^2 \\ v_{2,k+1}^2 \end{bmatrix}, \quad \bar{M}_r = (I - \bar{M}\bar{C}),$$

$$\bar{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ -B_1 K_{11} & (A_1 + B_1 K_{11}) & -B_1 K_{12} & B_1 K_{12} \\ B_2 K_{21} & -B_2 K_{21} & (A_2 + B_2 K_{22}) & -B_2 K_{22} \\ 0 & 0 & 0 & A_2 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{21} & 0 & 0 \\ 0 & 0 & M_{12} & 0 \\ 0 & 0 & 0 & M_{22} \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{21} & 0 & 0 \\ 0 & 0 & C_{12} & 0 \\ 0 & 0 & 0 & C_{22} \end{bmatrix}.$$

Note that from (11), if the initial estimation errors have zero means $E(\tilde{x}_{j,0|0}^i) = 0$, it follows that $E(\tilde{x}_{2,k|k}^1) = E(\tilde{x}_{1,k|k}^2) = 0$, meaning that *Estimator 1@2* and *Estimator 2@1* are unbiased estimators. Also note that the known constant terms l_1 and l_2 vanished in the filtering error equation (11).

Consider the steady state covariance $P = cov(\tilde{X})$, it follows that

$$P = \bar{M}_r \bar{A} P \bar{A}^T \bar{M}_r^T + \bar{M}_r \bar{B} Q \bar{B}^T \bar{M}_r^T + \bar{M} R \bar{M}^T, \quad (12)$$

$$\text{where } Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, R = \begin{bmatrix} R_{11} & 0 & 0 & 0 \\ 0 & R_{21} & 0 & 0 \\ 0 & 0 & R_{12} & 0 \\ 0 & 0 & 0 & R_{22} \end{bmatrix}.$$

First of all, it is required that *Estimator 1@2* and *Estimator 2@1* should be stable. If a feasible matrix P can be found for the following matrix inequality:

$$\begin{aligned}-P + \bar{M}_r \bar{A} P \bar{A}^T \bar{M}_r^T + \bar{M}_r \bar{B} Q \bar{B}^T \bar{M}_r^T + \bar{M} R \bar{M}^T &< 0, \\ P &> 0,\end{aligned}\quad (13)$$

then it provides an upper bound of the algebraic solution P of (12) and hence the estimators are stable. The "<" sign is used in (13) instead of "≤" to avoid the semi-definite problem. Actually, the solution of (13) from a numerical solver is arbitrarily close to the solution of (12). Taking the Schur complement of (13) yields

$$\begin{bmatrix} -P & \bar{A} + \bar{M}\bar{C} & B + \bar{M}D \\ \bar{A}^T + \bar{C}^T \bar{M}^T & -P^{-1} & 0 \\ \bar{B}^T + D^T \bar{M}^T & 0 & -I \end{bmatrix} < 0. \quad (14)$$

where

$$C = -\bar{C}\bar{A}, B = [\bar{B}Q^{\frac{1}{2}} \ 0], D = [-\bar{C}\bar{B}Q^{\frac{1}{2}} \ R^{\frac{1}{2}}].$$

Multiplying (14) on the left by $T = \text{blockdiag}(P^{-1}, I, I)$ and on the right by $T^T = T$ yields the following linear matrix inequality(LMI).

Theorem 1. Matrix L and symmetric matrix Y satisfying the LMI

$$\begin{bmatrix} -Y & (Y-L\bar{C})\bar{A} & (Y-L\bar{C})\bar{B}Q^{\frac{1}{2}} & LR^{\frac{1}{2}} \\ \bar{A}^T(Y-\bar{C}^TL^T) & -Y & 0 & 0 \\ Q^{\frac{1}{2}}\bar{B}^T(Y-\bar{C}^TL^T) & 0 & -I & 0 \\ R^{\frac{1}{2}}L^T & 0 & 0 & -I \end{bmatrix} < 0, \quad (15)$$

exist if and only if $P = Y^{-1}$, $\bar{M} = PL$, and $\bar{M}_r = I - \bar{M}\bar{C}$ satisfy matrix inequality (13).

Note that this is a standard construction in optimal filtering derived using LMIs and minimization (Skelton *et al.*, 1998), and (Colaneri *et al.*, 1997). The appeal of (15) versus (13) is that this latter inequality is linear in the matrix variables Y and L , which makes its solution and optimization tractable. From the perspective of formulating and solving non-classical information architecture control problems, however, an incipient problem arises through the inability to explore a block diagonal structure on the computed solution \bar{M} from (14) without also imposing such a structure on Y and P . Evidently from the structure of \bar{A} a block diagonal P is not typically of interest. Indeed it is the cross-covariance between terms such as $\tilde{x}_{1,k}^1$ and $\tilde{x}_{2,k}^1$ that captures the information architecture. Without a structural condition on \bar{M} , the solution of minimizing $\text{tr}(Y^{-1})$ subject to (15) versus (13) would yield the classical, fully-shared-measurement Kalman filtering solution. To explore the development of an LMI approach to finding feasible solutions to the non-standard information architecture problem, we employ a result of (de Oliveira *et al.*, 1999).

Lemma 1. (de Oliveira *et al.*, 1999) The following statements are equivalent.

(i) There exists a symmetric matrix $P > 0$ such that

$$A^T P A - P < 0.$$

(ii) There exist a symmetric matrix P and a matrix G such that

$$\begin{bmatrix} -P & A^T G^T \\ G A & -G - G^T + P \end{bmatrix} < 0.$$

Now we use Lemma 1 to establish Theorem 2.

Theorem 2. Matrices G, Y , and L satisfying

$$\begin{bmatrix} -G - G^T + Y & G A + L C & G B + L D \\ A^T G^T + C^T L^T & -Y & 0 \\ B^T G^T + D^T L^T & 0 & -I \end{bmatrix} < 0, \quad (17)$$

yield $P = Y^{-1}$ and $\bar{M} = G^{-1}L$ which satisfy (13). Conversely, P and \bar{M} satisfying (13) provide $Y = G = P^{-1}$, $L = P^{-1}\bar{M}$ which satisfy (17).

Corollary 1. If G and L are constrained to be block diagonal matrices in (17), then $P = Y^{-1}$ and $\bar{M} = G^{-1}L$ are also feasible in (13) with \bar{M} block diagonal.

Corollary 2. If matrices G, Y , and L , with G and L block diagonal conformably with \tilde{X}_k , can be found satisfying (17) then the state estimators (7)–(10), with the gains given by the block diagonal elements of $\bar{M} = G^{-1}L$, are stable and their covariances are bounded above by the corresponding diagonal blocks of P .

By inspecting the finer structure of matrices A and B , the matrix on the left hand side of (17) is linear in the unknowns Y, L , and G . A feasible solution from (17) will give us feasible values of P, M_{12} and M_{21} .

Furthermore, one may aim to seek a feasible solution of (17), which minimizes $\text{tr}(P) = \text{tr}(Y^{-1})$. To do this we introduce a new variable W such that

$$W > Y^{-1}, \quad (18)$$

and then minimize the $\text{tr}W$. The Schur compliment of (18) is

$$\begin{bmatrix} -W & I \\ I & -Y \end{bmatrix} < 0. \quad (19)$$

This yields the following convex LMI optimization problem to provide a solution for the observer gains for coordinated control with non-standard information structure.

Min Cov:

$$\min_{G,L,W,Y} \text{tr}W$$

subject to:

$$\begin{bmatrix} -G - G^T + Y & G A + L C & G B + L D \\ A^T G^T + C^T L^T & -Y & 0 \\ B^T G^T + D^T L^T & 0 & -I \end{bmatrix} < 0, \quad (20)$$

$$\begin{bmatrix} -W & I \\ I & -Y \end{bmatrix} < 0,$$

where G and L are block diagonal Y and W are symmetric.

Note that the covariances Σ_1^1 and Σ_2^2 from (7) and (8) are constant. Hence, once we get solutions G, Y and L from (20), the Kalman gains \bar{M}_{11} and \bar{M}_{22} will be replaced by M_{11} and M_{22} .

3. EXAMPLE

In this section, the result from Section 2 will be demonstrated with a *scalar* example. The notations remain the same as in Section 2 with only the matrix operation eased to the scalar calculation.

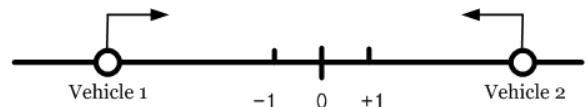


Fig. 1. Two mobile beads cooperation task

Consider two coordinated autonomous mobile beads on the real line described by

$$x_{k+1}^1 = x_k^1 + u_k^1 + w_k^1, \quad (21)$$

$$x_{k+1}^2 = x_k^2 + u_k^2 + w_k^2. \quad (22)$$

At each sampling time k , each solves the same optimization problem based on their local information:

$$\min_{u^i} J_k = (x_{k+1}^1 + x_k^2)^2 + (x_{k+1}^1 + 1)^2 + (x_{k+1}^2 + x_k^1)^2 + (x_{k+1}^2 - 1)^2. \quad (23)$$

The coordination tasks captured by J_k are:

- (i) to drive *vehicle 1* to -1 and *vehicle 2* to $+1$;
- (ii) to maintain the 2-vehicle formation symmetric about the origin, if the initial positions are symmetric.

Note that, emphasizing the estimation part of the problem, the coordination control task (ii) is merely illustrative. This task is a simple one-step-ahead LQG problem. The solution is:

$$u_k^1 = -\hat{x}_{1,k|k}^1 - \frac{1}{2}\hat{x}_{1,k|k}^2 + 1,$$

$$u_k^2 = -\frac{1}{2}\hat{x}_{2,k|k}^1 - \hat{x}_{2,k|k}^2 - 1.$$

Following the procedure in Section 2, the \bar{M} solved from (20) is

$$\bar{M} = \text{diag}(0.7085, 0.7085, 0.7085, 0.7085).$$

Since we can have M_{11} and M_{22} pre-computed from the Kalman filter equations (They both equal 0.6180 in this example.), we replace $\bar{M}(1,1)$ and $\bar{M}(4,4)$ with 0.6180. Thus, the gain matrix applied is:

$$\bar{M}_{app} = \text{diag}(0.6180, 0.7085, 0.7085, 0.6180).$$

The achieved steady state covariance matrix via applying \bar{M}_{app} is

$$P_{achv} = \begin{pmatrix} 0.6180 & 0.1788 & -0.0245 & 0 \\ 0.1788 & 0.6569 & -0.0371 & -0.0245 \\ -0.0245 & -0.0371 & 0.6569 & 0.1788 \\ 0 & -0.0245 & 0.1788 & 0.6180 \end{pmatrix}.$$

To demonstrate the validity of our technique, covariances, in comparison with P_{achv} , were computed with various $\bar{M}_{app}(2,2)$ and $\bar{M}_{app}(3,3)$ values. The Figure 2 shows the Error Covariances of *Estimator 2@1* by changing the observer gains $M_{12} = M_{21}$. The achieved steady state covariance of *Estimator 2@1* is 0.6569; the optimal covariance, suggested in Figure 2, is 0.6423.

At this stage, one may explore some design issues. For example, the controller in (Yan and Bitmead, 2003) deals with probabilistic constraints and, once the constraints become active, string instability occurs. In this

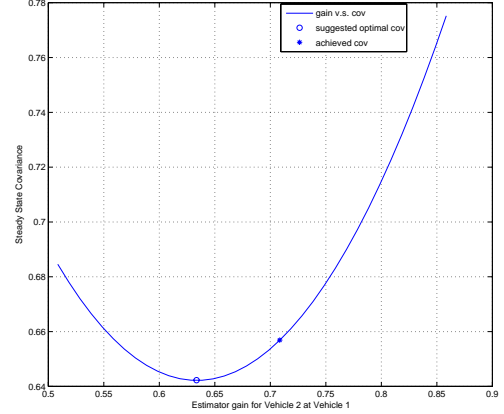


Fig. 2. Error Covariances of *Kalman filter 2@1* depending on M_{12} and M_{21}

problem, collision between the two beads is not allowed. Suppose that bead 1 and bead 2 are already in steady state and at time k bead 2 computes $\hat{x}_{2,k|k}^1$ and $\hat{x}_{2,k|k}^2$. One may ask what value of R_{21} can guarantee that the probability of collision is smaller than a certain level say 5%, i.e. $P(x_k^2 \geq x_k^1) \leq 5\%$. By the same method in (Yan and Bitmead, 2003), one can obtain

$$\Sigma_2^1 \leq \frac{l^2}{\alpha^2} - \Sigma_2^2, \quad (24)$$

where α is the solution of $\Phi(\alpha) = 1 - 5\%$ and $l = \hat{x}_{2,k|k}^1 - \hat{x}_{2,k|k}^2$, $\Phi(\cdot)$ is the standard normal distribution function. Solving (24) provides a choice of R_{21} such that if bead 2 observes that its distance from bead 1 is bigger than l then the probability of collision is smaller than 5%; if it sees the opposite case, it should take some emergency response procedure.

4. CONCLUSION

This paper studies the local estimator formulation in the vehicle coordinated control problem. Though the non-classical information architecture complicates the structure of the local estimator, LMI techniques can help the design. The result makes the study of more design issues in the coordinated control context possible.

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REFERENCES

- Colaneri, P., Jose C. Geromel and Arturo Locatelli (1997). *Control Theory and Design (RH₂ and RH_∞ viewpoint)*. Academic Press.

- de Oliveira, M.C., J. Bernussou and J.C. Geromel (1999). A new discrete-time robust stability condition. *Systems and Control Letters* **37**, 261–265.
- Lee, J.T., E. Lau and Y.C. Ho (2001). The witsenhausen counterexample: A hierarchical search approach for nonconvex optimization problems. *IEEE Transactions on Automatic Control* **46**, 382–397.
- Nair, G.N., I.M.Y. Marrels R.J. Evans and W. Moran (2004). Topological feedback entropy and nonlinear stabilization. *IEEE Transactions on Automatic Control* **49**, 1585–1603.
- Skelton, R.E., T. Iwasaki and K. Goriadis (1998). *A Unified Approach to Linear Control Design*. Taylor and Francis.
- Tatikonda, S. and S. Mitter (2004). Control under communication constraints. *IEEE Transactions on Automatic Control* **49**, 1056–1068.
- Witsenhausen, H.S. (1968). A counterexample in stochastic optimum control. *SIAM J.Control* **6**, 131–147.
- Yan, J. and R.R. Bitmead (2003). Coordinated control and information architecture. *Proceeding of 42th IEEE Conference on Decision and Control*.
- Yan, J. and R.R. Bitmead (2005). Incorporating state estimation into model predictive control and its application to network traffic control. *Automatica* **41**, 595–604.