SET MEMBERSHIP ESTIMATION OF PARAMETERS AND VARIABLES IN DYNAMIC NETWORKS BY RECURSIVE ALGORITHMS WITH MOVING MEASUREMENT WINDOW

Kazimierz Duzinkiewicz [**] **Mietek A. Brdys** [*,**]

[**] *Department of Automatic Control, Gdansk University of Technology, G.Narutowicza 11/12, 80 9 52 Gdansk, Poland, Email: kduzin@ely.pg.gda.pl,*

[*] *Department of Electronic, Electrical and Computer Engineering, School of Engineering, The University of Birmingham, Birmingham B15 2TT, UK, Email: m.brdys@bham.ac.uk*

Abstract: The paper considers a set-membership joint estimation of variables and parameters in complex dynamic networks based on parametric uncertain models and limited hard measurements. The recursive estimation algorithm with moving measurement window is derived that is suitable for on line network monitoring. The window allows stabilising the classic recursive estimation algorithm and significantly improves the estimate tightness. The estimator is validated on a case study water distribution network. Tight set estimates of unmeasured pipe flows, nodal heads and pipe resistances are obtained.

Keywords: set membership estimation, uncertainty, complexity, networks, water

1. INTRODUCTION

Joint estimation of variables and model parameters is a routine activity that is carried out on-line during network operation. Estimation of heads, flow rates, chlorine concentrations, pipe parameters or chlorine reaction rates across drinking water distribution network (Brdys and Chen, 1996; Brdys and Lisiak, 1999; Brdys *et al*., 2001; Duzinkiewicz 2005) and estimation of biological state and model parameters for integrated wastewater system (Rutkowski *et al*., 2004) can serve the examples. It is crucial to properly integrate a priori knowledge including mathematical models with measurement information provided by hard sensors in order to obtain robust and quality estimates. An uncertainty exists due to the modelling errors, measurement noise and disturbance inputs. In the paper in order to obtain robust estimates with guaranteed estimation error and at the same time to simplify modelling effort, which is necessarily due to a network complexity a set membership model of the uncertainty is employed (Schweppe, 1978; Milanese *et al., 1996;* Walter and Pronzato, 1997; Chang *et al*., 2004; Duzinkiewicz, 2005). For linear case, the solution set is a convex polyhedron which can be very complicated and other simple-shaped forms, such as ellipsoids, parallelotopes have been used to give an enclosure of the exact solution set (Milanese *et al.*, 1996; Walter and Pronzato, 1997). When the model is nonlinear, the previous algorithms are no longer relevant (Raïssi *et al.,* 2004). In the paper the outer approximation is used defined by taking orthogonal projections of the exact set. The estimates are sets bounding uncertain parameters and tubes bounding uncertain variable trajectories. The point estimates can be selected from the set estimates depending on their future usage. Regardless of the selection the estimation error can be always assessed in a guaranteed manner. The Chebyshev centres of the sets serve the point estimates that minimise the estimation error in a worst case (Milanese *et al.*, 1996). In the paper recursive algorithms for the set membership estimation are derived. A moving measurement window is introduced in order to stabilize the estimates and also to compromise between the computational effort and the estimates tightness.

2. ESTIMATION PROBLEM

2.1 Information

We shall consider a dynamic network that is composed of interconnected static and dynamic elements. The element variables and its model parameters are related by equalities to obtain the element mathematical model. For a dynamic element we shall distinguish between the state variables and remaining non-state variables. The non-state variables are composed of the network inputs, outputs and intermediate variables. The latter variables connect the elements. It is a common property of the static network element models that all the variables are linked in an implicit manner through the model equalities. Hence, for a network as a whole we shall distinguish between state variables $\boldsymbol{s}$, non-state variables $\boldsymbol{y}$ and external inputs (controls and disturbances) $\boldsymbol{u}$. As all three types of variables are to be estimated (controls due to actuator error) the composed vector $\boldsymbol{x}$ of the estimated variables is introduced as:

$$\boldsymbol{x} = \left[\boldsymbol{s}^T, \boldsymbol{u}^T, \boldsymbol{y}^T\right]^T; \boldsymbol{x} \in \Re^{n_x}, \boldsymbol{u} \in \Re^{n_u}, \boldsymbol{y} \in \Re^{n_y} \qquad (1)$$

Let us denote by $\mathcal{F}$ the operator representing all the algebraic equations modelling the network static part. Hence, a static part of the network model can be written as:

$$\mathcal{F}(\boldsymbol{x}) = \boldsymbol{0} \qquad (2)$$

The operator $\mathcal{F}$ is not exactly known and only its approximate model $\mathcal{F}^M$ is available. The following holds:

$$\boldsymbol{0} = \mathcal{F}(\boldsymbol{x}) = \mathcal{F}^M(\boldsymbol{x}, \boldsymbol{\gamma}) + \boldsymbol{e}_s(\boldsymbol{x}, \boldsymbol{\gamma}) \qquad (3)$$

where $\boldsymbol{\gamma} \in \Re^{n_\gamma}$ is the model parameter vector and $\boldsymbol{e}_s(\cdot)$ describes the modelling error.

It is assumed that the modelling error bounds $\boldsymbol{e}_s^{max}$ and $\boldsymbol{e}_s^{min}$ are known so that

$$\boldsymbol{e}_s^{min} \le \boldsymbol{e}_s(\boldsymbol{x}, \boldsymbol{\gamma}) \le \boldsymbol{e}_s^{max} \qquad (4)$$

The static network model used via (2), (3) and (4) enables us to bound sets of possible variable and parameter values to such that satisfy the inequality:

$$-\boldsymbol{e}_s^{max} \le \mathcal{F}^M(\boldsymbol{x}, \boldsymbol{\gamma}) \le -\boldsymbol{e}_s^{min} \qquad (5)$$

The inequality (5) with $\boldsymbol{x}(j),\ j = 1,2,\ldots$, bounds the variables and parameters at the stage *j*. By adding to this inequality known pointwise a priori bounds on the estimated variables and parameters $\boldsymbol{x}^{min}(j) \le \boldsymbol{x}(j) \le \boldsymbol{x}^{max}(j)$ and $\boldsymbol{\gamma}^{min} \le \boldsymbol{\gamma} \le \boldsymbol{\gamma}^{max}$ the *static a priori knowledge* at time step *j* is obtained that can be shortly written as

$$\mathcal{D}(\boldsymbol{x}(j), \boldsymbol{\gamma}) \le \boldsymbol{0} \qquad (6)$$

where $\mathcal{D}$ is suitably defined operator.

The network dynamics is composed of the element dynamics linked by intermediate variables as:

$$\boldsymbol{s}(j+1) = f(\boldsymbol{x}(j), j);\ j = 1,2,\ldots \qquad (7)$$

As previously, an approximate model $f^M(\cdot)$ of $f(\cdot)$ together with the modelling error $\boldsymbol{e}_d(\boldsymbol{x}, \boldsymbol{\gamma}, j)$ bounds $\boldsymbol{e}_d^{max}(j)$ and $\boldsymbol{e}_d^{min}(j)$, respectively is only available. The inequalities can then be written that robustly bound the estimated quantities at *j* as:

$$\boldsymbol{e}_d^{min}(j) \le \boldsymbol{s}(j+1) - f^{<M}(\boldsymbol{x}(j), \boldsymbol{\gamma}, j) \le \boldsymbol{e}_d^{max}(j) \qquad (8)$$

yielding the *dynamic a priori knowledge* at stage *j*.
The models are assumed *parametric* as opposed to *point - parametric* models (Chang *et al*., 2004). It means that there exist constant parameter vector $\boldsymbol{\gamma}^*$ and the modelling error mappings $\boldsymbol{e}_s^*(\cdot)$ and $\boldsymbol{e}_d^*(\cdot)$ such that for any external inputs from an admissible set the following holds over a control horizon:

$$\begin{gathered}
\mathcal{F}(\boldsymbol{x}^*(j)) = \mathcal{F}^M(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*) + \boldsymbol{e}_s(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*) \\
f(\boldsymbol{x}^*(j), j) = f^M(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*, j) + \boldsymbol{e}_d(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*, j) \\
\boldsymbol{e}_s^{min} \le \boldsymbol{e}_s(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*) \le \boldsymbol{e}_s^{max} \\
\boldsymbol{e}_d^{min} \le \boldsymbol{e}_d(\boldsymbol{x}^*(j), \boldsymbol{\gamma}^*) \le \boldsymbol{e}_d^{max}
\end{gathered} \qquad (9)$$

where $\boldsymbol{x}^*(j)$ is the network variable trajectory over the control horizon. The vector $\boldsymbol{\gamma}^*$ will be further called the real parameter vector.

Only small part of the variable set is directly measured. Let z denote vector of the measured variables. Clearly, $z \subset \boldsymbol{x}$. The measurements are taken at the discrete time instants *j* corresponding to the time steps of the network dynamic model (6). The measurement set available at time stage *k* is:

$$\boldsymbol{Z}^m(k) = \left[z^m(1)^T, \ldots, z^m(j)^T, \ldots, z^m(k)^T\right]^T \qquad (10)$$

The measurements are contaminated by noise $\boldsymbol{e}^m(j)$ and $z^m(j) = z(j) + \boldsymbol{e}^m(j),\quad j \in \overline{1:k}$. The noise is lower and upper bounded with know L_∞ bounds $\boldsymbol{e}^{m,min}(j)$ and $\boldsymbol{e}^{m,max}(j)$, respectively. Hence, the following holds for $j \in \overline{1:k}$:

$$z^m(j) - e^{m,max}(j) \le z(j) \le z^m(j) - e^{m,min}(j) \quad (11)$$

The inequalities (11) over $j \in \overline{1:k}$ constitute overall information about the variables and parameters available at *k* from the measurements gathered till *k*. We shall call it the *measurement information at time step k*. Let $X(k)$ be a vector of trajectories of the estimated variables over $j \in \overline{1:k}$. That is

$$X(k) = \left[x(1)^T, \dots, x(j)^T, \dots, x(k)^T\right]^T \quad (12)$$

The inequalities (11) can now be briefly written as

$$\mathcal{M}(k)(X(k)) \le 0 \quad (13)$$

where $\mathcal{M}(\cdot)$ is a suitably defined operator. We shall also introduce operator $m(k)$ to define the measurement inequality $m(k)(x(k)) \le 0$ only at *k*:

$$z^m(k) - e^{m,max}(k) \le z(k) \le z^m(k) - e^{m,min}(k) \quad (14)$$

2.2 Dynamic Joint Variable and Parameter Batch Estimation

At the time instant *k* we shall define a set $\Omega(k)$ of all variable trajectories $X(k)$ and parameters γ that are consistent with the measurement and a priori information over time period $\overline{1:k}$ which is available at *k*. Hence, the following holds:

$$\begin{aligned} \Omega(k) = \{ & (X(k), \gamma, s(k+1)) : \\ & \mathcal{M}(k)(X(k)) \le 0; \quad \mathcal{D}(x(j), \gamma) \le 0, j = \overline{1:k}; \\ & e_d^{min}(j) \le s(j+1) - f^M(x(j), \gamma, j) \le e_d^{max}(j) \} \end{aligned} \quad (15)$$

The state $s(k)$ is estimated as a component of the vector $x(k)$. The state $s(k+1)$ participates in the definition of the set $\Omega(k)$ as the inequality (8) with $j = k$ contributes to knowledge at *k* about $x(k)$ and $s(k+1)$ is present in (8). However, as opposed to $x(k), \gamma$, estimating $s(k+1)$ at *k* means predicting the state. Let $(X(k), \gamma, s(k+1)) \in \Omega(k)$. Regarding available at *k* the static, dynamic and measurement information, a set

$$\Omega_{x(k),\gamma,s(k+1)}(k) \stackrel{\Delta}{=} \{(x(k), \gamma, s(k+1))\} \quad (16)$$

is the smallest set where it is guaranteed that the estimated quantities $\{x^*(k), \gamma^*, s^*(k)\}$ belong to. Unfortunately, a topological structure of this set is complicated even for linear networks. Hence, for practical reasons we shall take its outer approximation that can be realistically determined as the set estimate at *k*. The outer approximation is defined by taking orthogonal projections of $\Omega_{x(k),\gamma,s(k+1)}(k)$ on the subspaces of $x_i(k)$, $s_i(k+1)$ and γ_i. This results in producing the intervals $[x_i^{min}(k), x_i^{max}(k)]$, $[s_i^{min}(k+1), s_i^{max}(k+1)]$, and $[\gamma_i^{min}(k), \gamma_i^{max}(k)]$ bounding at *k* the $x_i^*(k)$, $s_i^*(k+1)$ and γ_i^*. The intervals are determined by solving the following optimisation tasks:

$$\begin{aligned} & min\,(max)\,\{ f_0(X(k), \gamma, s(k+1)) = x_i(k)\ (or\ \gamma_i \\ & \qquad\qquad or\ s_i(k+1))\} \\ & subject\ to \quad (X(k), \gamma, s(k+1)) \in \Omega(k) \end{aligned} \quad (17)$$

The Cartesian products of these intervals yield the sets $\Omega_{x(k)}(k)$, $\Omega_{s(k+1)}(k)$ and $\Omega_\gamma(k)$ respectively that are outer approximations of the smallest sets bounding $x^*(k), s^*(k+1)$ and γ^*. Clearly, the following also holds:

$$\Omega_{x(k),\gamma,s(k+1)}(k) \subset \Omega_{x(k)}(k) \times \Omega_\gamma(k) \times \Omega_{s(k+1)}(k) \quad (18)$$

3. RECURSIVE ESTIMATION

The formulation (15) and (17) called the batch estimation is not suitable for on line applications. We shall now derive a recursive formulation. Let us compare the sets $\Omega(k)$ and $\Omega(k-1)$ that constitute the information bases for the estimation at *k* and *k-1*, respectively. New information available at *k* is the measurement information gathered at *k*. Moreover, at *k* the new variables occur $x(k)$ and $s(k+1)$ to be estimated. This new information can be then expressed as:

$$\begin{aligned} \omega(k) = \{ & (x(k), \gamma, s(k+1)) : \\ & \mathcal{D}(x(k), \gamma) \le 0, m(k)(x(k)) \le 0, \\ & e_d^{min}(k) \le s(k+1) - f^M(x(k), \gamma, k) \le e_d^{max}(k) \} \end{aligned} \quad (19)$$

The set $\Omega(k)$ can now be written in the recursive form as:

$$\begin{aligned} \Omega(k) = \{ & (X(k), \gamma, s(k+1)) : \\ & (X(k-1), \gamma, s(k)) \in \Omega(k-1), \\ & (x(k), \gamma, s(k+1)) \in \omega(k) \} \end{aligned} \quad (20)$$

and the recursive form of the estimation tasks at *k* read:

$$\begin{aligned} & min(max)\{ f_0(X(k), \gamma, s(k+1)) = x_i(k)\ (or\ \gamma_i \\ & \qquad\qquad or\ s_i(k+1))\} \\ & subject\ to \quad (x(k), \gamma, s(k+1)) \in \omega(k) \\ & \qquad \gamma \in \Omega_\gamma(k-1) \\ & \qquad s(k) \in \Omega_{s(k)}(k-1) \end{aligned} \quad (21)$$

where the set estimates $\Omega_\gamma(k-1)$ and $\Omega_{s(k)}(k-1)$ are known from the estimation at *k-1*.

Notice that due to (18) there is a conservatism in the estimates defined by (21) when compared to the batch estimates and this is the price to be paid for the recursive structure of the estimation algorithm. This conservatism may produce an unstable process of generating the points that are not consistent with the available information. The stability may be regained by directly introducing a number of past measurements into the estimation task that is performed at k. They form a measurement window with L past measurements. In the recursive algorithm (20), $L=0$. Since the measurement information is coupled with the priori information through the model (see (15)), overall information that is utilised for the estimation at k can be decomposed into the information used to perform the estimation at k-L-1 and the new information over the window $\boldsymbol{\omega}(k-L,k)$. Hence,

$$\begin{aligned}&\boldsymbol{\omega}(k-L,k)=\{(\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1)):\\&\mathcal{D}(\boldsymbol{x}(j),\gamma)\leq\boldsymbol{0},\ j\in\overline{k-L:k};\\&\mathcal{M}(k-L,k)(\boldsymbol{X}(k-L,k))\leq\boldsymbol{0};\\&\boldsymbol{e}_d^{min}(j)\leq\boldsymbol{s}(j+1)-f^M(\boldsymbol{x}(j),\boldsymbol{\gamma},k)\leq\boldsymbol{e}_d^{max}(j)\}\end{aligned}\tag{22}$$

where $X(k-L,k)$ is a set of all variable trajectories over $\overline{k-L:k}$.

Hence, the information base $\boldsymbol{\Omega}(k)$ for the estimation performed at k can be written as:

$$\begin{aligned}\boldsymbol{\Omega}(k)=\{&(\boldsymbol{X}(k),\boldsymbol{\gamma},\boldsymbol{s}(k+1)):\\&(\boldsymbol{X}(k-L-1),\boldsymbol{\gamma},\boldsymbol{s}(k-L))\in\boldsymbol{\Omega}(k-L-1)\\&(\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1))\in\boldsymbol{\omega}(k)\}\end{aligned}\tag{23}$$

where set $\boldsymbol{\Omega}(k-L-1)$ is the information base for the estimation task that is solved at k-L-1.

Notice that knowledge about the parameter γ a nd state $s(k-L)$ gathered as a result of the estimation performed at k-L-1 and based on the set $\boldsymbol{\Omega}(k-L-1)$ is expressed by the sets $\boldsymbol{\Omega}_{\boldsymbol{x}(k-L-1)}(k-L-1)$, $\boldsymbol{\Omega}_{\gamma}(k-L-1)$ and $\boldsymbol{\Omega}_{s(k-L)}(k-L-1)$. The first set has no influence on the estimation results at k. Hence, the recursive algorithm with moving measurement window of the length can be formulated at k as:

$$\begin{aligned}&min(max)\{f_0(\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1))=x_i(k)\\&\qquad\qquad(or\ \boldsymbol{\gamma}_i\quad or\ \boldsymbol{s}_i(k+1))\}\\&subject\ to\ (\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1))\in\boldsymbol{\omega}(k-L,k)\\&\qquad\boldsymbol{\gamma}\in\boldsymbol{\Omega}_{\gamma}(k-L-1)\\&\qquad\boldsymbol{s}(k-L)\in\boldsymbol{\Omega}_{s(k-L)}(k-L-1)\end{aligned}\tag{24}$$

The sets $\boldsymbol{\Omega}_{s(k-L)}(k-L-1)$ $\boldsymbol{\Omega}_{\gamma}(k-L-1)$ come from the estimation performed at k-L-1.

Let us notice that the original information inequalities but not their external approximations are processed over the window. This has, the pointed out above, stabilising impact on the estimates. The prediction $\boldsymbol{\Omega}_{s(k+1)}(k)$ of the state $\boldsymbol{s}(k+1)$ determined at k will be used during the estimation carried out at k+L+1 as an estimate of the initial condition. At k+1 the window moves ahead by one time step and the state $\boldsymbol{s}(k+1)$ will enter the window. Hence, the set $\boldsymbol{\Omega}_{s(k+1)}(k)$ can be used at k+1 as an additional constraint bounding $\boldsymbol{s}(k)$. Introducing the constraint

$$\boldsymbol{s}(k)\in\boldsymbol{\Omega}_{s(k)}(k-1)\tag{25}$$

into the constraint set of (24) does this.

Remaining within the window for some time creates an opportunity for smoothing the past estimates based on new measurement information. However, this requires solving additional optimisation tasks. The smoothing although expensive may vastly improve the estimate tightness. We shall limit our consideration to exploiting only one possibility. During estimation performed at k the state $\boldsymbol{s}(k-L+1)$ becomes an initial state condition for the estimation to be carried out at k+1. It is therefore beneficial for the estimation performance at k+1 to estimate this state also at k. By including this and (25) into (24) the following estimation algorithm is obtained:

$$\begin{aligned}&min(max)\{f_0(\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1))=x_i(k)\\&\qquad or\ s_i(k-L+1)\ \ or\ \boldsymbol{\gamma}_i\ \ or\ \boldsymbol{s}_i(k+1)\}\\&subject\ to\\&\qquad(\boldsymbol{X}(k-L,k),\boldsymbol{\gamma},\boldsymbol{s}(k+1))\in\boldsymbol{\omega}(k-L,k)\\&\qquad\boldsymbol{\gamma}\in\boldsymbol{\Omega}_{\gamma}(k-L-1)\\&\qquad\boldsymbol{s}(k-L)\in\boldsymbol{\Omega}_{s(k-L)}(k-L-1)\\&\qquad\boldsymbol{s}(k)\in\boldsymbol{\Omega}_{s(k)}(k-1)\end{aligned}\tag{26}$$

As that then $\boldsymbol{\Omega}_{s(k-L)}(k-L-1)=\boldsymbol{\Omega}_{s(k)}(k-1)$ if L=0. A routine state prediction is then sufficient in (26). Hence, the beneficial smoothing of the initial condition can be achieved only when the moving measurement window is applied to the estimation algorithm.

4.CASE STUDY SIMULATION RESULTS

A case study drinking water network will be investigated. Modelling the network hydraulics for operational purposes is presented in (Brdys and Chen, 1996). The network variables to be monitored are flows through the pipes, pressures at the network and tank nodes and water demands. The pipe resistances are the model parameters. The measurements are typically limited due to cost of sensor maintenance and unmeasured quantities need to be estimated by using the model and hard measurements. A skeleton of the system located in Lebork (Poland) is shown in Fig. 1.

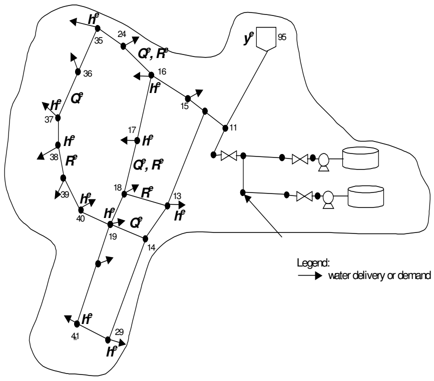

Fig. 1. Lebork case study network

A simple recursive algorithm with L=0 and a recursive estimation with moving measurement window of the window length L=2 were applied to estimate pipe flows, nodal heads and the pipe resistances. The results are shown in Figs. 2-11.

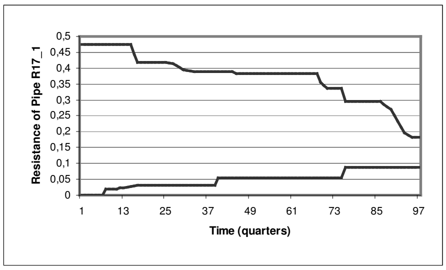

Fig. 2. Estimated bounds on resistance of pipe 17_18; L = 0

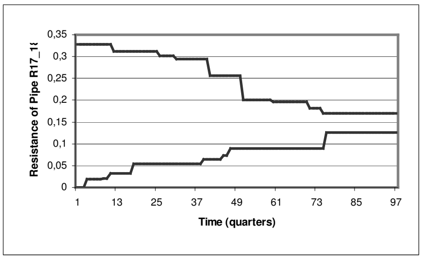

Fig. 3. Estimated bounds on resistance of pipe 17_18; L = 2

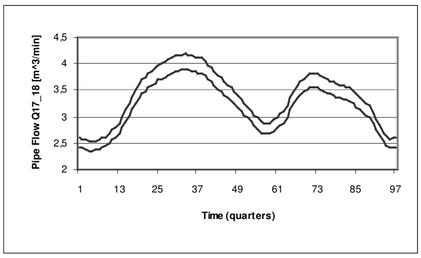

Fig. 4. Estimated bounds on pipe flow Q17_18; L= 0

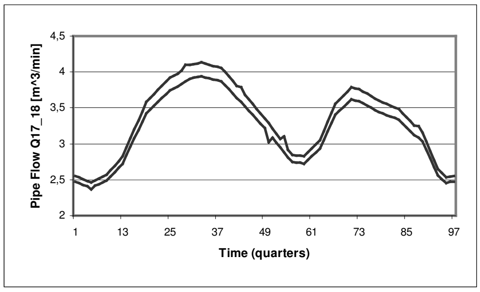

Fig. 5. Estimated bounds on pipe flow Q17_18; L = 2

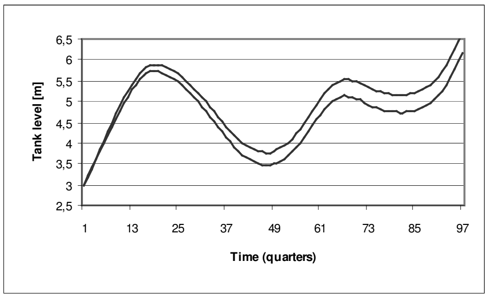

Fig. 6. Estimated bounds on tank level; L = 0

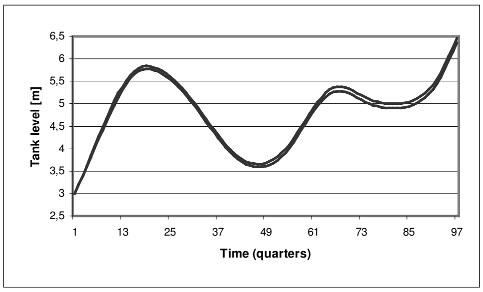

Fig. 7. Estimated bounds on tank level; L = 2

The optimisation tasks (26) were first converted into an approximated piecewise linear form and then solved by applying a mixed integer linear solver (Brdys *et al.*, 2001) to produce wanted global optima. The linearisation error was handled by introducing the error into formulation of the estimation problem priori information as a modelling error.

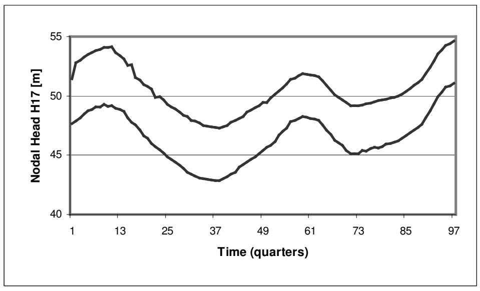

Fig. 8. Estimated bounds on nodal head H17; L = 0

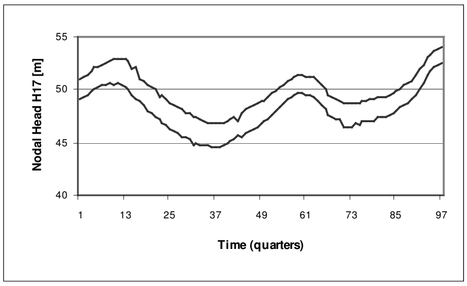

Fig. 9. Estimated bounds on nodal head H17; L = 2

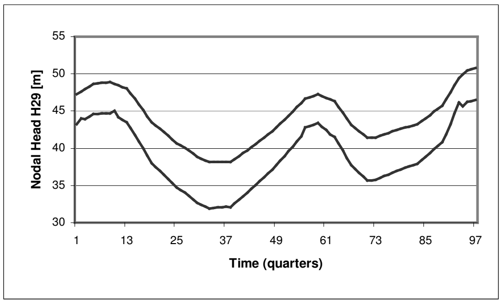

Fig. 10. Estimated bounds on nodal head H29; L = 0

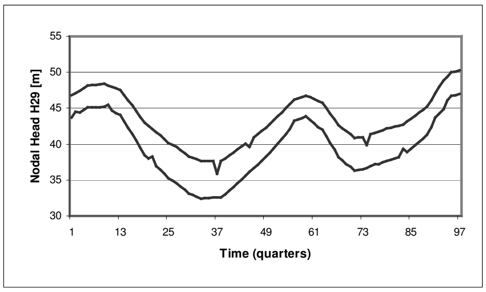

Fig. 11. Estimated bounds on nodal head H29; L = 2

As expected, better estimates are produced for L=2. Nevertheless, an approach to selection of the appropriate values for windows length L needs further investigations.

6. CONCLUSIONS

The paper has considered set-membership joint estimation of variables and parameters in dynamic networks. The recursive estimation algorithm suitable for on line network monitoring has been derived. It has been validated on a case-study water distribution network. The results have shown significant improvements of the estimates produced by estimation with the moving measurement window. Analysis of rigorous criteria for selection of the window length in order to reach a desired trade off between the estimation accuracy and computing effort are under research. Application of the recursive algorithm with moving measurement window to joint estimation of integrated quality and quantity in complex drinking water distribution networks has been continuing in (Duzinkiewicz, 2005).

ACKNOWLEDGEMENT

This work has been supported by the Polish State Committee for Scientific Research under grant No.4 T11A 008 25. The authors wish to express their thanks for the support.

REFERENCES

Brdys, M. A. and K. Chen (1996). Joint estimation of states and parameters of integrated quantity and quality models of dynamic water supply and distribution systems. *Proc. of the 13th IFAC World Congress, Vol.1, pp.73-78*, San Francisco.

Brdys, M. A., K. Duzinkiewicz, M. Grochowski, T. Rutkowski (2001). Robust estimation of integrated hydraulics and parameters in water distribution systems. *Proc. of the 4th ASCE Annual Water Distribution Systems Analysis, 2001 World Water and Environmental Resources Congress,* Orlando.

Chang, T., K. Duzinkiewicz, M. A. Brdys (2004). Bounding approach to parameter estimation of point-parametric models. *10th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Complex Systems, Osaka, Japan 2004.*

Duzinkiewicz, K. (2005). *Integrated Control of Drinking Water Supply and Distribution Systems*. DSc Thesis, AGH University of Science and Technology Publishers, Cracow.

Milanese, M., Norton, J.P., Piet-Lahanier, H., Walter, E. (1996). *Bounding approaches to system identification*. Plenum Press, New York and London.

Raïssi, T., Ramdani, N., Candau, Y. (2004). Set membership state and parameter estimation for systems described by nonlinear differential equations. *Automatica*, 40 (2004) 1771-1777.

Rutkowski, T., M. A. Brdys, K. Konarczak, T. Gmiński (2004). Set-bounded joined parameter and state estimation for model predictive control of integrated wastewater treatment plant systems at medium time scale. *10th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Complex Systems,* Osaka.

Schweppe, F. C.(1978). *Uncertain Dynamic Systems*. Prentice Hall Inc., USA.

Walter, E. and L. Pronzato (1997). *Identification of Parametric Models from Empirical Data*, Springer-Verlag, Berlin Heidelberg, New York Masson, Paris, Barcelona.