STATE FEEDBACK CONTROLLERS SYNTHESIS USING BMI OPTIMISATION FOR LARGE SCALE WEB HANDLING SYSTEMS

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Abstract: In web transport systems, the main concern is to control independently speed and tension in spite of perturbations such as radius variations and changes of set point. In this paper we present multivariable $\mathcal{H}_2$ controllers, with and without integrator, applied to winding systems. Different controller structures are considered: a centralized and a semi-decentralized controller with or without overlapping. The use of decentralized control leads to performance deterioration as compared to centralized control, but brings other advantages: on one hand, it allows for a local implementation close to each actuator, on the other, sensor or actuator failures create only local disturbances. Our approach consists in designing state feedback control, with bilinear matrix inequality (BMI) optimisation. Simulation results are given based on a nonlinear model for a web handling system, identified on an experimental bench.

Keywords: winding systems, large scale systems, $\mathcal{H}_2$ control, decentralized control, overlapping control, state feedback control, BMI optimisation.

1. INTRODUCTION

The systems handling web material such as textile, paper, polymer or metal are very common in the industry. The modelling and the control of web handling systems have been studied already for several decades. The increasing requirement on the control performances, however, and the handling of thinner web material led us to search for more sophisticated control strategies. One of the objectives in such systems is to increase web velocity as much as possible, while controlling web tension over the entire production line. This requires decoupling between web tension and speed, so that a constant tension can be maintained during speed changes. Robustness with respect to web elasticity variations is another important requirement. Achieving robustness not only provides a safe control of the web throughout the whole industrial process, but also permits to use the same controller for different types of web. So far, many industrial web transport systems have used decentralized PI-type controllers (as illustrated in Fig. 1). However, for higher control requirements more efficient control strategies must be used, e.g. LQG or $\mathcal{H}_\infty$ solutions (Zhou *et al*, 1995).

Most modern control law designs need the elaboration and validation of the plant model. A detailed description of the model we use being given in (Koç *et al.*, 2002), only the principal laws on which it is based will be recalled here.

Robust control has already been applied to web handling for reduced-size systems, containing no more than 3 motors, with multivariable $\mathcal{H}_\infty$ centralized controllers (Koç *et al.*, 2002). Due to the wide-range variation of the roller radius during the winding process, the dynamic behaviour of the system changes considerably with time. Thus, a substantial improvement has been obtained by the use of LPV controllers. Nevertheless, winding systems are generally of a larger scale (i.e. with a higher number of motors) and it is not suitable to use a centralized controller for such processes. These systems are therefore an adequate application for the decentralized control theories (Siljak, 1991). Recently, multivariable decentralized control strategies have been proposed for industrial metal transport systems (Geddes and Postlethwaite, 1998), (Grimble and Hearns, 1999) and for elastic web with $\mathcal{H}_\infty$ controllers (Van Antwerp, 2000), (Knittel *et al.*, 2002). In this last paper the global system is split into several subsystems, of an appropriate size allowing the synthesis for each one of its own multivariable controller. Each subsystem can be disjoined from its neighbours or can have overlapped parts with them. Both of these structures are considered in the present study. Off-diagonal elements of the plant ignored by a decentralized control lead to performance deterioration as compared with a centralized control. The stability analysis for such decentralized structures is presented in (Benlatreche *et al.*, 2004).

$\mathcal{H}_2$ control represents a new alternative for this category of industrial processes. (Doyle *et al.*, 1989) were among the first ones to give simple expressions for the design of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ controllers. Following works have dealt with various types of $\mathcal{H}_2$ optimisation problems, as for instance in (Chen *et al.*, 1993) via state feedback. The problem considered in that work belongs to the general singular type, which have a left invertible transfer matrix from reference input to controlled output. The authors construct and parameterize all static and dynamic $\mathcal{H}_2$-optimal state-feedback solutions. The paper of (Whorton *et al.*, 1994) was the first to present a novel homotopy algorithm to synthesize fixed order mixed $\mathcal{H}_2/\mathcal{H}_\infty$ compensators. The standard $\mathcal{H}_2$-optimal control problem is interpreted as a pole placement by (Kucera *et al.*, 2000). Since 2001 several works (Peaucelle *et al.*, 2001), (Arzelier *et al.*, 2002a and 2002b), have dealt with the resolution of BMI problems via an iterative method for $\mathcal{H}_2$, $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis via static output feedback.

The outline of the present paper is the following. Section 2 gives the development of the nonlinear model dedicated to the bench simulation and validated on our three-motor setup. Its linearization is used for controller design. A gain scheduling technique which can be used in adjunction with any controller, providing robustness to radius variations, is presented in section 3. Section 4 presents results in the synthesis of a state feedback controller using BMI optimisation for winding systems. The decentralized structure for a large scale web handling system is described in section 5. Simulation results are given in section 6. Finally, section 7 will conclude our work.

2. PLANT MODELING

A scheme of a three-motor setup with PI controllers is represented on Fig. 1. The inputs to system G_0 (defined by the dashed box) are the torque reference signals (u_u, u_v, u_w) of the brushless motors; its measurements are the web tensions T_u and T_w and the web velocity V. This velocity is imposed by the master traction motor whereas the web tension is controlled by the unwinding and winding motors.

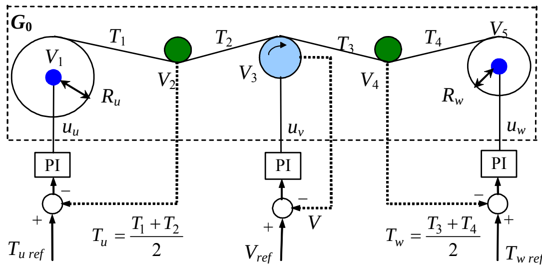

Fig. 1. Distributed PI control for a winding process.

The nonlinear model (Koç *et al.*, 2002) of a web transport system is built from the equations of web tension behaviour between two consecutive rolls and the equations describing the velocity of each roll.

2.1 *Web tension calculation.*

Modeling of web transport systems is based on three laws, which allow the calculation of web tension between two rolls.

Hooke's law: the tension T of an elastic web is a function of the web strain ε:

$$T = E\,S\,\varepsilon = E\,S\,\frac{L - L_0}{L_0} \tag{1}$$

where E is the Young elasticity modulus, S the web section, L the web length under stress and L_0 the web length without stress.

Coulomb's law: the study of a web tension on a roll can be considered as a problem of friction between solids (Koç, 2000).

Equation of Continuity (Koç *et al*., 2002) applied to the web gives:

$$L\frac{dT_{k+1}}{dt} = ES(V_{k+1}-V_k)+T_kV_k-T_{k+1}\left(2V_k-V_{k+1}\right) \quad (2)$$

2.2 Web velocity calculation.

The linear velocity V_k of roll k is obtained from the torque balance (Koç *et al*., 2002):

$$\frac{d}{dt}\left(J_k\frac{V_k}{R_k}\right) = R_k(T_k-T_{k-1})+K_kU_k+C_f \quad (3)$$

where K_kU_k is the motor torque assumed equal to the reference value and C_f is the friction torque. Note that both inertia J_k and radius R_k of unwinder and winder are time dependent and vary substantially during processing.

2.3 State space representation (Koç *et al*., 2002).

The nonlinear state-space model is composed of equation (2) for the different web spans and equation (3) for the different rolls. Under the assumption that J_k/R_k is slowly varying, which is the case for thin webs, V_k can be chosen as state variable in (3), leading to the following linear model:

$$E(t)\frac{dx}{dt} = A(t)x+B(t)u \qquad y = Cx \quad (4)$$

where: $x = \left[V_1\ T_1\ V_2\ T_2\ V_3\ T_3\ V_4\ T_4\ V_5\right]^{\mathrm{T}}$

$$y = [T_u\ V_3\ T_w]^{\mathrm{T}},\quad u = \left[u_u\ u_v\ u_w\right]^{\mathrm{T}}$$

3. GAIN SCHEDULING CONTROL

Let us consider the unwinder and the winder separately. With quasi-static assumption on radius variations, the transfer function between control signal and web tension appears to be inversely proportional to the radius:

$$\lim_{s\to 0}\frac{T_1(s)}{u_u(s)} \cong \frac{K_u}{R_u} \qquad \lim_{s\to 0}\frac{T_4(s)}{u_w(s)} \cong \frac{K_w}{R_w} \quad (5)$$

Based on this observation, a new plant G_R is obtained by multiplying the controller output signals u_u and u_w by the radius R_u and R_w respectively. This new plant has the advantage of making the gain at low frequency less dependent on radius and inertia. Therefore, gain scheduling improves robustness to radius variations and will be used in control strategies presented in the sequel.

4. SYNTHESIS OF STATE FEEDBACK CONTROLLER USING BMI OPTIMISATION FOR WINDING SYSTEMS

Here we consider the problem of controlling the 3-motor plant (Fig. 1) by using linear constant state feedback with limits on the feedback gains and the poles assignment region. In this application, the state variables can be measured directly. Indeed, the web tensions are measured directly by means of load cells and its speed by measuring the angular velocity of the roll, where it is assumed that there is no slip between web and roll. In this controller synthesis, the goal is to compute K satisfying (Hassibi *et al*., 1999):

$$\begin{aligned}&\min\ \alpha,\quad \text{subject to:}\\ &\left|K_{ij}\right| \le K_{ij,\max}\\ &(A+BK)^{\mathrm{T}}P+P(A+BK)<\alpha P\\ &P>0\end{aligned} \quad (6)$$

where A, B are the state matrices of the plant, K the controller and α the closed-loop decay rate. Also, the controller K is an $\mathcal{H}_2$ state-feedback controller, and will satisfy the following BMI:

$$\begin{aligned}&\min\ \gamma,\quad \text{subject to:}\\ &\begin{bmatrix}(A+BK)^{\mathrm{T}}P+P(A+BK)+C^{\mathrm{T}}C & PB\\ B^{\mathrm{T}}P & -\gamma I\end{bmatrix}<0\\ &P>0\end{aligned} \quad (7)$$

The unity static gain of the closed loop is ensured by a gain compensation matrix S, which is equal to the inverse static gain of the closed loop:

$$S = \left[C(BK-A)^{-1}B\right]^{-1} \quad (8)$$

Instead of using such a gain matrix S, the steady state accuracy can also be guaranteed by introducing one or more integrators. This is important for the nonlinear model. The new controller to design is then composed of two elements, K_1 and K_2, as shown on Fig. 2. This already known method is however new for BMI designed state-feedback control.

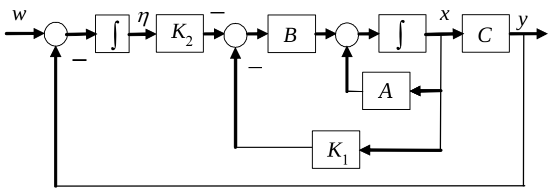

Fig. 2. State feedback control with integral action

The state space representation of this closed loop system is given by:

$$\begin{pmatrix} \dot{x} \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} A - BK_1 & -BK_2 \\ -C & 0 \end{pmatrix} \begin{pmatrix} x \\ \eta \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} w$$
$$y = \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} x \\ \eta \end{pmatrix} \tag{9}$$

Adapting equations (6) and (7) to the case of the feedback loop with integrator (Fig. 2) consists then in replacing the matrices *A*, *B*, *C* and *K* by $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{K}$ respectively, with:

$$\tilde{A} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}, \quad \tilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix},$$
$$\tilde{C} = \begin{pmatrix} C & 0 \end{pmatrix}, \quad \tilde{K} = \begin{pmatrix} K_1 & K_2 \end{pmatrix} \tag{10}$$

The problem of solving inequalities (6) and (7), with the data of equation (10), is known to be NP-hard. Initially, these BMIs were solved locally with the path-following method in (Hassibi *et al.*, 1999). In our case, we solved this problem by using the toolbox PENBMI of TOMLAB. Results are presented in section 6. The next step will consist in adapting this approach to a semi-decentralized strategy for our 9-motor web handling plant.

5. ROBUST CONTROL OF LARGE SCALE SYSTEMS

In industrial processes including a large number of actuators it may be inconvenient to use a global multivariable controller.

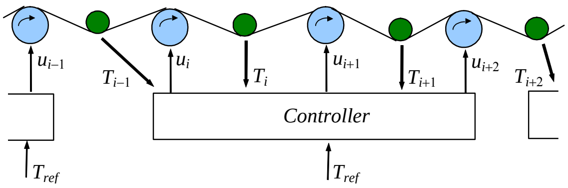

Fig. 3. Semi-decentralized control strategy for large scale systems

An alternative solution is to use semi-decentralized controllers, which reduces the controller dimensions (Knittel *et al.*, 2003a). The validation is made by simulation on a 9-motor plant. The system is split into three parts: each subsystem contains three motors and is controlled independently by its own controller (Fig. 3). This choice results from a trade-off between the number of subsystems and their respective size: a small number of subsystems reduces correlatively the number of controllers, which guarantees usually good global system performance, in terms of reference tracking and disturbance rejection, whereas a large number of subsystems, thus also of controllers, makes the system more robust to sensor and actuator failures.

To reduce the coupling existing between two consecutive subsystems, it may be worthwhile to introduce overlapping (Siljak, 1991), by letting two consecutive controllers share some inputs and outputs. For instance, input signals of tractors located at the boundary of two subsystems come from two controllers (Fig. 4). Such a decentralized overlapping control strategy has already given good results in the case of a vehicle platoon (Stankovic *et al.*, 2000).

In (Knittel, 2003b) it is shown that the control signal for the overlapped actuator results neither from the average nor from the sum of the outputs of the two controllers. However, the sum (*a* = 1 and *b* = 1 in Fig. 4) is a good approximation of the overlapped control strategy presented in (Siljak, 1991).

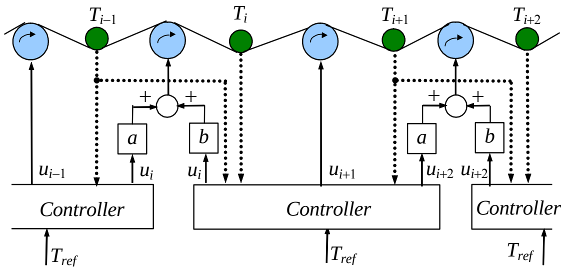

Fig. 4. Semi-decentralized weighted overlapped control strategy for a large scale system.

It is worth to point out that this is to our knowledge the first time that a decentralized control, with or without overlapping, is applied to a state-feedback controller.

6. SIMULATION RESULTS

Simulations using the nonlinear model have been performed in order to evaluate the improvements given by the control approaches exposed previously.

6.1 3-motor centralized control with and without integrators.

We have first applied the state-feedback controller, with and without integrators, synthesized by BMI optimisation as described in section 4, to the model of our three-motor nonlinear system. In Fig. 5 are shown the time responses for the web tension and the web velocity, with and without integral action.

It can be seen that the use of either a gain compensation matrix or an integral action yield both excellent results, by cancelling entirely static errors as well on the tension as on the velocity. In the case of the integral action, however, a slower transient response is observed, due to the extra poles added to the closed loop by the integrators. Also the additional output feedback loops induce in that case a stronger coupling between tension and velocity.

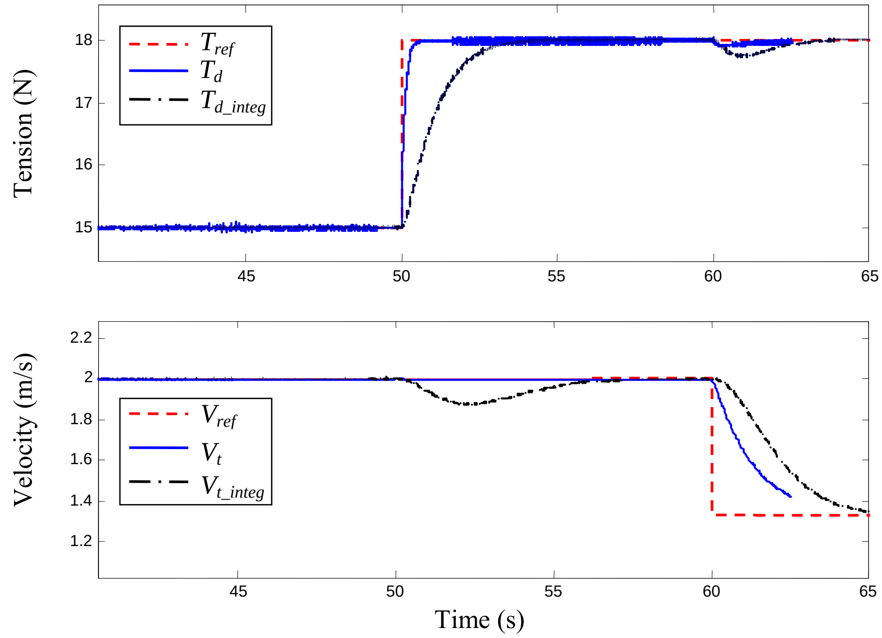

Fig. 5. Three-motor centralized control with and without integrators

6.2 9-motor decentralized control without overlapping.

For the 9-motor system the control strategy without integrator has been chosen, due to the excellent results which it had achieved on the 3-motor model and the additional computational load that the inclusion of these integrators would impose on the resolution of the BMI problem.
The nonlinear 9-motor system has been divided into three subsystems each of them including two tension controllers and one velocity controller dedicated to the traction motor. For this application, the gain compensation matrix (8) had to be calculated for the global system and not for the semi-decentralized structure.

Fig. 6 shows the time responses of the web tension and velocity for our simulation model of a nine-motor web handling system without overlapping.

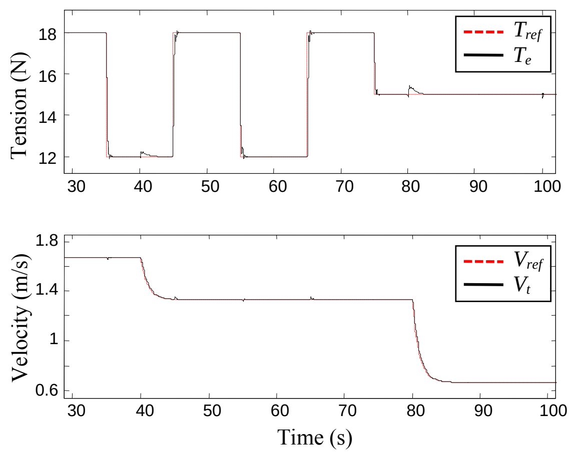

Fig. 6. Nine-motor decentralized control without overlapping

6.3 9-motor decentralized control with overlapping

In Fig. 7 are illustrated the time responses of the tension and the velocity of the web obtained with our nine-motor simulation model with overlapping.

It is obvious from Fig. 7 that the decentralized control with overlapping yields very good results, namely a good reference tracking, a very fast transient response, no overshoot, a vanishing static error and principally a very week coupling between web tension and velocity resulting from step changes.

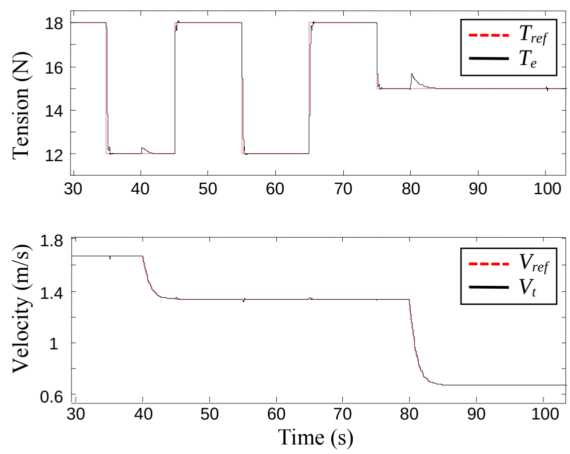

Fig. 7. Nine-motor decentralized control with overlapping

The results are as good as those presented in the precedent subsection. This is, in our opinion, due to the good performance achieved by the state feedback control synthesized by BMI optimisation.

Comparable performance has not yet been achieved by the use of an $\mathcal{H}_\infty$ structure (Knittel *et al.*, 2003a).

7. CONCLUSION

In this paper, a state feedback control with or without integrator, has been synthesised by BMI optimisation, with the aim of minimizing $\mathcal{H}_2$ performance of the closed loop. This control has then been applied first to the simulation model of a 3-motor web handling system. Very good results have been obtained, especially a good reference tracking with a vanishing static error.

The state feedback control, including a gain compensation term, has then been applied to a large scale web handling system, containing in our case nine motors. For that purpose two decomposition schemes have been investigated: a decomposition into disjoined subsystems and another one with overlapping of those subsystems. The control law applied to these two schemes has shown very good performance, which again would have been hard to achieve by a dynamical output feedback designed by $\mathcal{H}_\infty$ optimisation.

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