AN OPTIMAL DECISION MODEL FOR COASTAL AQUIFERS MANAGEMENT

Marco Antonellini[c,d], Giovanni Gabbianelli[c], Riccardo Minciardi[a,b], Michela Robba[a,b], Roberto Sacile[a,b], Elisa Ulazzi[c]

[a] DIST, Department of Communication, Computer and System Sciences, University of Genova
[b]CIMA, Interuniversity Center of Research in Environmental Monitoring
[c] Interdept. Center for Environmental Sciences Research, University of Bologna, 48100 Ravenna, Italy -
[d]Stanford University (CA, USA) – Department of Geological and Environmental Sciences, Stanford, USA

Abstract: A general decision model for sustainable groundwater management is described through the identification of the decision variables, and the formalization of the objective function and the constraints. The objectives of the decision problem represent the goals that are pursued by the management strategies, according to the specific exigencies of the decision makers, while the constraints represent limits to be respected, exigencies to be fulfilled, and can also be used to take into account the various aspects of the problem. The decision model is applied to the coastal freatic aquifer of the Cervia city that is polluted by salt water.

Keywords: Decision Support Systems, Water pollution, Opimization problems, Decision making, Management Systems.

1. INTRODUCTION

Water resources should be managed in a sustainable way in order to respect the ecosystems and to preserve the resource availability. Water management includes a wide set of correlated problems that should be taken into account because they strictly interact with water demand, water availability, and water quality. Specifically, the water demand for the different uses (agriculture, industry, drinking water, public use) should be satisfied, water quality standards (that should be different for the various water uses) must be respected, the ecosystem should be preserved, sustainable policies and regulations should be developed, technological solutions must be tested, etc. Bear (2000) underlines the main problems regarding seawater intrusion with specific attention to the necessary modelling efforts. Figure 1 shows the external "pressures" on groundwater quality that should be considered when building a decision model. Thus, it is essential to develop and apply integrated approaches able to take into account the multiplicity of aspects and objectives of the considered decision problems.

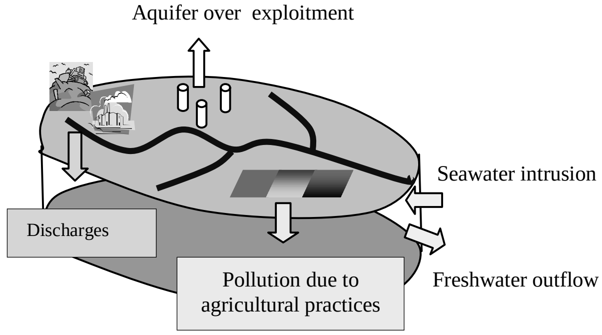

Figure1. The pressures on groundwater quality.

In this connection, to support the so-called integrated water management (IWM) problem an approach has been proposed based on the design of Decision Support Systems (DSSs) (Lombardo et al., 2003) integrating decision models, simulation models (used to analyze the water system behaviour under different viewpoints, i.e., hydraulic, chemical, biologic, etc...), and Geographic Information Systems (GIS) tools. In the literature, different techniques have been introduced to find viable solutions to IWM problems, most of which based on different mathematical programming techniques: linear programming, nonlinear programming, mixed-integer programming, optimal control techniques, differential dynamic programming, stochastic programming, combinatorial optimization and multiple objective programming (Das and Datta, 2001; Shamir and Bear, 1984; Psilovikos, 1999). The main challenge is to find appropriate simulation-optimization methods that are able to find helpful solutions to management problems. Das and Datta (1999) used the nonlinear finite-difference form of the steady state density dependent miscible flow and salt transport model for seawater intrusion in coastal aquifers, embedding it within the constraints of the management model. Besides, a number of nonlinear optimization-based multiple-objective management models for sustainable utilization of coastal aquifers have been formulated and solved. In this paper, a general decision model for coastal aquifer management is proposed. First of all, specific decision variables are described and then the objective functions and the constraints are formalized. The model has been applied to the case study of the coastal freatic aquifer of the Cervia municipality, that is polluted by salt water because in most of the aquifer the hydraulic head is not able to contrast the saltwater intrusion at its base. A major impact of this pollution regards the death of the pinewoods present in the area. Indeed, this is not a very special case, since similar problems arise for other kinds of vegetation.

2. THE SYSTEM MODEL

The state equations that can be used to represent the behaviour of the groundwater system are relevant to the dynamics of the water flows (hydraulic model), and of the concentration of pollutants (chemical model). The characteristics and activities relevant to the territory under concern can be represented through a schematic partition of the considered territory into cells. Figure 2 shows the positive (precipitation P, irrigation water I, leakage L, surface water interactions S) and negative (evapotranspiration E, extracted water Q) contributions to the water balance of two generic neighbouring cells *i* and *j*.

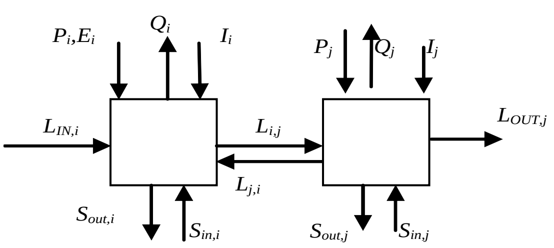

Figure2. Water flows relevant to two generic neighbouring cells.

In the present work a generalized model is formalized for a generic cell *m*, *m=1,…, M*, where *M* is the number of cells. In the following, a description of a general discrete physical/chemical model is proposed, considering the aquifer divided in cells and quantifying the positive and negative contributions to every cell. For the sake of simplicity, every cell is described by two kinds of state equations: the first kind describes the water balance, whereas the second one describes the mass balance of the various pollutant concentrations. As regards the mass balance the only pollutant that is taken into account is salt concentration.

2.1 Water balance state equations.

The state variables appearing in the generalized water balance state equations are represented by H_m^t (the hydraulic head in cell *m* at time *t* [m], *m=1,…,M*). The control variables appearing in the water balance state equations are

- Q_m^t: the water extracted from cell *m* in time interval (t, t+1) [m^{3}s^{-1}];
- I_m^t: the irrigation water that enters cell *m* in time interval (t, t+1)
- S_m^t: the controllable water flow in the surface canals in cell *m*; in fact, because of their permeability, water infiltrates in the aquifer.

Some of the variables affecting the behaviour of the state variables relevant to cell *m* are functions of the state variables of different cells. They are:

- $L_{IN,m}^t$: the overall water flow entering the generic cell *m* in time interval *(t, t+1)* [m^{3}s^{-1}], *m=1,…,N*;
- $L_{OUT,m}^t$: the overall water flow leaving generic cell *m* in time interval (*t*, *t+1*) [m^{3}s^{-1}], *m=1,…,N*;

Finally, the following quantities are assumed to be known for every cell *m*: a parameter ε_m [m^{-2}], which is a function of the parameters that describe the geometric characteristics of the cell and soil properties, and E_m^t [m^{3}s^{-1}], that is a flow taking into account a certain set of physical effects (precipitation, evapotranspiration, water run off, etc.) that take place in cell *m*, within time interval (*t*, *t+1*). Then, the state equation representing the water balance for a generic cell *m* can be written as (Minciardi et al., 2004)

$$H_m^t = H_m^{t-1} + \varepsilon_m \left(\begin{array}{l} \sum_{k=1}^{M} \max\left(\bar{\varepsilon}_{km}\left[H_k^{t-1} - H_m^{t-1}\right], 0\right) + \\ -\sum_{k=1}^{M} \max\left(\bar{\varepsilon}_{km}\left[H_m^{t-1} - H_k^{t-1}\right], 0\right) + \\ -Q_m^{t-1} + S_m^{t-1} + E_m^{t-1} \end{array} \right) \Delta t$$

$$m=1,\dots,M \qquad t=1,\dots,T \qquad (1)$$

where $\bar{\varepsilon}_{km}$ is a parameter, to be estimated, that depends on the geological and geometric characteristics of the boundary layer between cell *k* and *m*. It is important to note that, for a freatic aquifer $\bar{\varepsilon}_{km}$ is function of the hydraulic heads.

2.2 Mass balance state equations.

The mass balance equations may be obtained by considering the water balance (state) equations **(1)** and multiplying each water flow appearing in such equations by the corresponding pollutant concentration. This computation gives rise to a new set of state equations. The complete formalization of the state equation representing the water quality balance for cell *m* (*m=1,…,M*) and pollutant *p* is reported in Minciardi et al. (2004).

3. THE MANAGEMENT PBROBLEM

As mentioned in the previous section, the control variables of the system are: the overall extracted water, the flow for agriculture, and the water flow that in the surface canals because it percolates in the aquifer. As regards the state variables, they are: the hydraulic head and the salt concentration in every cell *m* at time *t*.

Several objectives should be taken into account when managing water resources exploitation within an integrated framework. Such objectives are:

1. minimizing the economical costs and maximizing the benefits;
2. minimizing water demand dissatisfaction (with respect to the expressed aspiration levels);
3. containing the overall concentration of salt in the aquifer;
4. minimizing the difference between the actual and the ideal (as regards the pinewood health) values for the hydraulic head and the salt concentration.

Objective 1: minimizing the costs. The costs to be considered are pumping costs. Pumping costs include the costs due to the energy used to lift water from the well. It is reasonable to represent the pumping costs CP_m [€/year] relevant to the cell *m* as

$$CP_m = \sum_{t=1}^{T} CPu_m Q_m^t (\bar{H}_m - H_m^t) \quad m=1,\dots,M \qquad (2)$$

where $\bar{H}_m$ is a parameter representing the height of ground level above an impermeable soil layer, and CPu_{m} is the unit cost for the energy used to lift water in cell m . Thus, Objective 1 is given by

$$J_1 = \sum_{m=1}^{M} CP_m \qquad (3)$$

Objective 2: minimizing water demand dissatisfaction. The main kinds of water demand to be satisfied regard irrigation, drinking water and industrial/public use. The target is to satisfy, if possible, the water demands, estimated on the basis of local exigencies. Specifically, let us use the notation D_m^t for the overall water demand in cell *m* at time t. Objective 2 to be minimized is given by

$$J_2 = \sum_{t=1}^{T}\sum_{m=1}^{M} \max\ (D_m^t - Q_m^t, 0) \qquad (4)$$

Objective 3: containing the overall pollutant concentration in the aquifer. The concentration of pollutant in every cell of the aquifer $C_{m,p}^t$ should be compared with a reference value $\bar{C}_p$ that corresponds to an ideal concentration limit. Objective 3 is expressed by

$$J_3 = \sum_{t=1}^{T}\sum_{m=1}^{M} \max\left(\frac{C_{m,p}^t}{\bar{C}_p} - 1, 0\right) \qquad (5)$$

Objective 4 and 5: ecological requirements. The health of the vegetation that characterizes coastal areas is strongly influenced by the salt concentration and by the hydraulic head that characterize the groundwater system. Specifically, vegetation suffers for a high salt concentration, and for too big and too low hydraulic heads. As a consequence, Objective 4 and 5, to be minimized, can be expressed as

$$J_4 = \sum_{t=1}^{T}\sum_{m \in \bar{M}} (H_m^t - \hat{H}_m)^2 \qquad (6)$$

$$J_5 = \sum_{t=1}^{T}\sum_{m \in \bar{M}} C_{m,p}^t \qquad (7)$$

where $\bar{M}$ represents the cells with an ecosystem in which vegetation should be preserved and $\hat{H}_m$ is ideal hydraulic head for vegetation. The other symbols are weight coefficients.

The overall objective function. The overall objective function is given by the weighted objectives previously formalized. Specifically, the overall objective function to be minimized is

$$Min \quad \sum_{i=1}^{5} \gamma_i J_i \qquad (8)$$

where γ_i is the weight for the *i-th* objective.

3.3 The constraints

The constraints that are necessary to build up a decision model for planning purposes belong to different classes: chemical constraints (as regards the pollutant concentration), constraints corresponding the state equations, water demand constraints, and technological constraints. In the following, all the mentioned classes of constraints are discussed and formally introduced.

Concentration constraints. Such constraints impose that the pollutant concentration in every aquifer cell is less or equal to a pre-defined value $C*_m$, that is to say

$$C_m^t \le \overline{C}* \qquad m=1,\ldots,M \quad t=1,\ldots,T \qquad (9)$$

Water demand constraints. Indicating with $D_m^{\min}$ the minimum water demand to be satisfied n cell *m*, the following constraints can be formalized,

$$\sum_{m=1}^{M} Q_m^t \ge D^{\min,\,t} \qquad t=1,\ldots,T \qquad (10)$$

Technological constraints. This class of constraints allows considering technological characteristics of the treatment plants and the pumps. Specifically, for the pump-sizing, it can be stated that the pumped water must be comprehended between a range of values,

$$\overline{Q}^{\min} \le Q_m^t \le \overline{Q}^{\max} \qquad m=1,\ldots,M \;\; t=1,\ldots,T \qquad (11)$$

where $\overline{Q}^{\min}$ and $\overline{Q}^{\max}$ represent, respectively, the minimum and the maximum allowable rates.

State equation constraints. The state equations, embedded as constraints, come from water and water quality balances (see Minciardi 2004).

4. APPLICATION TO A CASE STUDY

The model has been applied to the Cervia municipality, where the coastal freatic aquifer is polluted by salt water. The area of study has an extension of 20 km^2. Starting from the 50's (the coastal area underwent an intensive urban development that brought to 50% coverage of the land by buildings and paved surfaces. The remaining territory consists of farmland and pinewood trees that belong to the Regional Park of the Po (see Figure 3).
Groundwater winning from deep confined aquifers and development of offshore gas fields caused a total subsidence in the period 1950-2003 of 0.42 meters.
A dense network of land reclamation drainage canals that is connected to large water scooping machines characterizes the territory. The most important drainage canals are the Scolo Cupa and the Canale Mesola di Montaletto that cuts through the study area and enters the harbor canal of Cervia.
The roman age Cervia's saltworks are located 1.5 km inland (Figure 3) Salt water is brought to the saltworks via the *Canale del Pino*. The harbor canal and the *Canale del Pino* allow for surface salt water intrusion in the area. We individuated 504 water wells in activity within the area investigated; 357 wells have a depth ranging from 3 to 10 m, whereas the other wells have larger depth. About half of the wells are used to water gardens and the other half are used to tap freshwater in the beach seaside resorts during the summer period.

Figure 3. The study area

Climate data concerning temperatures and rainfall for the hydrologic balance have been supplied by the meteorological service of the Emilia-Romagna Region. The other hydrological data have been recovered from published data.
The monthly field monitoring campaign over a period of 1 year (June 2002 – may 2003) has been done on a total of 187 points (piezometers and canals) and included water table depth, temperature and electric conductivity (converted in salinity values using Lewis e Perkins (1981) equation (Unesco 1983)). The collected data have been introduced in a GIS system. The freatimetric maps evidence a water table depth located a few centimetres above the mean sea level. Consequently, in most of the aquifer the hydraulic head does not contrast the salt water intrusion at its base. The only areas placed above sea level are along the coastline and along the present canals because of their influence on the water table. The water table changes seasonally with a mean range of approximately 0.9 m, with the maximum value of 0.82 m a.s.l in winter and minimum value in summer of -1.06 m m.s.l.. During the fall-winter season the water table presents more areas above the m.s.l., thanks to the greater precipitations and the larger amount of water present in the drainage canals. The surface salinity maps evidence that salt water intrusion from the harbor canal and the Canale del Pino, and is also apparent in proximity to the sea outlets of the drainage canals. The salinity is also high in proximity of the pinewoods and to the water scooping machines, as a consequence of the salt water pull up from the bottom caused by the water table

falling due to the pinewood and the waterwork systems. The aquifer system has been divided in cells, which are homogeneous characteristics of the state variable values. The 2D representation of the system is provided in Figure 4. Specifically, there are four typologies of cells that are related to land use considerations (1=agricultural area; 2=pinewood; 3,5=urbanization; 4= beach). Moreover, in Figure 4, two kinds of canals of surface water (S-freshwater canals and R-saltwater canals) are represented, which interact with the cells in which they pass through. The position of pumps and water machines that extract water in the studied area are also indicated.

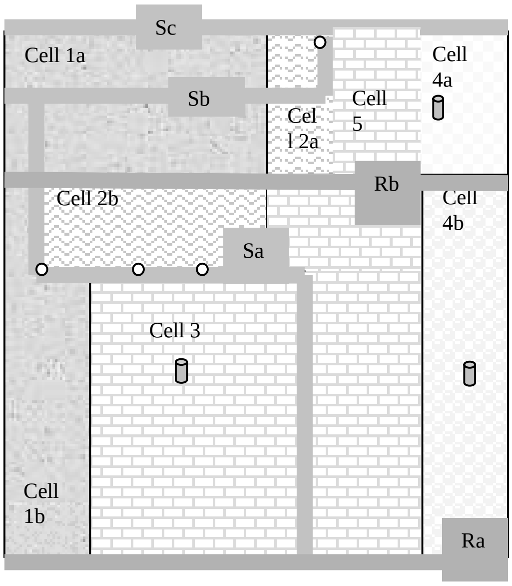

Figure 4. The division of the coastal aquifer in cells.

For the sake of simplicity, as a preliminary approach, the optimization problem has been solved for cells 1b, 2b, 3, and 4b, since they have the highest impact on the dynamics of the system. With the aim of testing the physical/chemical model, at first the decision variables have been imposed to be equal to the values correspondent to a known management period (of which also the values for hydraulic head were known). Figure 5 and 6 plot the simulated values for hydraulic heads in cells 4b and 2b, respectively with respect to the collected data.

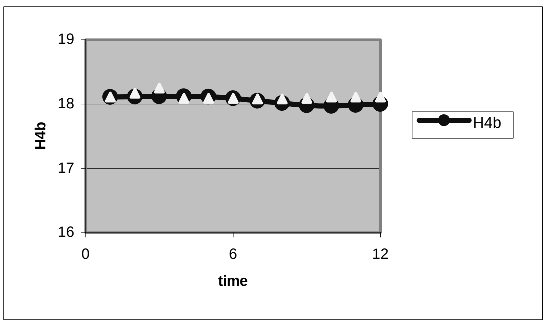

Figure 5. Simulated data for hydraulic head in cell 4b versus collected data in cell 4b.

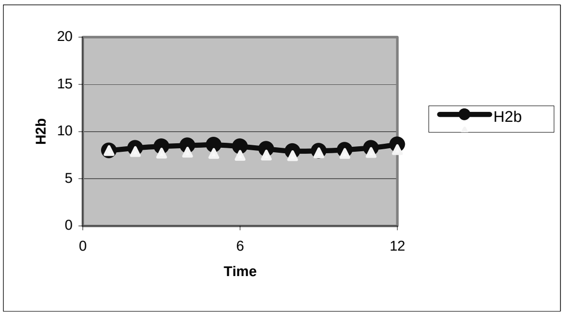

Figure 6. Simulated data for hydraulic head in cell 2b versus collected data in cell 2b.

The optimization problem has been solved by taking into account Objectives 1, 4 and 5 because they seemed to be the most meaningful for the case study. In Table 1 are reported the values of the parameters present in equations (6), (8)-(11).

Table 1.Parameters present in equations (6), (8)-(11).

$\gamma_1 = \gamma_5 = 1$	$\hat{H}_m = \hat{H}_{2b} = 7$	$\overline{Q}^{\min} = 0$
$\gamma_2 = \gamma_3 = 0$	$\overline{C}*_p \leq 22$ (only for m=2b)	Only for cell 4b $D^{\min,5} = 8000$ $D^{\min,6-9} = 10000$
$\gamma_4 = 10^7$	$\overline{Q}^{\max} = 100000$	

The control variables for the optimization problem are $Q_{4b}, Q_{1b}, Q_{2b}, Q_3$ (for cells 4b and 3 these variables correspond to water pumped by wells, while for cells 1b and 2b they correspond to water extracted by water machines). Table 2 reports their optimal values for a time horizon of twelve months (the time interval is one month).

Table 2. The optimal values of the control variables.

Time	Q4b	Q1b	Q3	Q2b
1	0	0	0	100000
2	0	0	0	100000
3	0	0	0	69616
4	0	0	0	40011
5	8000	0	0	0
6	10000	0	0	0
7	10000	0	0	0
8	10000	0	0	8524
9	10000	0	0	19024
10	0	0	0	38200
11	0	0	0	59048
12	0	0	0	0

Figures 7 and 8 report the values of H_{2b} and the concentration for all the cells.

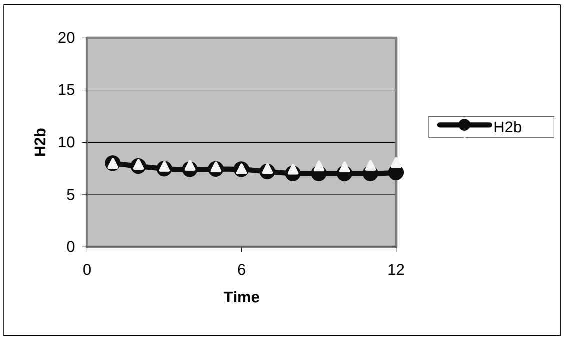

Figure 7. Optimal values for the hydraulic head in cell 2b.

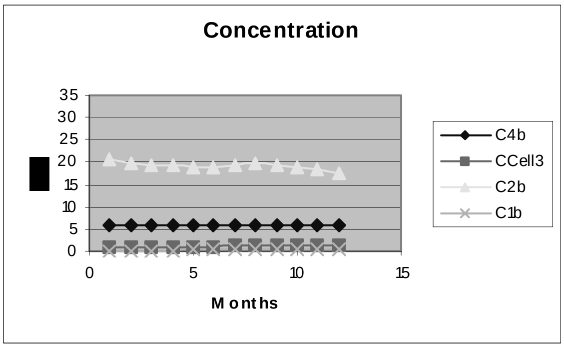

Figure 8. Optimal values for concentration in the considered cells.

Finally, the opimization problem has been solved changing the weight coefficients in the objective function in order to give an higher weight to the water quality of the cell with the pinewood (cell 2b). Specifically, $\gamma_1 = \gamma_4 = 1$ and $\gamma_5 = 10^7$. Figure 9 and Table 3 present the results of the optimization problem in this case. It is evident that, in order to diminish salt concentration, it is necessary to change the pumping schedule of water extracted in cell 2b.

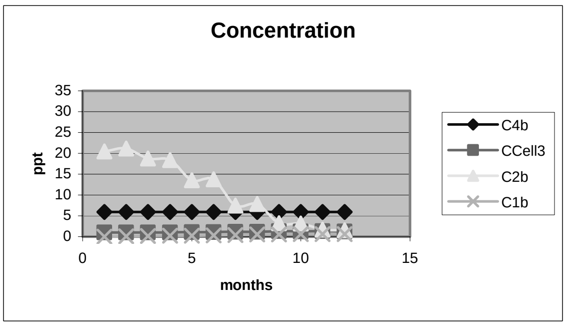

Figure 9. Optimal values of salt concentration in the considered cells.

Table 3. The results of the optimization problem.

Time	Q4b	Q1b	Q3	Q2b
1	0	0	0	100000
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	8000	0	0	0
6	10000	0	0	100000
7	10000	0	0	0
8	10000	0	0	100000
9	10000	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0

5. CONCLUSIONS

In this work, a decision model for groundwater management, capable of integrating different models (hydraulic model, chemical model) and different issues (economic, environmental, ecological), has been presented. The physical/chemical model is based on water and mass balances, considering a multicell scheme, with rectangular cells, following a general formalization approach that provides the opportunity of choosing the desired degree of accuracy of the discretization. The model has been applied to the coastal freatic aquifer of the Cervia municipality, where the saltwater affects the aquifer and creates problems to the preservation of the pinewoods present in the area. The developed DSS represents a first prototype able to manage groundwater resources in coastal aquifers. Future developments regard a calibration of the physical-chemical models, a formalization of the problems considering multiple decision makers, and a deep sensitivity analysis for the considered case study.

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