DESIGN CONSIDERATIONS FOR ELECTROMECHANICAL ACTUATION IN PRECISION MOTION SYSTEMS

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Abstract: In this paper the actuation system, often encountered high-precision motion systems, is studied. To find out the limitations of the actuation system, a integrated electromechanical model of the amplifier, actuator and mechanics is derived. This is done in a system's theory-approach, so that amplifier feedback circuit design can be tuned using loop-shaping techniques. Stability analysis is carried out for the amplifier feedback network design, including relevant uncertainties in the actuator under working conditions. The important limiting properties of the amplifier are discussed, and translated into constraints for setpoint design. Further, ideas for improvement of existing actuation systems are given.

Keywords: setpoint design, motion control, mechatronic modeling, transconductance amplifier, voice-coil actuator

1. INTRODUCTION

In current high-level motion control applications speed and accuracy are the major concerns. Mechanical design is optimized to ensure reproducibility and to prevent performance limiting artifacts as backlash and friction. Apart from proper feedback design, feedforward control and setpoint design become more and more important in high-performance applications (Lambrechts *et al.*, 2005), as it can provide over 90 % of the system's tracking accuracy. To obtain this high performance, exact translation from feedforward signal to the actual actuator force is desired. Although often regarded as ideal, the servo amplifier and corresponding electronic feedback circuit can be the limiting factor on this respect. It can decrease the intended performance improvement of a sophisticated feedforward design. In this paper, the important limiting properties of the actuator systems are discussed, and translated into constraints for setpoint design. Starting point is a detailed model of the actuation system, i.e. a current-controlled Lorentz' actuator with amplifier. Next stability analysis for the amplifier's feedback design is discussed. Section 3.1 depicts in what way the actuation system can be adjusted to deal with more severe input signals. The theory is illustrated with practical examples.

2. ELECTROMECHANICAL MODELING

In motion control applications, the amplifier is often considered as a one-sided coupled static system with a control voltage u_i going in, and a current i_o coming out. However, in reality the amplifier circuit shows internal dynamics and there exists mutual coupling between the amplifier, actuator and the mechanics of the motion system Fig. 1. The amplifier creates a voltage u_a across the actuator coil, which results in a current i_a. This current is controlled by the amplifier's feedback circuit. As a result of the actuation force

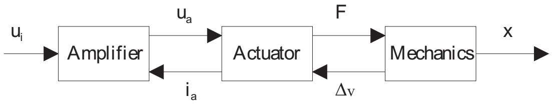

Fig. 1. Bilateral couplings in the actuation chain in terms of flow and effort.

F, the mechanics start moving over distance x. The speed difference in the actuator Δv results in an induced voltage, which also influences the behavior of the amplifier. For a correct analysis of the amplifier dynamics, both mechanical and electrical subsystems should be modeled. First actuator, amplifier circuit and mechanics are modeled individually, after which the integrated model is discussed.

2.1 Lorentz' actuator modeling

The Lorentz' actuators, also called air core actuators, are often used in precision applications. They enable high accelerations and have a smooth force characteristic. They are available in different topologies (e.g. Fig. 2), but all have the same principle of force production. The force produced

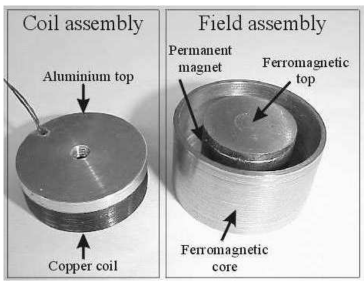

Fig. 2. Typical Lorentz' actuator topology.

by interaction of the coil current and the magnetic field can by calculated by Lorentz' formula:

$$F = lBi_a = K_F i_a \tag{1}$$

where l is the total length of the wire, B is the average magnetic flux density in the radial direction in the volume of the coil, i_a is the current of the wire. K_F is called the force constant of the actuator. Unfortunately, Eq. 1 does not include the complete dynamical behavior of the actuator. The current i_a is dynamically related to the input voltage u_a, as follows from the simplified electrical circuit of a voice coil actuator shown in Fig. 3. The actuator coil has a certain resistance R_c and inductance L_c. Both effects result in a voltage drop: $u_r = iR_c$ and $u_l = L_c\frac{di}{dt}$. Furthermore, the speed difference δv between the field assembly and the coil results in a back-electromotive voltage u_{EMF}, quantified by the constant K_B. The relation between the input voltage and the current is given by Kirchoff's voltage law:

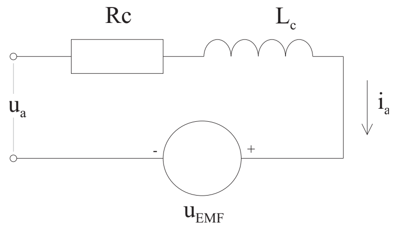

Fig. 3. Equivalent circuit representation of the Lorentz' actuator (adopted from (Magnetics Devision, 2002)).

Parameter	Min.	Max.	Max. dev.	Dep.
R_c [Ω]	1.95	2.93	40 %	T
L_c [μH]	739	639	14 %	x
K_F [$\frac{N}{A}$]	2.98	3.83	25 %	T,x

Table 1. Parameter deviations of a typical Lorentz'actuator.

$$\begin{aligned} u_a &= u_R + u_L + u_{EMF} \\ &= R_c i_a + L_c\frac{di_a}{dt} + K_B\Delta v \end{aligned} \tag{2}$$

Unfortunately, the parameters in this linear model are not constant during duty of the actuator. The force at a constant current deviates from its maximum value by 10% up to 20% over the complete stroke of the actuator. Furthermore, the magnetic flux produced by the permanent magnet is thermal dependent, resulting in non-constant K_F (see Fig. 4). The parameters R_c, L_c are also

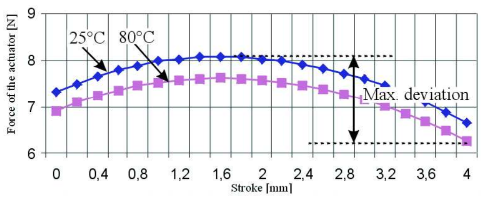

Fig. 4. Typical force characteristic for an Lorentz' actuator: position and thermal dependent

changing during the operation of the actuator. An example of the most important parameter tolerances, due to position x or temperature T dependency, are listed in Table 1.

2.2 Amplifier modeling

The amplifier converts the output signal of the digital control system u_i into a proportional current i_o. This type of amplifier is called voltage-to-current convertor (V/I convertor) or transconductance amplifier. Although there exists specific transconductance amplifier designs (Mills and Hawksford, 1989), often a standard power amplifier is adapted to do the job. The nominal behavior of amplifiers with good buffering properties, high input impedance and low output impedance, can be studied using Black's unilateral amplifier model ((Nordholt, 1983) p.6). It makes use of the absence of significant coupling between the source and load of the amplifier. Op-amp (operational amplifier) circuit design is based on this model and is ideally suited for a system's-theory based analysis (e.g. see (Dostál, 1993)).

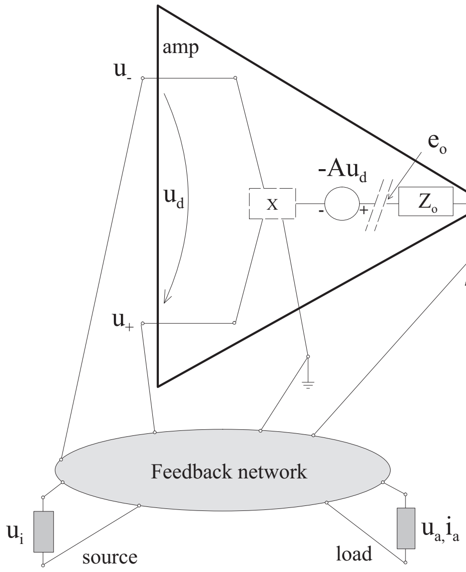

Fig. 5. Idealized amplifier model. Internal dynamics are simplified to the linear transfer $A(s)$ and output impedance Z_o. Other dynamics (captured in X) are assumed to be ideal.

The functionality of op-amp circuits, but also amplifiers in general, is mainly dependent on the dynamics of the feedback network, which is designed around the inputs and outputs of the amplifier and is also connected to both the source and the load (Fig. 5). The open-loop gain $A(s)$ of the amp is defined as the transfer from the differential input voltage u_d $(u_+ - u_-)$ to the output voltage e_o (see Fig. 5). Since load conditions of the amplifier are severe in this application, the output impedance Z_o is included, and the actual output voltage becomes u_o. In general, the static gain A_0 is high and there is one low-frequent dominant pole τ_1, limiting the open-loop bandwidth (Eq. 3). Within the bandwidth of the closed-loop system, the amplifier dynamics can be often approximated by $\frac{A_0}{\tau_1 s}$.

$$A(s) = \frac{E_o(s)}{U_d(s)} = \frac{A_0}{(1+\tau_1 s)(1+\tau_2 s)\dots} \approx \frac{A_0}{\tau_1 s} \quad (3)$$

By breaking the loop between amplifier and feedback network somewhere, the resulting open-loop system can be used for analysis using basic feedback theory. An intersection before the output voltage e_o is a good choice for easy analysis (i.e. using Black's model), since only unilateral coupling is present here (see Fig. 5). The open-loop transfer from e_o to the differential input voltage u_d is defined as the *feedback factor* $\beta(s)$. It may be clear that the open-loop is given by $A(s)\beta(s)$. The closed-loop transfer from u_i to the actual output voltage or current can be simplified to Eq. 4, in which G_∞ is defined as the closed-loop gain with an ideal amplifier $A \to \infty$. Here the direct/feedforward coupling through the feedback circuit is neglected since it is very small (see (Dostál, 1993) p.128 for details).

$$G(s) \approx G_\infty(s)\frac{\beta(s)A(s)}{1+\beta(s)A(s)} \quad (4)$$

The most basic way to turn a power amp into a transconductance amplifier is depicted in Fig. 6. By the feedback circuit, the current through the actuator $i_a = i_o$ is converted into a feedback voltage u_- by a shunt resistor ($Z_s = R_s$). Ideally, the input voltage u_i is now linked to the output current by $i_o = \frac{1}{Z_s}u_i$, so $G_\infty = \frac{1}{Z_s}$. The feedback

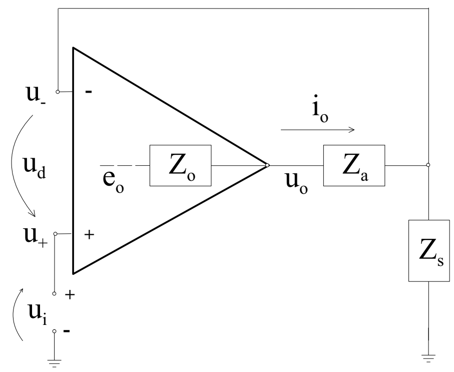

Fig. 6. Op-amp as a transconductance amplifier, with Z_{act} the actuator impedance and Z_s the shunt impedance.

factor $\beta(s)$ for this topology, which includes the actuator dynamics, is given by:

$$\beta = \frac{Z_s}{Z_s + Z_{act} + Z_o} \quad (5)$$

2.3 Mechanical modeling

Under normal working conditions, the motion controller attenuates the effect of resonances. Despite of this, the resonant behavior is included in the model to see if this effects the performance of the actuation system. The transfer from an input force F_a at the actuator position p_a to the displacement at this position x_a is given in Eq. 6.

$$H_{aa}(s) = \frac{X_a(s)}{F_a(s)} = \frac{1}{m_{aa}s^2} + \sum_{r=1}^{\infty}\frac{\phi_r(p_a)^2}{s^2+w_r^2} \quad (6)$$

$$= \frac{1}{m_{aa}s^2}H_{res}(s) \quad (7)$$

Since input and output positions are equal, the modal contributions ($r = 1\dots\infty$) are all positive ($\phi^2(p_a)$), which results in a collocated transfer. In multiplicative form it can be given as a pure mass system multiplied by a residual transfer $H_{res}(s)$ with unity gain for low frequencies Eq. 7 and phase between 0 and $-180°$.

2.4 Integrated model

The mechanical model is used to express the relation between the current i_a and u_{EMF}. By definition, this can be regarded as the EMF-impedance Z_{EMF} (using Eq. 2). Using $\mathcal{L}\Delta v = sX_a(s)$, the EMF-impedance can be expressed as:

$$Z_{EMF} = \frac{U_{EMF}(s)}{I_a(s)} = \frac{K_B s X_a(s)}{\frac{1}{K_F}F_a(s)} = \frac{K_B K_F}{m_{aa}s}H_{res}(s)$$

With this relation and Eq. 2, it is possible to rewrite the feedback factor Eq. 8. In general, the two poles are well separated (as in Fig. 7).

$$\beta(s) \approx \frac{R_s s}{L_c s^2 + (R_s + R_c + R_o)s + \frac{K_B K_F}{m_{aa}}H_{res}(s)} \tag{8}$$
$$\approx \frac{\beta_0 s}{(1+\tau_m s)(1+\tau_e s)}$$

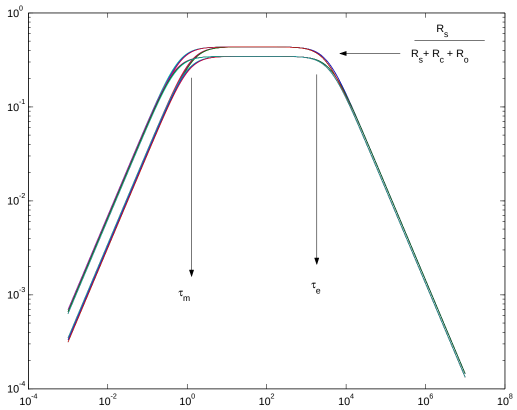

Fig. 7. Amplitude plot of $\beta(s)$.

2.5 Stability analysis

The feedback factor $\beta(s)$ is crucial for the stability of the amplifier loop $G(s)$, since the open-loop is given by $A(s)\beta(s)$. Like other SISO control problem, this loop can be constructed by loopshaping techniques. By analysis of the open-loop (see Fig. 8, it is clear that the influence of the mechanics is a 90° phase lead at low frequencies. Therefore, the resonance behavior H_{res} will not cause low-frequent stability problems. Since it does not significantly affect the behavior at higher frequencies $> \frac{1}{\tau_e}$ (see Eq. 8), H_{res} is neglected in the remainder of this paper. The bandwidth is preferably chosen as high as possible in order to approximate the ideal gain, which seems to be easy because of the intrinsically high gain A_0 of the amplifier. However, serious stability problems can occur: the bandwidth of amplifier will exceed the τ_1 and also τ_e, so the open-loop will be around $-180°$ at 0dB (see Fig. 8). Furthermore, with the uncertainties

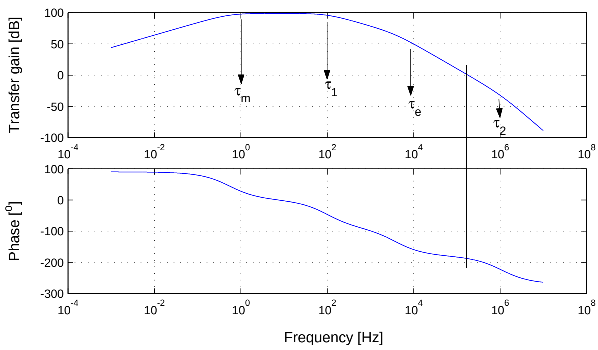

Fig. 8. Open-loop $A(s)\beta(s)$. Without compensation, the closed-loop can easily be unstable or very badly damped.

of the actuator parameters it is hard to predict the $0dB$ point exactly. The topology Fig. 6 should be adapted to create a robustly stable closed-loop without a badly damped resonance peak. Here two approaches are presented which can also be combined:

- *Decreasing loop-gain.* The loop-gain cannot be changed by the shunt resistance, since this determines the gain (transconductance) of the amplifier $G_t = G_\infty$. By using local voltage feedback around the original power amplifier (`pa` in Fig. 9), the effective gain $A(s)$ of the amplifier is decreased. A voltage invertor circuit can be used for this purpose (e.g. see p.140 (Dostál, 1993)). The disadvantage of this method is a lower bandwidth of the closed-loop.
- *Phase compensation.* Creating phase-lead in the loop can also solve our problem, without decreasing the bandwidth drastically. A modified feedback circuit (`fb`) as presented in Fig. 9 can do this job. The new transfer of the feedback factor can be approximated:

$$\beta_{new}(s) \approx \frac{R_3}{R_2+R_3}\frac{1+R_2Cs}{1+\frac{R_2R_3}{R_2+R_3}Cs}\beta_{old}(s) \tag{9}$$

  This circuit will automatically decrease the low-frequent gain by $\frac{R_2}{R_2+R_3}$, which is also inversely related to the ratio between the pole and the zero location of the phase-lead. Since this gain also affects the gain of the amplifier G_∞, an additional op-amp `oa` with local feedback can be used to set G_∞.

With these modification options is should be possible to create a well-damped transconductance amplifier with reasonable bandwidth and an easily adjustable gain G_t.

3. LIMITATIONS

In general the bandwidth of the amplifier is much higher than the overal motion control loop. Furthermore, we will assume that the frequency content of the designed feedforward signal F_{ff} is also

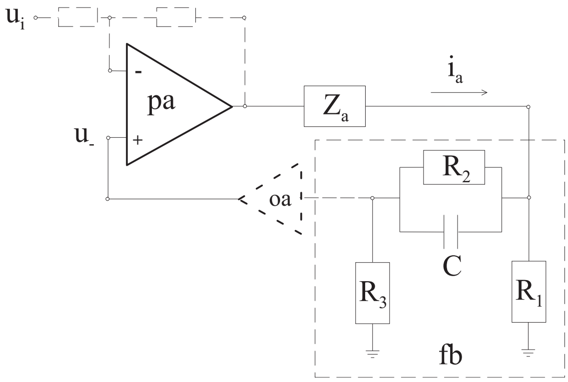

Fig. 9. Adapted transconductance feedback circuit.

limited within the bandwidth of the amplifier, which means we can assume that the actuation force is proportional to the input voltage:

$$F_a = K_F i_a = K_F G_t u_i \tag{10}$$

In other words, within the linear range of the amplifier, given by their *small signal* behavior, no limiting effect are expected. The *large signal* properties of an electric system represents its behavior outside its linear range. In general these properties are the result of internal signal limitations (i.e. currents and voltages) in the amplifier. Unfortunately, this behavior can directly limit the performance of feedforward control. In high-precision applications, setpoint design is often carried out by addition of piecewise polynomial functions. In this way, the reference signal $x(t)$ can be limited by limiting the maximum values for velocity v, acceleration a, jerk j and even snap or djerk s. The corresponding feedforward signal can directly be related to these values (i.e. neglecting resonances):

$$F_{ff} = m_{aa}a \qquad \Leftrightarrow \qquad i_a = \frac{m_{aa}}{K_F}a \tag{11}$$

The three most important limitations and consequences for feedforward design are listed below (see also Fig. 10).

- *Limited output current*, $\pm i_{max}$. Often the current is regulated by means of an internal safety circuit. This is directly related to the maximum actuation force, which limits the maximum acceleration directly (using Eq. 1):

$$i_{max} = \frac{m_{aa}}{K_F}a_{max} \tag{12}$$

- *Limited output-voltage swing*, $\pm V_s$. Since an amplifier is fed by a limited voltage of the power supply, the maximum swing of the output voltage e_o is limited. Using $e_o = Z_s + Z_{act} + Z_o$ (for symplicity, the original system Fig. 6 is used), e_o can be expressed in terms of trajectory parameters velocity, acceleration and jerk:

$$\begin{aligned} e_o &= \\ &(R_o + R_s + R_c)i_o + L_c\frac{di_o}{dt} + \frac{K_F K_B}{m_{aa}}\int i_o dt \\ &= \frac{m_{aa}}{K_F}\Big((R_o + R_s + R_c)a + L_c j + \frac{K_F K_B}{m_{aa}}v\Big) \end{aligned} \tag{13}$$

If the feedforward signal uses more voltage swing than available, *clipping* occurs, which distorts the output signal (see Fig. 11). Note that the relation between e_o and i_o equals $\frac{\beta}{R_s}$, so low- frequent and high-frequent content is amplified in e_o due to speed difference (and corresponding u_{EMF}) and coil inductance respectively.

- *Limited slew-rate*, S. Internally, the amplifier has several stages with limited current capabilities. In general, the current limitation in the input stage is easily reached, due to a high differential input voltage u_d. As as result, the amplifier cannot follow and the output will increase under a constant slew $\frac{de_o}{dt}$. The maximum output-voltage slope for a given setpoint can be calculated:

$$\begin{aligned} \frac{de_o}{dt} &= \\ \frac{m_{aa}}{K_F}&\Big((R_o + R_s + R_c)j + L_c s + \frac{K_F K_B}{m_{aa}}a\Big) \end{aligned} \tag{14}$$

Severe slew-rate requirements take place on a very short time-base. Therefore, the effect of a violation is hard to see in the current signal i_o, and will have even less influence on the mechanics. In general, the slew-rate will not limit the trajectory design.

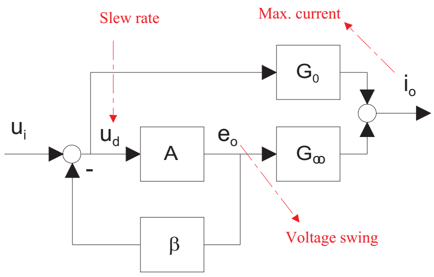

Fig. 10. Block scheme of the amplifier and feedback circuit. The most important signal limitations are depicted.

Although the parameter uncertainties listed in Table 1 do influence the amplifier-loop dynamics, the resulting effect on the actuator gain Eq. 1 is limited by the uncertainty of only K_F due to current steering. This uncertainty in actuator gain is both temperature and position dependent. In precision applications, feedforward can be compensated (i.e. by using lookup-tables or temperature measurements). At higher frequencies the simple model of Eq. 2 is not valid any more. The consequences are not further discussed but can cause performance limitations (i.e. skin-effect and Eddy-current losses).

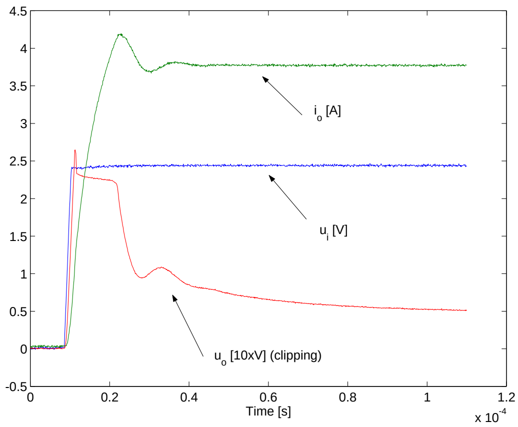

Fig. 11. Measurement of a corrupted feedforward signal due to limiting V_s (clipping). Clearly the current i_o cannot follow the input voltage u_i.

3.1 Adjustments

The most important limitations of the amplifier are the limited voltage swing and the maximum output current. For class of intended setpoints and corresponding feedforward signals, the maximum voltage swing of e_o and maximum current i_o can be calculated. If these values violate the V_s or i_{max} specifications, the actuation system has to be adjusted. The can be done in various ways, e.g.:

- selecting an amplifier with larger power capabilities. In general this means higher output current but also a higher voltage swing, so we increase both i_{max} and V_s. Disadvantage of a larger amplifier is the increasing cost and a higher noise level, which can make this solution unwanted. Design of dedicated amplifiers is possible, but also very expensive.
- improving the quality of the magnet assembly. This will increases K_F, which directly decreases the requirements on both i_{max} and V_s (Eq. 12 and Eq. 13).
- altering the number of windings of the coil. The increase in windings n is given by the ratio $r_w = \frac{n_{new}}{n_{old}}$. Keeping the total volume of the coil constant, we can express e_o and i_o in terms of r_w and the original parameters:

$$i_o^{new} = \frac{i_o^{old}}{r_w}, \qquad e_o^{new} = \frac{m_{aa}}{K_F}\Big(\frac{R_o + R_s + r_w^2 R_c}{r_w} a + L_c j + r_w \frac{K_F K_B}{m_{aa}} v\Big) \tag{15}$$

By decreasing the number of windings, the requirement on V_s is exchanged by a higher current demand and vise-versa. Thicker wires have a better thermal conductivity than thin wires, but the corresponding higher currents also cause higher distortion levels in the amplifier.

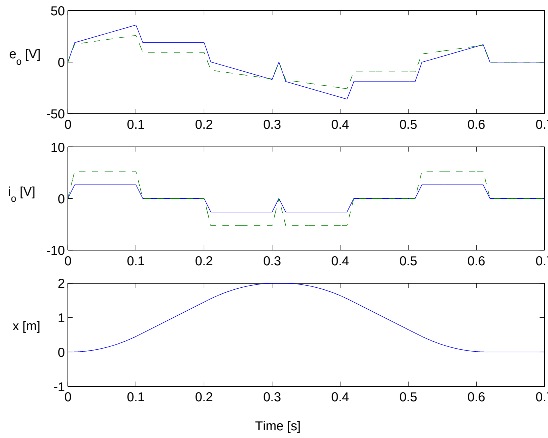

Fig. 12. Influence on e_0 and i_o for changing number of turns of coil wiring for a given trajectory x. For the dashed line, the number of turns is halved, where the cross-section is doubled.

4. CONCLUSIONS

This paper gives several guidelines to robustly stabilize a tranconductance amplifier which loads a Lorentz' actuator. The critical limitations of the amplifier can be expressed in terms of trajectory parameters. Although it is an indirect calculation, since we assume the constant gain relation $u_i = G_t i_o$, the result will be useful in most cases. Only for very fast setpoints, the input voltage will significantly differ from the output current. Furthermore, several options for adjusting existing actuation systems are given.

REFERENCES

Dostál, Jiří (1993). *Operational Amplifiers*. 2nd ed.. Butterworth–Heinemann. Research Institue for Mathematical Machines, Prague, Czechoslovakia.

Lambrechts, Paul, Matthijs Boerlage and Maarten Steinbuch (2005). Trajectory planning and feedforward design for electromechanical motion systems. *Control Engineering Practice* **13**, 145–157.

Magnetics Devision, Kimco (2002). Voice coil actuators, an application guide. Technical report. BEI Technologies, Inc., USA.

Mills, P.G.L. and M.O.J. Hawksford (1989). Transconductance power amplifier systems for current-driven loudspeakers. *Journal of the Audio Engineering Society* **37**(10), 809–822.

Nordholt, E.H. (1983). *Design of High-Performance Negative-Feedback amplifiers*. Elsevier. Amsterdam. Revision of the author's thesis.

Self, D. (2002). *Audio Power Amplifier Design Handbook*. third ed.. Newnes. Oxforf.