FUZZY ROBUST TRACKING OF A BIOREACTOR

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Abstract: A model-based robust control scheme is applied to a continuous bioreactor operating in an non stationary regime. The proposed control algorithm is a fuzzy robust error feedback controller that allows tracking several profiles of the substract concentration, while maintaining the stability conditions of the process. A numerical case study is simulated to test the robustness properties of the proposed controller in the face of parameter uncertainties and changes on load disturbances.

Keywords: Fuzzy controller, robust tracking, bioreactor

1. INTRODUCTION

Periodically time-varying systems are often used to model natural or forced periodic phenomena occurring in various engineering applications. Many processes experience periodic disturbances due to natural cycle times of upstream processes or other cyclical environmental influences such as diurnal temperature fluctuations. In wastewater treatment plants, for example, the feed flow rate and its composition can exhibit strong diurnal variations (Butler et al., 1995). In some applications, forced periodic operations can be used either to improve selectivity and yields (Bailey, 1973) or to make a continuous operation feasible (Ruthven et al., 1994; Wu et al., 1998). A potential drawback of adopting such an operation mode is that operation becomes more complex and difficult to control. Also, processes with periodic characteristics may show strong non-stationary or cyclo-stationary behavior. Additionally, because of the wide envelope of operating space covered by a periodic operation, the process dynamics can be strongly nonlinear. These factors make the control of periodic processes more complicated than the designed for constant operation processes.

Lazar and Ross (1990) and Chen et. al. (1995) have demonstrated that the average concentration of the product through time from a biochemical reaction occurring in a CSTR is magnified substantially by forced oscillations. Sterman and Ydstie (1991) have theoretically demonstrated that low frequency periodic perturbations in the reaction temperature of the CSTR enhance the yield of a certain set of chemical reactions and that the maximum yield can be thrice that of the steady state operation. Cinar et. al. (1987) have study the modification of dynamic properties of linear and nonlinear systems by introduction of fast, zero-average oscillations in a system's parameters, in particular they varied reactant flow rates to a CSTR and found a modification of the S-shaped

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steady-state curve, resulting in a higher production rate or lower energy expenditure compared to a steady operation with shifted input conditions. A common characteristic of these studies relies on studying the effect of applying those periodic changes rather than trying to find the appropriate periodic operating conditions to meet the stringent quality requirements and the suitable control system to track the nominal oscillating trajectories.

In this study, a nonlinear robust model-based control technique is implemented in a continuous biological reactor to regulate substrate concentration by tracking varying operating conditions. This robust regulator, is a fuzzy error feedback controller which relies on the existence of an internal model, obtained by finding, if possible, an immersion of the exosystem dynamics into an observable one, which allows to generate all the possible steady state inputs for the admissible values of the system parameters (Castillo-Toledo and Di Gennaro, 2002). This controller is a generalization and can be employed to track several references. We will show that these control schemes are capable of maintaining the output tracking error within predefined bounds while ensuring the stability of the closed-loop system. This paper is organized as follows: in section 2 an overview of the theory behind the robust control scheme is presented; a model for a biological reactor is presented in section 3; the error feedback controller is developed in section 4, and a study case is solved and discussed in section 5. Finally, the paper is closed with some concluding remarks.

2. ROBUST REGULATION PROBLEM FOR NONLINEAR SYSTEMS

Let us consider the following nonlinear time-invariant system

$$\dot{x} = f\left(x, u, \omega, \mu\right), \tag{1}$$

$$\dot{\omega} = S\omega, \tag{2}$$

$$e = h\left(x, \omega, \mu\right), \tag{3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are the state and input variables of the plant,.respectively. $\mu \in \mathbb{R}^s$ denotes a parameter vector which may have some values in a neighborhood $\wp \subset \mathbb{R}^s$ of the nominal ones. $\omega \in \mathbb{R}^r$ represents the state of an external signal generator, which models the reference and disturbance signals affecting the plant, called the exosystem, which is linear and it is supposed that it does not depend on μ, which is often verified in practical problems. Finally, the tracking error $e \in \mathbb{R}^p$ which, in many cases, is given as a difference between the system output and the reference signal.

The *Error Feedback Regulation Problem* for the aforementioned system is defined as the problem of tracking the reference signals and/or rejecting the disturbance signals, and maintaining the closed-loop stability property. For the case of robust regulation, we also impose the requirement that these conditions hold when the parameters vary in a neighborhood of the nominal values. This problem may be solved by determining a certain submanifold of the state space (x, ω), where the tracking error is zero, which is rendered attractive and invariant by feedback; that is, the *nonlinear robust regulation problem* (NRRP) consists in finding, if possible, a dynamic controller of the form

$$\begin{aligned} \dot{z} &= \varphi\left(z, \omega, e\right) \\ u &= \vartheta\left(z, \omega\right) \end{aligned}$$

such that, for all admissible values μ in a neighborhood $\wp$ of the nominal values, the following conditions hold

N1 Stability: The equilibrium point $(x, z) = (0, 0)$ of the closed-loop system without disturbances

$$\begin{aligned} \dot{x} &= f\left(x, \vartheta\left(z, 0\right), 0, \mu\right) \\ \dot{z} &= \varphi\left(z, 0, h\left(x, 0, \mu\right)\right) \end{aligned}$$

is asymptotically stable.

N2 Regulation: For each initial condition $(x(0), z(0), \omega(0))$ in a neighborhood of the origin, the solution of the closed-loop system

$$\begin{aligned} \dot{x} &= f\left(x, \vartheta\left(z, \omega\right), \omega, \mu\right), \\ \dot{z} &= \varphi\left(z, \omega, h\left(x, \omega, \mu\right)\right), \\ \dot{\omega} &= S\omega, \end{aligned}$$

satisfies the condition $\lim_{t\to\infty} e(t) = 0$.

A local solution to this problem has been given by Isidori (1995) which is stated in terms of the existence of nonlinear mappings $x_{ss} = \pi(\omega, \mu)$ and $z_{ss} = \sigma(\omega, \mu)$ satisfying the regulator equations

$$\frac{\partial \pi\left(\omega, \mu\right)}{\partial \omega} s\left(\omega\right) = f\left(\pi\left(\omega, \mu\right), \vartheta\left(\sigma\left(\omega, \mu\right), \omega\right), \omega, \mu\right) \tag{4}$$

$$\frac{\partial \sigma\left(\omega, \mu\right)}{\partial \omega} s\left(\omega\right) = \varphi\left(\sigma\left(\omega, \mu\right), \omega, 0\right) \tag{5}$$

$$0 = h\left(\pi\left(\omega, \mu\right), \omega, \mu\right) \tag{6}$$

for all admissible values of $\mu \subset \wp$. The next theorem states the conditions for the existence of a solution to the NRRP.

Theorem 1. (Isidori 1995) The Nonlinear Robust Regulation Problem is solvable if and only if there exist mappings

$$x_{ss} = \pi\left(\omega, \mu\right), \text{ and } u_{ss} = \gamma\left(\omega, \mu\right) = \begin{pmatrix} \gamma_1\left(\omega, \mu\right) \\ \vdots \\ \gamma_m\left(\omega, \mu\right) \end{pmatrix},$$

with $\pi(0,\mu) = 0$ and $\gamma(0,\mu) = 0$, both defined in a neighborhood of the origin, satisfying the equations

$$\frac{\partial \pi(\omega,\mu)}{\partial \omega} s(\omega) = f(\pi(\omega,\mu), \gamma(\omega,\mu), \omega, \mu) \quad (7a)$$

$$0 = h(\pi(\omega,\mu), \omega, \mu), \quad (7b)$$

for all (ω,μ), and such that for each $i = 1,\ldots,m$ the exosystem is immersed into a system

$$\dot{z} = \varphi(z,\omega) \quad (8a)$$

$$\gamma(\omega,\mu) = \psi(z) \quad (8b)$$

defined on a neighborhood Ξ^0 of the origin, in which $\varphi(0,0) = 0$ and $\psi(0) = 0$. and the two matrices

$$\Phi = \left[\frac{\partial \varphi}{\partial \xi}\right]_{\substack{\xi=0,\\ \omega=0}}, \qquad H = \left[\frac{\partial \psi}{\partial \xi}\right]_{\xi=0}$$

are such that the pair

$$\begin{pmatrix} A_0 & 0 \\ NC_0 & \Phi \end{pmatrix}, \qquad \begin{pmatrix} B_0 \\ 0 \end{pmatrix}$$

is stabilizable for some choice of the matrix N, and the pair

$$\begin{pmatrix} C_0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} A_0 & B_0 H \\ 0 & \Phi \end{pmatrix}$$

is detectable. Where

$$A_0 = \left[\frac{\partial f(x,u,\omega,0)}{\partial x}\right]_{\substack{x=0,\\ \omega=0,\\ u=0}}, \quad B_0 = \left[\frac{\partial f(x,u,\omega,0)}{\partial u}\right]_{\substack{x=0,\\ \omega=0\\ ,u=0}},$$

$$C_0 = \left[\frac{\partial h(x,\omega,0)}{\partial x}\right]_{\substack{x=0,\\ \omega=0}}.$$

■

Remark 1. Equations (7a)-(7b) are known as the Francis-Isidori-Byrnes equations (FIB) (Byrnes et al., 1997) used to find the subset Z on the Cartesian product $\mathbb{R}^n \times \mathbb{R}^m$ called, so far, the zero tracking error submanifold. The mapping $x_{ss} = \pi(\omega,\mu)$ represents the steady state zero output submanifold and the mapping $u_{ss} = \gamma(\omega,\mu)$ is the steady state input which makes invariant this steady state zero output submanifold.

The main problem using the nonlinear immersion (8a) consists on finding the observer matrix which in general is calculated with the linear approximation Φ, which may differ from the nonlinear immersion in certain zones, for this reason a fuzzy approach may introduce some advantages. If the nonlinear immersion (8a) exists and can be expressed by the fuzzy system

$$\dot{\varsigma}(t) = \sum_{i=1}^{d} m_i(\varsigma,\omega)\, \Phi_i \varsigma(t) \quad (9)$$

$$\gamma(\omega,\mu) = H\varsigma(t), \quad (10)$$

then the controller which solves the NRRP is given by

$$\dot{\xi}(t) = A_0 \xi(t) - B_0 (H\varsigma(t) - u) - \sum_{i=1}^{d} m_i(\varsigma,\omega)\, G_{1i} (C_0 \xi(t) - e(t)) \quad (11a)$$

$$\dot{\zeta}(t) = \sum_{i=1}^{d} m_i(\varsigma,\omega)\, \Phi_i \varsigma(t) - \sum_{i=1}^{d} m_i(\varsigma,\omega)\, G_{2i} (C_0 \xi(t) - e(t)) \quad (12)$$

$$u(t) = K\xi(t) + H\zeta(t), \quad (13)$$

where K and G_{1i}, G_{2i} make stable matrices $(A_0 + B_0 K)$ and

$$A_i - G_i C, \qquad i = 1,2,\ldots,d$$

respectively, with

$$A_i = \begin{pmatrix} A_0 & -B_0 H \\ 0 & \Phi_i \end{pmatrix}, \quad G_i = \begin{pmatrix} G_{1i} \\ G_{2i} \end{pmatrix}, \quad \text{and}$$

$$C = \begin{pmatrix} C_0 & 0 \end{pmatrix}.$$

Obviously, the stabilizability an detectability of the pairs

$$[A_0, B_0] \quad \text{and} \quad [A_i, C], \qquad i = 1,2,\ldots,d.$$

Observer matrices G_i, can be calculated by the linear matrix inequalities

$$A_i^T P + P A_i - \left(C^T M_i^T + M_i C\right) < 0, \quad i = 1,2,\ldots,d$$

$$P > 0$$

where

$$M_i = P G_i$$

Notice that the main feature guaranteeing the zero output tracking error is the immersion (9) which incorporates the nonlinearity of the steady state input.

3. THE BIOREACTOR MODEL

Consider a continuous stirred tank biological reactor where a substrate is consumed by an microoganisme. The bioreactor dynamics is given by the following set of equations which results from the mass biomass and substrate balances

$$\frac{dx}{dt} = [\mu(s) - D]\, x \quad (14)$$

$$\frac{ds}{dt} = (s_i - s) D - k\mu(s)\, x \quad (15)$$

where x, and s are, the biomass and substrate concentration in the bioreactor, respectively, while s_i, and D are the input biomass concentration and the dilution rate. Finally, the microbial growth rate, $\mu(s)$, is given by a Monod kinetic,

$$\mu(s) = \frac{\mu_{\max} s}{K_s + s} \tag{16}$$

this model is widely used in biological processes.

4. ROBUST CONTROL DESIGN

In this section it is presented a generalized case of tracking an arbitrary operating substrate condition. To facilitate the design and application of the nonlinear robust control law, let us rewrite the bioreactor modeling equations (14 and 15) in terms of the state space

$$\dot{x}_1 = \mu(x_2)\, x_1 - x_1 u \tag{17a}$$

$$\dot{x}_2 = -\mu(x_2)\, x_1 + (s_i - x_2)\, u \tag{17b}$$

where the states $x_1 = kx$, and $x_2 = s$, are the biomass yield and, the substrate concentration in the reactor. The manipulated variable is the dilution rate, D. Consider a given reference which can be described by a lineal dynamic system called exosystem of the form

$$\dot{\omega} = S\omega, \qquad \omega(0) = \omega_0 \tag{18a}$$

$$x_{2r} = \omega_1 \tag{18b}$$

where x_{2r} is the reference substrate concentration, $\omega = \begin{pmatrix}\omega_1 & \cdots & \omega_r\end{pmatrix}^T$ are the states of exosystem, and the dynamic matrix $S \in \mathbb{R}^{r\times r}$ has the special form

$$S = \begin{pmatrix} 0 & 1 \; 0 \cdots 0 \\ 0 & \phi \end{pmatrix},$$

where $\phi \in \mathbb{R}^{(r-1)\times(r-1)}$ could be any matrix. This restriction states that the reference signal has the special form $x_{2r} = c + f(t)$, where c is a constant and $f(t)$ could be the integral of any solution for the system $\dot{w} = \phi w$.

Thus, the tracking errors can be expressed as

$$e_1(t) = x_2(t) - \omega_1(t)\,. \tag{19a}$$

To find the steady state mappings, we proceed as follows. The tracking error is zero when

$$\begin{aligned} \pi_2(\omega) &= \omega_1(t)\,, \qquad (20)\\ \dot{\pi}_1 &= \left[\mu(\omega_1) - \frac{\dot{\omega}_1}{s_i - \omega_1}\right]\pi_1 - \frac{\mu(\omega_1)}{s_i - \omega_1}\pi_1^2,\\ \gamma(\omega) &= \frac{\dot{\omega}_1 + \mu(\omega_1)\,\pi_1}{s_i - \omega_1}, \end{aligned}$$

the solution for $\pi_1(\omega)$ is

$$\pi_1(\omega) = \frac{(s_i - \omega_1)\,\pi_{1_0}}{\pi_{1_0} + (s_i - \omega_{1_0})\, e^{-\int_0^t \mu(\omega_1)dt}}. \tag{21}$$

which for a large enough time, $\pi_1(\omega) \approx s_i - \omega_1$, and steady state input approach to

$$\gamma(\omega) = \frac{\dot{\omega}_1}{s_i - \omega_1} + \mu(\omega_1)\,,$$

this input, for a given exosystem of the form (18a) is generated by the *generalized immersion*

$$\begin{aligned} \dot{z}_1 &= \phi z_1 + z_{11} z_1,\\ \dot{z}_2 &= S\hat{z}_2 - \frac{2\dot{\omega}_1}{\omega_1}\hat{z}_2 + z_{22} z_2 \qquad (22)\\ \gamma(\omega) &= z_{11} + z_{21} \end{aligned}$$

where $z_1 = (z_{11}, \ldots, z_{1r-1})^T$, $z_2 = (z_{21}, \ldots, z_{2r})^T$, and $\hat{z}_2 = (z_{22}, \ldots, z_{2r})^T$. As it can be seen, immersion (22) is time-varying nonlinear The fuzzy version for this nonlinear immersion consists in 8 rules which depend on z_{11}, z_{21}, and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ (whose estimated ranges are $-a_1 \le z_{11} \le a_1, -a_2 \le z_{21} \le a_2, -a_3 \le z_{22} - \frac{2\dot{\omega}_1}{\omega_1} \le a_3$) as follow

(1) **IF** z_{11} is negative, z_{21} is small and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is negative
THEN $\dot{z} = \Phi_1 z,\ \gamma(\omega) = Hz$
(2) **IF** z_{11} is negative, z_{21} is small and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is positive
THEN $\dot{z} = \Phi_2 z,\ \gamma(\omega) = Hz$
(3) **IF** z_{11} is negative, z_{21} is large and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is negative
THEN $\dot{z} = \Phi_3 z,\ \gamma(\omega) = Hz$
(4) **IF** z_{11} is negative, z_{21} is large and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is positive
THEN $\dot{z} = \Phi_4 z,\ \gamma(\omega) = Hz$
(5) **IF** z_{11} is positive, z_{21} is small and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is negative
THEN $\dot{z} = \Phi_5 z,\ \gamma(\omega) = Hz$
(6) **IF** z_{11} is positive, z_{21} is small and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is positive
THEN $\dot{z} = \Phi_6 z,\ \gamma(\omega) = Hz$
(7) **IF** z_{11} is positive, z_{21} is large and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is negative
THEN $\dot{z} = \Phi_7 z,\ \gamma(\omega) = Hz$
(8) **IF** z_{11} is positive, z_{21} is large and $z_{22} - \frac{2\dot{\omega}_1}{\omega_1}$ is positive
THEN $\dot{z} = \Phi_8 z,\ \gamma(\omega) = Hz$

where

$$\Phi_i = \begin{pmatrix} \Phi_{1i} & 0 \\ 0 & \Phi_{2i} \end{pmatrix}$$

$$\Phi_{1i} = \phi + I_1 (-1)^{ceil(i/4)}\, a_1$$

$$\Phi_{2i} = \begin{pmatrix} 0 & (-1)^{ceil(i/2)} a_2 \; 0 \cdots 0 \\ 0 & \phi \end{pmatrix} + I_2 (-1)^i a_3,$$

I_1 and I_2 are identity matrices of dimension $r-1$ and r, respectively, while $ceil(x)$, means the nearest integer towards zero. Hence the global fuzzy immersion has the form

$$\dot{z} = \sum_{i=1}^{8} m_i(z,\omega)\,\Phi_i z \tag{23}$$

where the membership functions are

$$m_i(z,\omega) = h_{\frac{3+(-1)^{ceil(n/4)}}{2}}(z) \cdot h_{\frac{7+(-1)^{ceil(n/2)}}{2}}(z) \cdot \cdot h_{\frac{11+(-1)^n}{2}}(z,\omega)$$

$$\begin{aligned} h_1(z) &= \frac{1}{2}\left(1 - \frac{z_{11}}{a_1}\right), \\ h_2(z) &= 1 - h_1(z), \\ h_3(z) &= \frac{1}{2}\left(1 - \frac{z_{21}}{a_2}\right), \\ h_4(z) &= 1 - h_3(z), \\ h_5(z,\omega) &= \frac{1}{2}\left(1 - \frac{z_{22}}{a_3} + \frac{2\dot{\omega}_1}{a_3\omega_1}\right), \\ h_6(z) &= 1 - h_5(z). \end{aligned}$$

5. NUMERICAL SIMULATION RESULTS

In this section we illustrate the features of the robust error feedback controller (REFC) tracking not constant operating conditions in the bioreactor. For simulation purposes this controller was applied to a waste water anaerobic biological reactor, whose system parameters were taken from Alcaraz Gonzalez et. al. (2003). For the sake of completeness, these values are reported in Table 1.

We selected the following tracking case to test the REFC:

$$\text{Reference:} \quad s_r = 2 + 0.5\sin\left(\frac{\pi}{2}t\right),$$

which can be generated by the exosystem

$$\begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix} = \begin{pmatrix} 0 & c & 0 \\ 0 & 0 & c \\ 0 & -c & 0 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix},$$

hence the nonlinear immersion (22) becomes

$$\begin{aligned} \dot{z}_{11} &= cz_{12} + z_{11}^2 \\ \dot{z}_{12} &= -cz_{11} + z_{11}z_{12} \\ \dot{z}_{21} &= z_{21}z_{22} \\ \dot{z}_{22} &= cz_{23} - 2\frac{c\omega_2}{\omega_1}z_{22} + z_{22}^2 \\ \dot{z}_{23} &= -cz_{22} - 2\frac{c\omega_2}{\omega_1}z_{23} + z_{22}z_{23} \end{aligned}$$

The fuzzy version for this nonlinear immersion is calculate as in equation (23), and the robust controller has the form of equations (11a), (12) and (13). In order to test the disturbance and parametric changes rejection properties of the REFC for this controller, some changes in various kinetic parameter values and inlet conditions were introduced in the system. These changes took into account the presence of realistic perturbations that are unavoidable in industry as perturbations in the inlet substrate concentration and error in the values of the kinetic constants of Monod expression which play a very important role in the modeling because it is contained both in the substrate and biomass balances. Table 2 lists these changes of the aforementioned parameters (with respect to the nominal values) and the time where those changes were introduced during the progress of the simulation.

Figures 1, 2 and 3 show, respectively, the output, input and error produced in the system. As it can be seen the bioreactor tracks the oscillatory signal despite perturbations and parametric variations. The controller is able to overcome saturation and to achieve stabilization, finding the zero error input-submanifold.

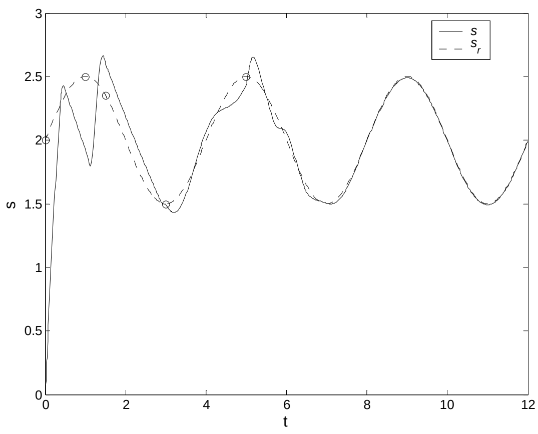

Fig. 1. Substrate concentration in the bioreactor (initial condition $s = 0$)

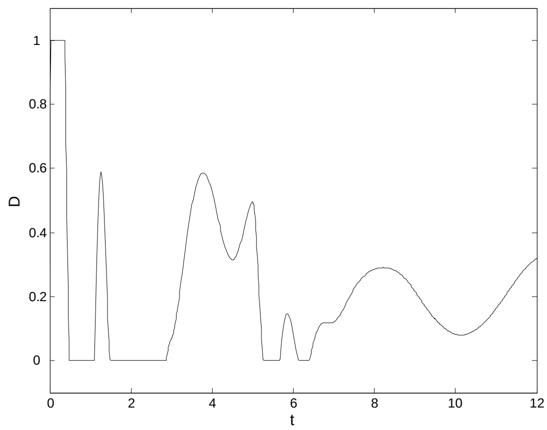

Fig. 2. Dilution rate.

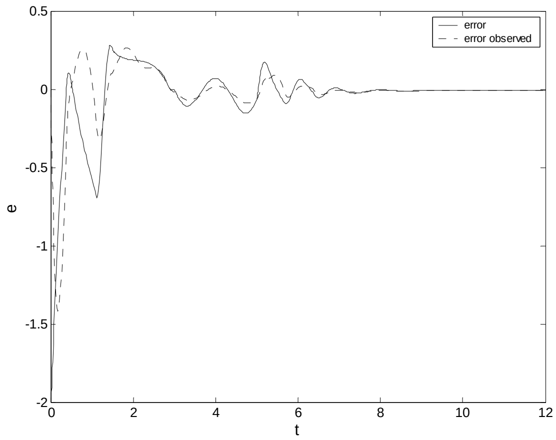

Fig. 3. Output error, $e = s - \omega_1$ and observer calculated error.

6. CONCLUSIONS

A scheme for the robust control of a biological reactor was presented. It is composed of an error feedback controller and a fuzzy estimator which allows to assure stability for nonlinear immersion. The performance of the robust regulator has been examined through numerical simulation under various uncertainties and external disturbances. The proposed structure has shown to maintain good properties, even when confronting significant modeling errors, such as parameter uncertainties and load disturbances. It must be pointed out the applicability of the proposed controller is not restricted to biological systems. In fact, our design methodology can be easily extended to higher dimensional (partially or fully) linearizable systems, such as chemical reactors and distillation columns. Results in this direction will be reported in the near future.

Table 1. Parameter nominal values, changes and input disturbances

Parameter	Nominal value	% deviation from nominal value $0 \le t < 1$	$1 \le t < 1.5$	$1.5 \le t < 3$	$3 \le t < 5$	$t > 5$
$\mu_{\max}$	0.69	20%	−20%	−30%	50%	10%
K_s	4.95	10%	−10%	30%	20%	−20%
k	6.6	−10%	0%	20%	−15%	0%
S_i	10	0%	10%	−10%	−20%	20%

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