OBSERVER-BASED CONTROL OF A PVTOL AIRCRAFT

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Abstract: In this paper, we propose an observed-based control for the PVTOL aircraft considering that the angular position is not measurable. The control strategy presented here, is robust with respect to observer errors and takes into account constraints imposed by the nonlinear observer. The observer design is based on the assumption that the horizontal position is the only available measure. We present simulation results in closed-loop when the controller uses the estimated state provided by the observer.

Keywords: Nonlinear Observer, Aircraft control, Nonlinear control systems, Stabilizing controllers, Saturation functions

1. INTRODUCTION

The Planar Vertical Take Off and Landing (PVTOL) aircraft is the well-known simplified aircraft model that provides a practical and simple representation of several helicopters and some special airplanes. The nonlinear dynamical model of the PVTOL aircraft proposed in (Hauser *et al.*, 1992) is given by the following equations

$$\ddot{x} = -\sin(\theta)\, u_1 + \varepsilon \cos(\theta)\, u_2 \tag{1}$$

$$\ddot{y} = \cos(\theta)\, u_1 + \varepsilon \sin(\theta)\, u_2 - 1 \tag{2}$$

$$\ddot{\theta} = u_2 \tag{3}$$

where x, y denote the horizontal and vertical position of the aircraft center of mass and θ is the angular position that the aircraft makes with the horizon. The control inputs u_1 and u_2 are respectively the thrust (directed out the bottom of the aircraft) and the angular acceleration (rolling moment). The parameter ε is a small coefficient which characterizes the coupling between the rolling moment and the lateral acceleration of the aircraft. The coefficient " -1" is the normalized gravitational acceleration (see Figure 1).

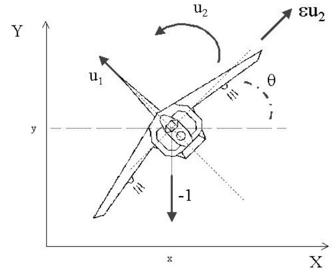

Fig. 1. The PVTOL aircraft(front view)

Several control laws have been proposed in the literature. To cite a few of them, the studies include (Hauser *et al.*, 1992; Teel, 1996; Martin *et al.*, 1996; Marconi *et al.*, 2002; Olfati-Saber, 2002; Zavala *et al.*, 2003) but there are very few experimental tests published in the literature.

A critical problem encountered when implementing control laws on real experiments is that the state is not completely measurable. We have realized in our real experiments (Castillo *et al.*, 2004) that one of the main difficulties is to measure the angle of the PVTOL aircraft. In order to resolve this problem, we propose a bounded observer-based control based on Lyapunov analysis.

The methodology that we use in the present paper is derived from (Sánchez *et al.*, 2004) and (Lozano *et al.*, 2004). The proposed observer is related to high gains observers that provide faster convergence rates. We present the robustness of the proposed control law in presence of observer errors. This controller also satisfies the restrictions of the designed observer. Simulations have shown that the observer-based control in the closed-loop system performs well.

The paper is organized as follows. The control law and the robustness to small errors in the estimation are explained in section 2. In section 3, the construction of the nonlinear observer is presented. Simulations are shown in section 4. Finally, some conclusions are given in section 5.

2. ROBUST CONTROL LAW WITH RESPECT TO OBSERVER ERRORS

We have already developed several solutions for the stabilization of the PVTOL aircraft (Fantoni *et al.*, 2002; Zavala *et al.*, 2003), etc... The control strategy used here, comes from one of these papers, namely (Lozano *et al.*, 2004). The selection of the controller has been determined according to the observer requirements. We will prove, in this section, that the control law is robust in the sense that it remains stable with respect to small observer errors. In addition, the controller allows us to satisfy the restrictions imposed by the observer (for example, to avoid $u_1 = 0$).

We will consider a simplified model for the PVTOL aircraft system, i.e. with $\varepsilon = 0$. Indeed, we have neglected the coupling between the rolling moment and the lateral acceleration of the aircraft. This choice is due to the fact that the coefficient ε is very small ($\varepsilon << 1$) and not always well-known. Furthermore, Olfati-Saber (Olfati-Saber, 2002) has shown that by an appropriate change of coordinates, we can transform the system to an equivalent system of the same form (1-3) but with $\varepsilon = 0$. Therefore, in this paper, we will consider the following system

$$\ddot{x} = -\sin(\theta)\, u_1 \tag{4}$$

$$\ddot{y} = \cos(\theta)\, u_1 - 1 \tag{5}$$

$$\ddot{\theta} = u_2 \tag{6}$$

2.1 *Boundedness of the vertical displacement* y

The objective is to find a control law such that we obtain a desired behavior for the vertical displacement y

$$\ddot{y} = \sigma_{m_2}\left(-a_1\dot{y} - \sigma_{m_1}\left(a_2\dot{y} + a_3 y\right)\right) \tag{7}$$

where $\sigma(s)$ is a continuous saturation function that have been introduced to constrain the acceleration and the velocity of the aircraft, which will be defined below and a_i for $i = 1, 2, 3$ are positive constants.

The desired behavior is achieved when all the measurements of the state vector are available (Castillo *et al.*, 2002). On the other hand, when we use an estimation of the nonavailable variables, estimation errors could occur.

We then propose the following control strategy

$$u_1 = \frac{1 + \sigma_{m_2}\left(-a_1\dot{y} - \sigma_{m_1}\left(a_2\dot{y} + a_3 y\right)\right)}{\cos\sigma_p(\hat{\theta})} \tag{8}$$

where $0 < p < \frac{\pi}{2}$, $m_2 < 1$ since $u_1 = 0$ has to be avoided because of the observer requirements (see (Sánchez *et al.*, 2004)), $\hat{\theta}$ is the estimation of θ and σ_η, for some $\eta > 0$ is a continuous saturation function (see Figure (2)) defined as

$$\sigma_\eta(s) = \begin{cases} -\eta & s < -f_1 \\ -\sqrt{r^2 - (s + f_1)^2} - v & -f_1 \le s < -f_2 \\ s & -f_2 \le s < f_2 \\ \sqrt{r^2 - (s - f_1)^2} + v & f_2 \le s < f_1 \\ \eta & s \ge f_1 \end{cases} \tag{9}$$

where r is the radius of the circle tangential to linear part in the interval $[-f_2, f_2[$ and to the constant part ($-\eta$ or η), $f_1 = \eta + r\left(\sqrt{2} - 1\right)$, $f_2 = \eta + r\left(\frac{1}{\sqrt{2}} - 1\right)$ and $v = \eta - r$.

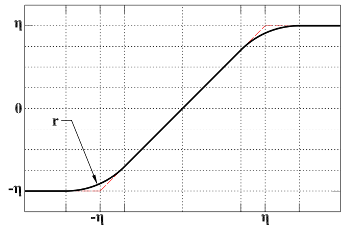

Fig. 2. Linear saturation $\sigma_\eta(s)$

Note that the time derivative of $\sigma_\eta(s)$ is also continuous (see Figure (3))

$$\frac{\partial\sigma_\eta(s)}{\partial s} = \begin{cases} 0 & s < -f_1 \\ \dfrac{s+f_1}{\sqrt{r^2-(s+f_1)^2}} & -f_1 \le s < -f_2 \\ 1 & -f_2 \le s < f_2 \\ \dfrac{-(s-f_1)}{\sqrt{r^2-(s-f_1)^2}} & f_2 \le s < f_1 \\ 0 & s \ge f_1 \end{cases} \tag{10}$$

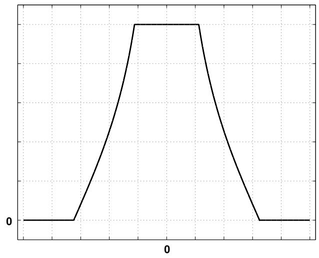

Fig. 3. Saturation partial derivative $\frac{\partial\sigma_\eta(s)}{\partial s}$

When (8) is introduced in (4)-(6), we obtain

$$\ddot{x} = -\frac{\sin(\theta)}{\cos\sigma_p(\hat{\theta})}(1+\sigma_{m_2}) \tag{11}$$

$$\ddot{y} = \frac{\cos(\theta)}{\cos\sigma_p(\hat{\theta})}(1+\sigma_{m_2}) - 1 \tag{12}$$

$$\ddot{\theta} = u_2 \tag{13}$$

where $p < \frac{\pi}{2}$. The estimation variable of θ is defined as $\hat{\theta} = \theta + e_\theta$. Let us assume that e_θ is bounded and that after a finite time T_3, $\theta(t)$ and $\hat{\theta}(t)$ belong to the interval $I_{\frac{\pi}{2}} = \left(-\frac{\pi}{2}+\epsilon, \frac{\pi}{2}-\epsilon\right)$ for some $\epsilon > 0$. We will show in Section 2.2 that this will be the case.

By using some general computations and standard mathematical inequalities, we can prove that the function

$$g_\theta(\theta,\hat{\theta}) \triangleq \frac{\cos(\theta)}{\cos\sigma_p(\hat{\theta})} \tag{14}$$

is a strictly positive function for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ which satisfies

$$1-\alpha \le g_\theta \le 1+\alpha$$

where α is a positive constant which depends on the range of values of θ and on the observer error.

The y-subsystem (12) can be rewritten as

$$\dot{y}_1 = y_2 \tag{15}$$

$$\dot{y}_2 = \bar{u} \tag{16}$$

where $y = y_1$ and $\bar{u} = g_\theta(1+\sigma_{m_2}) - 1$.

We propose the following positive definitive function

$$V_1 = \frac{1}{2}y_2^2 \tag{17}$$

Differentiating V_1 with respect to the time, we obtain

$$\dot{V}_1 = y_2(g_\theta(\sigma_{m_2}(-a_1y_2 - \sigma_{m_1}(a_2y_2+a_3y_1))) + g_\theta - 1) \tag{18}$$

Then, $\forall |y_2| > \frac{m_1}{a_1} + \frac{\alpha}{1-\alpha} + \delta$, we have $\dot{V}_1 < 0$, for some $\delta > 0$ arbitrarily small.

This implies that in finite time , $\exists T_0$ such that

$$|y_2| \le \bar{y}_2 \triangleq \frac{m_1}{a_1} + \frac{\alpha}{1-\alpha} + \delta \tag{19}$$

for $t \ge T_0$. Therefore, for $t \ge T_0$,

$|a_1y_2 + \sigma_{m_1}(a_2y_2+a_3y_1)| \le 2m_1 + \frac{a_1\alpha}{1-\alpha} + a_1\delta$.

Let us assume that m_1 and m_2 verify

$$1 > m_2 \ge 2m_1 + \frac{a_1\alpha}{1-\alpha} + a_1\delta \tag{20}$$

It then follows that, for $t \ge T_0$

$$\dot{y_2} = g_\theta(-a_1y_2 - \sigma_{m_1}(a_2y_2+a_3y_1)) + g_\theta - 1 \tag{21}$$

Let us define the variable z as follows

$$z = a_2y_2 + a_3y_1 \tag{22}$$

From (15), (21) and (22), we have

$$\dot{z} = a_2g_\theta(-a_1y_2 - \sigma_{m_1}(z)) + a_2(g_\theta - 1) + a_3y_2 \tag{23}$$

Let us now propose a second positive definite function

$$V_2 = \frac{1}{2}z^2 \tag{24}$$

Differentiating V_2, we obtain

$$\dot{V}_2 = -z(a_2g_\theta\sigma_{m_1}(z) + (a_1a_2g_\theta - a_3)y_2 + a_2(1-g_\theta)) \tag{25}$$

Then

$$\forall |z| > \bar{z} \triangleq \frac{(a_3 + a_1a_2(1+\alpha))\bar{y}_2}{a_2(1-\alpha)} + \frac{\alpha}{1-\alpha} + \delta \tag{26}$$

we have $\dot{V}_2 < 0$, for some $\delta > 0$ arbitrarily small. This implies that in finite time, $\exists T_1$ such that $|z| \le \bar{z}$ for $T_1 \ge T_0$. Therefore

$$|y_1| \le \frac{\bar{z} + a_2\bar{y}_2}{a_3} \tag{27}$$

for $T_1 \ge T_0$.

Finally, $y(t)$ and $\dot{y}(t)$ are bounded by choosing a_i for $i = 1,2,3$ suitably. a_i for $i = 1,2,3$ have to be chosen such that $\bar{z} < m_1$ and such that (20) is satisfied.

2.2 *Boundedness of the angle θ and of the horizontal displacement x*

To prove the boundedness of the angle θ and of the horizontal displacement x, let us consider the following assumptions:

The estimation variables can be defined as $\hat{\theta} = \theta + e_\theta, \dot{\hat{\theta}} = \dot{\theta} + e_{\dot{\theta}}, \dot{\hat{x}} = \dot{x} + e_{\dot{x}}$ and assume that the estimation errors and their derivative are bounded

$|e_\theta| \le \varepsilon_\theta, |e_{\dot\theta}| \le \varepsilon_{\dot\theta}, |e_{\dot x}| \le \varepsilon_{\dot x}, |\dot e_{\dot\theta}| \le \varepsilon'_{\dot\theta}, |\dot e_{\dot x}| \le \varepsilon'_{\dot x}$ where $\varepsilon_\theta, \varepsilon_{\dot\theta}, \varepsilon_{\dot x}, \varepsilon'_{\dot\theta}$ and $\varepsilon'_{\dot x}$ are positive constants.

In order to establish a bound for $\dot\theta$, let us define u_2 as

$$u_2 = -\sigma_a(\dot{\hat\theta} + \sigma_b(z_1)) \tag{28}$$

where $a > 0$ is the desired upper bound for $|u_2|$ and z_1 will be defined later.

Let

$$V_3 = \frac{1}{2}\dot\theta^2 \tag{29}$$

Then it follows that

$$\dot V_3 = -\dot\theta\sigma_a(\dot\theta + e_{\dot\theta} + \sigma_b(z_1)) \tag{30}$$

Note that if $|\dot\theta| > b+\varepsilon_{\dot\theta}+\delta$ for some $b > 0$ and some $\delta > 0$ arbitrarily small, then $\dot V_3 < 0$. Therefore, after some finite time T_2, we will have

$$|\dot\theta(t)| \le b + \varepsilon_{\dot\theta} + \delta \tag{31}$$

Let us assume that b verifies

$$a \ge 2b + 2\varepsilon_{\dot\theta} + \delta \tag{32}$$

Then, from (13) and (28) we obtain for $t \ge T_2$

$$\ddot\theta = -\dot\theta - e_{\dot\theta} - \sigma_b(z_1) \tag{33}$$

In order to establish a bound for θ, let us define z_1 as

$$z_1 = z_2 + \sigma_c(z_3) \tag{34}$$

for some z_3 to be defined later and

$$z_2 = \hat\theta + \dot{\hat\theta} \tag{35}$$

From (33)-(35) we have

$$\dot z_2 = -\sigma_b(z_2 + \sigma_c(z_3)) + \dot e_{\dot\theta} \tag{36}$$

Let

$$V_4 = \frac{1}{2}z_2^2 \tag{37}$$

then

$$\dot V_4 = -z_2\left(\sigma_b(z_2 + \sigma_c(z_3)) - \dot e_{\dot\theta}\right) \tag{38}$$

Note that if $|z_2| > c+\varepsilon'_{\dot\theta}+\delta$ for some δ arbitrarily small and some $c > 0$, then $\dot V_4 < 0$. Therefore, it follows that after some finite time $T_3 \ge T_2$, we have

$$|z_2(t)| \le \bar z_2 \triangleq c + \varepsilon'_{\dot\theta} + \delta \tag{39}$$

From (31) and (39), it follows that for $t \ge T_3$

$$|\theta(t)| \le \bar\theta \triangleq \bar z_2 + \varepsilon_\theta + 2\varepsilon_{\dot\theta} + b + \delta \tag{40}$$

In order to fulfil the requirement stated when we have introduced the function g_θ in (14), we choose

$$\bar\theta \le \frac{\pi}{2} - \varepsilon_\theta - \epsilon \tag{41}$$

for some $\epsilon > 0$, then θ and $\hat\theta$ belong to $I_{\frac{\pi}{2}}$, for $t \ge T_3$.

Assume that b and c also satisfy

$$b \ge 2c + \varepsilon'_{\dot\theta} + \delta \tag{42}$$

Then, in view of (39), (36) reduces to

$$\dot z_2 = -z_2 - \sigma_c(z_3) + \dot e_{\dot\theta} \tag{43}$$

for $t \ge T_3$.

In order to establish a bound for $\dot x$, let us define z_3 as

$$z_3 = z_4 + \sigma_d(z_5) \tag{44}$$

where z_4 is defined as

$$z_4 = z_2 + \hat\theta - \dot{\hat x} \tag{45}$$

and z_5 will be defined later. From (11), (35) and (43) and the above it follows that

$$\begin{aligned}\dot z_4 = (1+\sigma_{m_2})\frac{\sin(\theta)}{\cos\sigma_p(\theta+e_\theta)} - \theta \\ -\sigma_c(z_4+\sigma_d(z_5)) - e_\theta + \dot e_{\dot\theta} - \dot e_{\dot x}\end{aligned} \tag{46}$$

Define

$$V_5 = \frac{1}{2}z_4^2 \tag{47}$$

then

$$\begin{aligned}\dot V_5 = -z_4(\sigma_c(z_4+\sigma_d(z_5)) + \theta \\ -\frac{\sin(\theta)(1+\sigma_{m_2})}{\cos\sigma_p(\theta+e_\theta)} + e_\theta - \dot e_{\dot\theta} + \dot e_{\dot x})\end{aligned} \tag{48}$$

Note that if

$$|z_4| > d + \bar g_0 + \bar\varepsilon_0 + \delta \tag{49}$$

where $\bar g_0 = \frac{\sin(\bar\theta)(\bar m_2+1)}{\cos(p)} - \bar\theta$, $\bar\varepsilon_0 = \varepsilon_\theta + \varepsilon'_{\dot\theta} + \varepsilon'_{\dot x}$, $\bar m_2$ is the value for which σ_{m_2} converges (see (12), (19) and (27)) and for some δ arbitrarily small and some $d > 0$, there exist a finite time $T_4 > T_3$, large enough such that $\dot V_5 < 0$. Therefore, after some finite time $T_5 > T_4$, we have

$$|z_4(t)| \le d + \bar g_0 + \bar\varepsilon_0 + \delta \tag{50}$$

Let us assume that d and c verify

$$c \ge 2d + \bar g_0 + \bar\varepsilon_0 + \delta \tag{51}$$

Thus, after a finite time T_5, (46) reduces to

$$\begin{aligned}\dot z_4 = (1+\sigma_{m_2})\frac{\sin(\theta)}{\cos\sigma_p(\theta+e_\theta)} - \theta \\ -z_4 - \sigma_d(z_5) - e_\theta + \dot e_{\dot\theta} - \dot e_{\dot x}\end{aligned} \tag{52}$$

Note that in view of (35), (45) and (50) it follows that $\dot x$ is bounded. In order to establish a bound for x, let us define z_5 as

$$z_5 = z_4 + \hat\theta - 2\dot{\hat x} - x \tag{53}$$

From (11), (35), (45) and (52) we get

$$\begin{aligned}\dot z_5 = -\sigma_d(z_5) + 3\left(\frac{\sin(\theta)(1+\sigma_{m_2})}{\cos\sigma_p(\theta+e_\theta)} - \theta\right) \\ -3e_\theta + \dot e_{\dot\theta} + e_{\dot x} - 3\dot e_{\dot x}\end{aligned} \tag{54}$$

Define

$$V_6 = \frac{1}{2} z_5^2 \tag{55}$$

then

$$\dot{V}_6 = -z_5(\sigma_d(z_5) - 3\left(\frac{\sin(\theta)(1+\sigma_{m_2})}{\cos\sigma_p(\theta + e_\theta)} - \theta\right) \\ +3e_\theta - \dot{e}_{\dot{\theta}} - e_{\dot{x}} + 3\dot{e}_{\dot{x}}) \tag{56}$$

Note that if

$$|z_5| > 3\bar{g}_0 + \bar{\varepsilon}_1 + \delta \tag{57}$$

where $\bar{\varepsilon}_1 = 3\varepsilon_\theta + \varepsilon_{\dot{x}} + \varepsilon'_{\dot{\theta}} + 3\varepsilon'_{\dot{x}}$, for some δ arbitrarily small and $d \geq 3\bar{g}_0 + \bar{\varepsilon}_1 + \delta$, there exist a finite time $T_6 > T_5$, large enough such that $\dot{V}_6 < 0$. Therefore, after some finite time $T_7 > T_6$, we have

$$|z_5(t)| \leq 3\bar{g}_0 + \bar{\varepsilon}_1 + \delta \tag{58}$$

Boundedness of x follows from (50), (53) and (58).

Finally, the control input u_2 is given by

$$u_2 = -\sigma_a(\dot{\hat{\theta}} + \sigma_b(\hat{\theta} + \dot{\hat{\theta}} + \sigma_c(2\hat{\theta} + \dot{\hat{\theta}} \\ -\dot{\hat{x}} + \sigma_d(3\hat{\theta} + \dot{\hat{\theta}} - 3\dot{\hat{x}} - x)))) \tag{59}$$

3. NONLINEAR OBSERVER FOR THE PVTOL AIRCRAFT

The observer design comes from the paper (Sánchez *et al.*, 2004). The observer allows us to estimate the angle θ of the PVTOL aircraft with the assumption that the horizontal position is the only available measure. In this section, we will recall the main lines of the observer synthesis.

Since for the estimation of the angle θ, we only need the measurement of x, we then define the following reduced state equations

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T = \begin{pmatrix} x & \dot{x} & \theta & \dot{\theta} \end{pmatrix}^T \tag{60}$$

The dynamic equations are then given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_2 \\ -u_1 \sin(x_3) \\ x_4 \\ u_2 \end{pmatrix} = f(x,u) \tag{61}$$

with the output

$$h = Cx = x_1 \tag{62}$$

Let us consider the following nonlinear change of coordinates

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \sin(x_3) \\ x_4 \cos(x_3) \end{pmatrix} = T(x) \tag{63}$$

The dynamic system has the following structure

$$\dot{z} = A(u) z + g(u,z) \tag{64}$$

where

$$A(u) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -u_1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$g(u,z) = \begin{pmatrix} 0 & 0 & 0 & \dfrac{u_2\left(1-z_3^2\right)^{3/2} - z_3 z_4^2}{1-z_3^2} \end{pmatrix}^T$$

An observer for system (64) is given by

$$\dot{\hat{z}} = A(u)\hat{z} + g(u,\hat{z}) - \Gamma^{-1}\Delta_\rho^{-1} K (C\hat{z} - h) \tag{65}$$

where $\Delta_\rho = \text{diag}\left\{1/\rho, 1/\rho^2, 1/\rho^3, 1/\rho^4\right\}$, $\Gamma = \text{diag}(1,1,-u_1,-u_1)$, $\rho > 0$ and K is such that $\left(\tilde{A} - KC\right)$ is stable where

$$\tilde{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{66}$$

The observer in the original states is therefore given by

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{pmatrix} = \begin{pmatrix} \hat{x}_2 - k_1\rho(\hat{x}_1 - h) \\ -u_1 \sin\hat{x}_3 - k_2\rho^2(\hat{x}_1 - h) \\ \hat{x}_4 + \dfrac{k_3\rho^3}{u_1\cos\sigma_p(\hat{x}_3)}(\hat{x}_1 - h) \\ u_2 + \dfrac{k_4\rho^4}{u_1\cos\sigma_p(\hat{x}_3)}(\hat{x}_1 - h) \end{pmatrix} \tag{67}$$

which does not have singularities since $\cos\sigma_p(\hat{x}_3)$ and u_1 are different from zero.

We then define the estimation error as $e = \hat{z} - z$, whose dynamics are given by

$$\dot{e} = \left(A(u) - \Gamma^{-1}\Delta_\rho^{-1}KC\right)e + g(u,\hat{z}) - g(u,z).$$

By considering a coordinate transformation of the form $\bar{e} = \Gamma\Delta_\rho e$, we obtain

$$\dot{\bar{e}} = \rho\left(\tilde{A} - KC\right)\bar{e} + G_e(u,\hat{z},z) + \dot{\Gamma}\Gamma^{-1}\bar{e} \tag{68}$$

with $\Gamma\Delta_\rho\left(g(u,\hat{z}) - g(u,z)\right) = \Gamma\Delta_\rho G_e(u,\hat{z},z)$, $\Gamma\Delta_\rho A(u)\Delta_\rho^{-1}\Gamma^{-1} = \rho\tilde{A}$ and $C\Delta_\rho^{-1}\Gamma^{-1} = \rho C$.

Let us define $V(\bar{e}) = \bar{e}^T P \bar{e}$, a Lyapunov function for the system (68), with P verifying $P(\tilde{A}-KC) + (\tilde{A}-KC)^T P = -I$.

By taking the time derivative of $V(\bar{e})$ along equation (68), we obtain

$$\dot{V}(\bar{e}) = 2\bar{e}^T P \dot{\bar{e}} \\ \leq -\rho\|\bar{e}\|_2^2 + 2\lambda_{\max P}\|\bar{e}\|_2\|\Gamma\Delta_\rho G_e(u,\hat{z},z)\|_2 \\ +2\lambda_{\max P}^{3/2}\left\|\dot{\Gamma}\Gamma^{-1}\right\|_2\|\bar{e}\|_2^2$$

with $\|x\|_P^2 = x^T P x$ and satisfying the inequalities

$$\lambda_{\min P}\|x\|_2^2 \leq \|x\|_P^2 \leq \lambda_{\max P}\|x\|_2^2$$

where $\lambda_{\min P}$ and $\lambda_{\max P}$ denote the smallest and largest eigenvalues of P respectively.

Let us assume that $\|\Gamma \Delta_\rho G_e(u, \hat{z}, z)\|_2 \leq l_1 \|\bar{e}\|_2$ and $\sup_{t \geq 0} \left\| \dot{\Gamma}\Gamma^{-1} \right\|_2 = l_2$ where l_1 and l_2 are positive constants.

Then, if ρ is sufficiently large such that $(\rho - 2l_1 \lambda_{\max P} - 2l_2 \lambda_{\max P}^{3/2}) > 0$, then $\dot{V}(\bar{e}) \leq 0$.

Finally, there exists $\rho_0 > 0$ such that for all $\rho > \rho_0$, system (67) is an exponential observer for system (61).

4. SIMULATION RESULTS

In order to validate the performance of the proposed control law with the observer design, we have carried out simulations. We have replaced, in the control law computation, the unknown state by the estimated state. The initial conditions are: $(x, y, \theta) = \left(20, 10, \frac{\pi}{3}\right)$, $\left(\dot{x}, \dot{y}, \dot{\theta}\right) = (2, 1, 1)$ and $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4) = (20, 2.2, 0, 0.8)$. The parameters of the observer are: $(k_1, k_2, k_3, k_4) = (4, 6, 4, 1)$ and $\rho = 1.2$. The parameters of the controller are: $(a, b, c, d) = (3.2, 1.6, 0.8, 1.92)$ and $(a_1, a_2, a_3) = (1, 0.5, 0.3)$. We finally observe in Figure (4) the satisfactory performance of the proposed observer-based control. In addition, since high gains observers provide faster convergence rates and since we have proved that the proposed control law is robust with respect to observer errors, we can expect some satisfactory performance of the proposed observer-based control even in presence of uncertainties. Some simulations in presence of noise in the output and model uncertainties (see (Sánchez *et al.*, 2004)) have supported this remark.

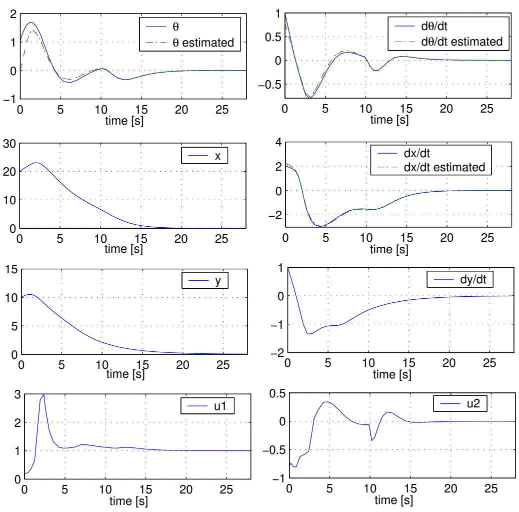

Fig. 4. System behavior applying the observer-based control law

5. CONCLUSIONS

A nonlinear observer design for the PVTOL aircraft model has been proposed in order to make an estimation of the angular position. It has been proved that the control strategy used here, is robust with respect to observer errors and that the observer-based control is stable. The methodology has been based on the use of embedded saturations and Lyapunov analysis of the closed-loop system. Simulations results have shown that the nonlinear observer performs well in closed-loop. Future works include the implementation of the observer and the controller on a real experiment at Heudiasyc laboratory.

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