COMPARISONS OF SUBSPACE IDENTIFICATION METHODS FOR SYSTEMS OPERATING ON CLOSED-LOOP

Weilu Lin *, **S. Joe Qin** *,[1], and **Lennart Ljung** **

* *Department of Chemical Engineering, The University of Texas at Austin, Austin, Texas 78712, USA*
** *Department of Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden*

Abstract: In this paper, we analyze two recently proposed closed-loop subspace identification methods, referred to as innovation estimation method and whitening filter approach respectively. The similarity and difference between them are investigated in detail. It turns out that all closed-loop subspace identification methods can be classified as one-step, two-step, or multi-stage projection methods. A SISO closed-loop simulation shows that to identify a consistent model the whitening filter approach might require longer future and past horizons than the innovation estimation method.

Keywords: Closed-loop Identification, Subspace Algorithms

1. INTRODUCTION

Subspace identification methods (SIMs) are attractive not only because of their numerical simplicity and stability, but also for their state space form that is very convenient for optimal estimation and control. However, it is well known that most traditional SIMs (e.g., N4SID, MOESP, CCA) are not applicable to systems operating under closed-loop conditions without special treatments. As pointed out by many researchers (Ljung, 1999), the fundamental problem with closed-loop data is the correlation between the process noise and the input.

Aimed at identifying a state space model with feedback, a couple of closed-loop SIMs have been proposed in the last decade (Verhaegen, 1993; Ljung and McKelvey, 1996; Overschee and Moor, 1997). More recent work is presented in (Qin and Ljung, 2003; Jansson, 2003), which has been regarded as a significant advance in subspace identification of feedback systems (Chiuso and Picci, 2005). The consistency of the algorithms has been investigated in (Chiuso and Picci, 2005; Lin *et al.*, 2004*a*). The main purpose of this work is to investigate the similarity and difference of these two approaches, and compare the performance based on a well known closed-loop example.

The rest of the paper is organized as follows. In Section 2, we state the problem. The similarity and difference of two recently proposed closed-loop SIMs is presented in detail in Section 3. In Section 4, a closed-loop SISO simulation is given to show the performance of different algorithms. Section 5 concludes the paper.

2. PROBLEM FORMULATION

In the paper, the system to be identified can be written in an innovation form as follows,

[1] Corresponding Author: qin@che.utexas.edu. Supported by Weyerhaeuser Company through sponsorship of the Texas-Wisconsin Modeling and Control Consortium.

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (1a)$$
$$y_k = Cx_k + Du_k + e_k \quad (1b)$$

where $y_k \in R^{n_y}$, $x_k \in R^n$, $u_k \in R^{n_u}$, and $e_k \in R^{n_y}$ are the system output, state, input, and innovation, respectively. A, B, C and D are system matrices with appropriate dimensions. K is the Kalman filter gain. The system described by equation 1 can also be represented as

$$x_{k+1} = A_K x_k + B_K u_k + Ky_k \quad (2a)$$
$$y_k = Cx_k + Du_k + e_k \quad (2b)$$

where $A_K = A - KC$, and $B_K = B - KD$. We consider the input u_k is determined through feedback which makes u_k correlated with past innovation e_k. We refer to equation 2 as the predictor form.

The system represented by equation 1 and the represented by equation 2 are equivalent, but system (1) uses the original process A matrix while system (2) uses the predictor A_K matrix. If the process to be identified is unstable, the predictor A_k matrix can still be stable. The closed-loop identification problem is: given a set of input/output and reference measurements under closed-loop, estimate the system matrices (A, B, C, D) and Kalman filter gain K up to a similarity transformation.

3. CLOSED-LOOP SUBSPACE IDENTIFICATION METHODS

3.1 Notation and Overview

Based on state space description in equation 1, an extended state space model can be formulated as

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \quad (3)$$

where the subscripts f and p denote future and past horizons, respectively. The extended observability matrix is

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \quad (4)$$

and H_f and G_f are Toeplitz matrices:

$$H_f = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \cdots & D \end{bmatrix} \quad (5a)$$

$$G_f = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & \cdots & I \end{bmatrix} \quad (5b)$$

The input and output data are arranged in the following Hankel form:

$$U_f = \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+N-1} \\ u_{k+1} & u_{k+2} & \cdots & u_{k+N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+f-1} & u_{k+f} & \cdots & u_{k+f+N-2} \end{bmatrix} \quad (6)$$

Similar formulations are made for Y_f and E_f. Subspace identification consists of estimating the extended observability matrix first and then the model parameters.

Solving x_k by iterating equation 2, it is straightforward to derive the following relation,

$$X_k = L_z Z_p + A_K^p X_{k-p} \quad (7)$$

where

$$X_k = \begin{bmatrix} x_k & x_{k+1} & \cdots & x_{k+N-1} \end{bmatrix} \quad (8a)$$
$$L_z \triangleq \begin{bmatrix} \Delta_p(A_K, K) & \Delta_p(A_K, B_K) \end{bmatrix} \quad (8b)$$
$$\Delta_p(A, B) \triangleq \begin{bmatrix} A^{p-1}B & \cdots & AB & B \end{bmatrix} \quad (8c)$$
$$Z_p \triangleq \begin{bmatrix} Y_p^T & U_p^T \end{bmatrix}^T \quad (8d)$$

Substituting equation 7 into equation 3, we obtain

$$Y_f = \Gamma_f A_K^p X_{k-p} + \Gamma_f L_z Z_p + H_f U_f + G_f E_f \quad (9)$$

If the past horizon p is large enough, the first term on the RHS tends to zero for stable A_K. The last two terms of the RHS of equation 9 are correlated for closed-loop systems. Therefore, most of the closed-loop SIMs try to decouple these two terms. The SIMPCA methods proposed in (Wang and Qin, 2002) and a later modification in (Huang *et al.*, 2005) move $H_f U_f$ to the LHS and use principal component analysis on the joint input/output data simultaneously. We refer to these approaches as one-step approaches since no pre-estimation is needed. Another approach that falls in the one-step approach category is the observer/Kalman filter ID (OKID) by (Phan and Longman, 1992) .

Since equation 9 is actually composed of f block rows in each term and the first block row gives an estimate of the innovation, Qin and Ljung (Qin and Ljung, 2003) propose an innovation estimation method (IEM) that partition equation 9 in to f block rows and use the estimated innovation from previous block rows to further estimate model parameters of the next block row sequentially. An alternative method known as IEM1 (Lin *et al.*, 2004*b*) estimates the innovation from the first block row and then treats $\hat{e}_k$ as known to estimate other model parameters. The SSARX approach proposed in (Jansson, 2003) uses the predictor form (equation 2) and pre-estimate a high order ARX model parameter to decouple the correlation between U_f and E_f.The well known CVA algorithm proposed by Larimore (Larimore, 1990) actually pre-estimate H_f using a high order ARX and the move $\hat{H}_f U_f$ to the LHS of equation 9. Shi and MacGragor (Shi and MacGregor, 2001) also use this technique.

These approaches are referred to as two-step approaches in which a pre-estimation step is needed to decouple the noise and control input. The pre-estimation step is usually done by a high-order ARX; only different information is used to carry out the main step.

Inspired from the SSARX approach, Chiuso and Picci (Chiuso and Picci, 2005) give a variation known as the whitening filter approach (WFA) that uses the predictor model form and carry out multi-stage projections row by row. In each block row projection causality is strictly enforced, similar to (Qin *et al.*, 2005). No pre-estimation is involved but the projections have to be done block-row wise to decouple noise from control input. We refer to these approaches as multi-stage projection approaches.

3.2 Innovation Estimation Method

Neglecting the first term in RHS of equation 9 and partitioning the resulting equation row-wise, we obtain

$$Y_{fi} = \Gamma_{fi} L_z Z_p + H_{fi} U_i + G^{-}_{fi} E_{i-1} + E_{fi} \tag{10}$$

where

$$\Gamma_{fi} = CA^{i-1} \tag{11a}$$

$$H_{fi} \triangleq \left[CA^{i-2}B \ \cdots \ CB \ D \right] \tag{11b}$$

$$G^{-}_{fi} \triangleq \left[CA^{i-2}K \ \cdots \ CK \right] \tag{11c}$$

$$Y_f = \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{ff} \end{bmatrix}; \ Y_i \triangleq \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{fi} \end{bmatrix}; i = 1, 2, \ldots, f \tag{12}$$

U_{fi}, U_i, E_{fi}, and E_i are defined in a similar way. For example, the first block row of equation 12 is

$$Y_{f1} = CL_z Z_p + DU_1 + E_{f1} \tag{13}$$

which is a high-order ARX model.

The innovation estimation method proposed in (Qin and Ljung, 2003; Lin *et al.*, 2004*b*) involves estimating innovation sequence row-wise and estimating Γ_f through a weighted singular value decomposition (SVD). A, B, C, D and K can also be obtained as illustrated in (Qin *et al.*, 2005).

3.3 Whitening Filter Approach

Based on state space description in equation 2, an alternative extended state space model can be formulated as

$$Y_f = \bar{\Gamma}_f X_k + \bar{H}_f U_f + \bar{G}_f Y_f + E_f \tag{14}$$

The modified extended observability matrix is

$$\bar{\Gamma}_f = \begin{bmatrix} C \\ CA_K \\ \vdots \\ CA_K^{f-1} \end{bmatrix} \tag{15}$$

and $\bar{H}_f$ and $\bar{G}_f$ are:

$$\bar{H}_f = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB_K & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA_K^{f-2}B_K & CA_K^{f-3}B_K & \cdots & D \end{bmatrix} \tag{16a}$$

$$\bar{G}_f = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CK & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA_K^{f-2}K & CA_K^{f-3}K & \cdots & 0 \end{bmatrix} \tag{16b}$$

Similar to the innovation estimation method, one can substitute equation 7 into equation 14, and partition the resulting equation row-wise as

$$\begin{aligned} Y_{fi} = & \bar{\Gamma}_{fi} L_z Z_p + \bar{\Gamma}_{fi} A_K^p X_{k-p} \\ & + \bar{H}_{fi} U_i + \bar{G}_{fi} Y_i + E_{fi} \end{aligned} \tag{17}$$

where

$$\bar{\Gamma}_{fi} = CA_K^{i-1} \tag{18a}$$

$$\bar{H}_{fi} \triangleq \left[CA_K^{i-2}B_K \ \cdots \ CB_K \ D \right] \tag{18b}$$

$$\bar{G}_{fi} \triangleq \left[CA_K^{i-2}K \ \cdots \ CK \ 0 \right] \tag{18c}$$

Therefore, through a multi-stage least squares similar to the innovation estimation method, one can estimate $\bar{\Gamma}_f L_z$, $\bar{H}_f$ and $\bar{G}_f$. $\bar{\Gamma}_f$ can be estimated through a weighted SVD. It is well known that A_K, C, D, B_K, and K can be obtained through the estimates of $\bar{\Gamma}_f$, $\bar{H}_f$ and $\bar{G}_f$. After that A and B can be backed out through the definition of A_K and B_K.

[Remark 1] The above analysis clearly illustrates the similarity between the innovation estimation method and the whitening filter approach. They all partition the extended state space row-wise and utilize multi-stage least square method to estimate system matrices. The innovation estimation method starts from a state space model in innovations form, while the whitening filter approach is based on a state space model in predictor form.

[Remark 2] There is another implementation of the whitening filter approach (Jansson, 2003). One can estimate the Markov parameters through the high order ARX, and subtracting the effect of future inputs and outputs.

[Remark 3] As pointed out by Chiuso and Picci (Chiuso and Picci, 2005), both approaches require that eigenvalues of A_K lie strictly inside the unit circle. For a finite past horizon, they are biased due to $A_K^p X_{k-p} \neq 0$.

[Remark 4] For finite data the predictor model form is time varying due to a time varying Kalman filter, even though the system is time-invariant. This may complicate the rank condition of $\bar{\Gamma}_f$ and the subsequent extraction of A_k and C from $\bar{\Gamma}_f$. From this point the IEM is superior to SSARX or WFA.

[Remark 5] The innovation estimation method uses the process A matrix to form the observability matrix, while the whitening filter approach uses the predictor matrix A_K. For open loop unstable systems the whitening filter approach can be numerically advantageous, as demonstrated in (Chiuso and Picci, 2005). However, for bounded systems such as stable or integrating systems, this advantage disappears. In the next section we compare these methods using the closed-loop example given in (Overschee and Moor, 1996) which has one integrating pole and four stable poles. The simulation results seems to favor the innovation estimation method.

4. SIMULATION EXAMPLE

The example in (Overschee and Moor, 1996) is adopted here for comparison. The model of the plant is given in a state space form:

$$A = \begin{bmatrix} 4.40 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4.00 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.0033 \\ -0.00002 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, K = \begin{bmatrix} 2.3 \\ -6.64 \\ 7.515 \\ -4.0146 \\ 0.86336 \end{bmatrix}, D = 0$$

The feedback mechanism is

$$u_k = -F(q)y_k + r_k$$

where

$$F(q) = \frac{(0.61q^4 - 2.03q^3 + 2.76q^2 - 1.83q + 0.49)}{q^4 - 2.65q^3 + 3.11q^2 - 1.75q + 0.39} \tag{20}$$

and r_k is a zero-mean white noise sequence with standard deviation 1. We take the number of data points $j = 1200$ and generate 100 data sets, each one with the same reference input r_k but with different noise sequence e_k. We choose $f = p = 20$ for innovation estimation methodes, and $f = p = 30$ for whitening filter approaches. In our simulation, we observe that to obtain unbiased estimation the whitening filter approach needs larger f and p than the innovation estimation method.

The pole estimation results for the closed-loop experiments are shown in Figs. 2, 4, 6, and 8. From the results we can see that all the methods can provide consistent estimates, while the whitening filter approach produce the worst results.

The estimates of the frequency response for the closed-loop simulations are shown in Figs. 1, 3, 5, and 7. We can see that the estimated frequency responses from all the methods match well with that of the real system at low frequency, but they all show bias at high frequency.

5. CONCLUSIONS

In this paper, we analyze two recently proposed closed-loop subspace identification methods, referred to as innovation estimation method and whitening filter approach respectively. The similarity and difference of them are investigated in detail. Both approach partition the extended state space model into block rows and use the information estimated from the first block row further estimate model parameters in the remaining rows. Through this partition the correlation between the process input and innovation due to feedback is decoupled. It turns out that although they are based on different representations of state space models all of them can be implemented through multi-stage least squares. All closed-loop SIMs can be classified into one-step, two-step and multi-stage approaches and each of them seems to have its own advantages.

REFERENCES

Chiuso, A. and G. Picci (2005). Consistency analysis of some closed-loop subspace identification methods. **41**, 377–391.

Huang, Biao, Steven X. Ding and S. Joe Qin (2005). Closed-loop subspace identification: an orthogonal projection approach. **15**, 53–66.

Jansson, M. (2003). Subspace identification and arx modelling. In: *Proceedings of the 13th IFAC SYSID Symposium*. Rotterdam, NL.

Larimore, W. E. (1990). Canonical variate analysis in identification, filtering and adaptive control. In: *IEEE Conference on Decision and Control*. pp. 596–604.

Lin, W., S. J. Qin and L. Ljung (2004*a*). A framework for closed-loop subspace identification with innovation estimation. *Submitted to Automatica*.

Lin, W., S. J. Qin and Lennart Ljung (2004*b*). On consistency of closed-loop subspace identification with innovation estimation. In: *43rd IEEE Conference on Decision and Control*. Atlantis, Bahamas. pp. 2195–2200.

Ljung, L. (1999). *System Identification - Theory for the User*. second ed.. Prentice Hall PTR.

Ljung, L. and T. McKelvey (1996). Subspace identification from closed loop data. *Signal Processing* **52**, 209–215.

Overschee, P. Van and B. De Moor (1996). *Closed-loop subspace system identification*. Technical Report ESAT-SISTA/TR 1996-52I, Katholieke Universiteit Leuven.

Overschee, P. Van and B. De Moor (1997). Closed-loop subspace system identification. In: *Proceedings of the 36th Conference on Decision and Control*. pp. 1848–1853.

Phan, M., Horta L.G. Juang J.-N. and R.W. Longman (1992). Improvement of observer/Kalman filter identification (OKID) by residual whitening. In: *AIAA Guidance, Navigation and Control Conference*. Hilton Head, South Carolina.

Qin, S. J. and Lennart Ljung (2003). Closed-loop subspace identification with innovation estimation. In: *Proceedings of the 13th IFAC SYSID Symposium*. Rotterdam, NL. pp. 887–892.

Qin, S. J., W. Lin and L. Ljung (2005). A novel subspace identification approach with enforced causal models. *Revised for Automatica*.

Shi, R. and J. F. MacGregor (2001). A framework for subspace identification methods. In: *Proceeding of ACC*. Arlington, VA. pp. 3678–3683.

Verhaegen, M. (1993). Application of a subspace model identification technique to identify lti systems operating on closed-loop. *Automatica,* **29**, 1027–1040.

Wang, J. and S. J. Qin (2002). A new subspace identification approach based on principle component analysis. *J. Proc. Cont.,* **12**, 841–855.

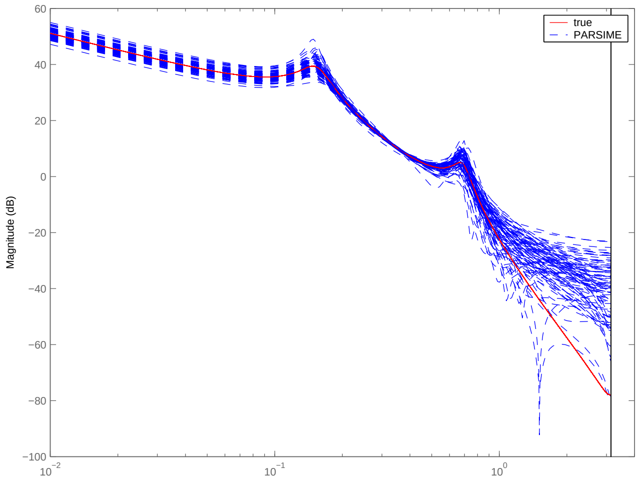

Fig. 1. The Bode magnitude plot of PARSIM-E for SISO closed-loop simulations.

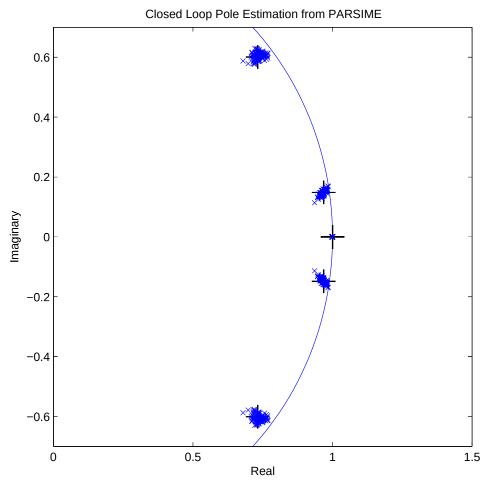

Fig. 2. The eigenvalues of estimated A matrix: $\times$ estimated pole, $+$ system pole.

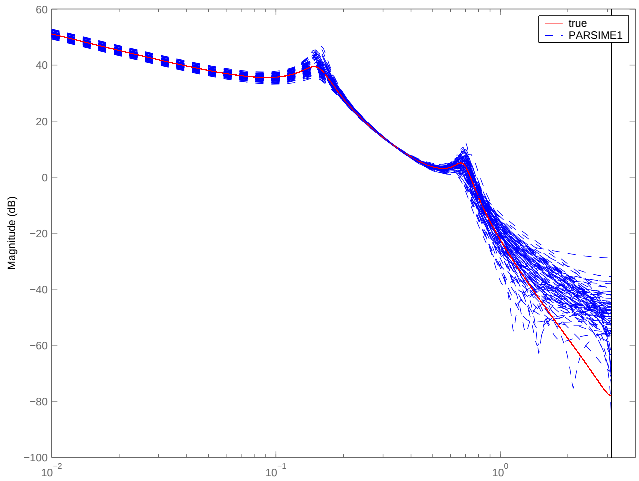

Fig. 3. The Bode magnitude plot of PARSIM-E1 for SISO closed-loop simulations.

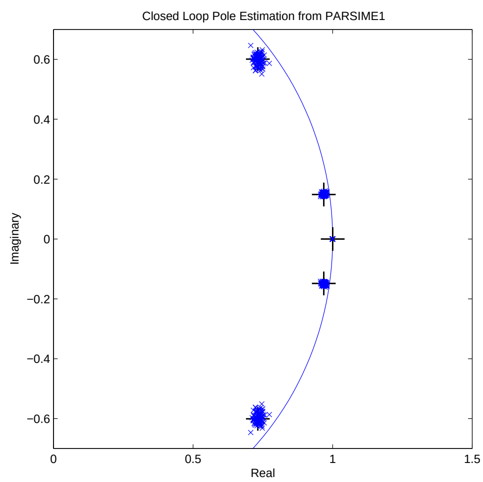

Fig. 4. The eigenvalues of estimated A matrix: $\times$ estimated pole, $+$ system pole.

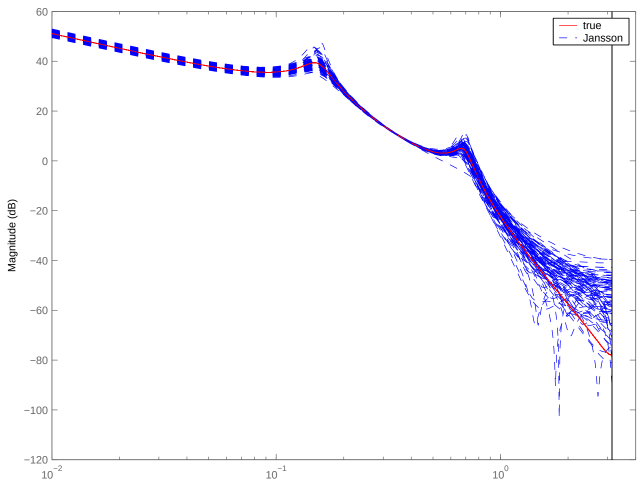

Fig. 5. The Bode magnitude plot of Jansson's approach for SISO closed-loop simulations.

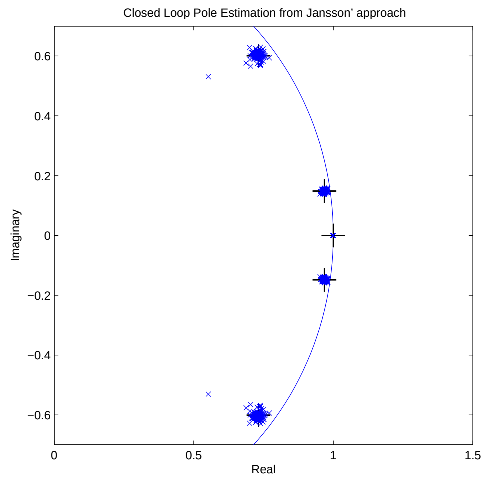

Fig. 6. The eigenvalues of estimated A matrix: $\times$ estimated pole, $+$ system pole.

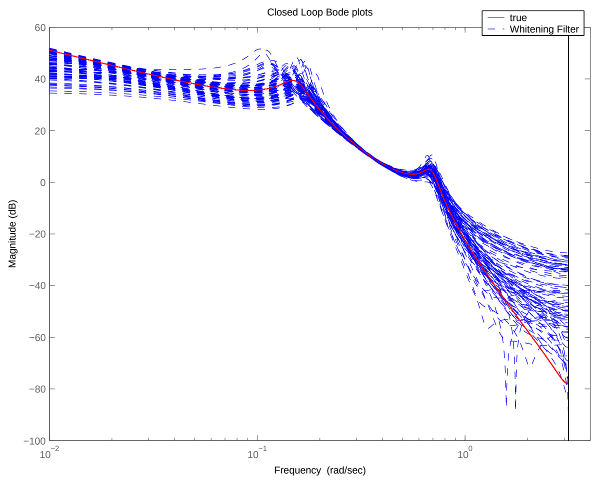

Fig. 7. The Bode magnitude plot of whitening filter approach for SISO closed-loop simulations.

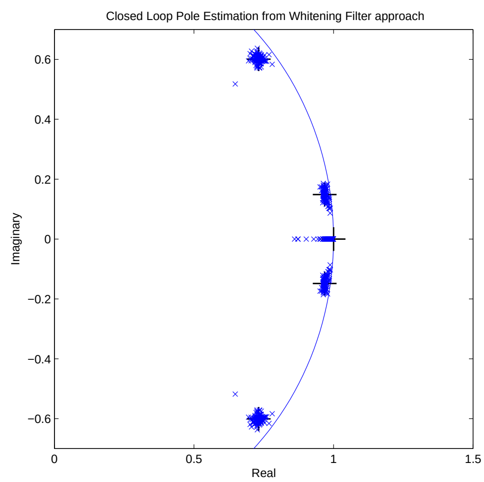

Fig. 8. The eigenvalues of estimated A matrix: $\times$ estimated pole, $+$ system pole.