OPTIMAL SHAPE DESIGN FOR TIME DEPENDENT SYSTEM - APPLICATION TO OPTIMAL POSITION OF ACTUATORS

Arnaud MÜNCH [*,1]

[*] *Laboratoire de Mathématiques de Besançon, UMR CNRS 6623, Université de Franche-Comté, 16, route de Gray 25030 Besançon cedex, FRANCE*

Abstract: We address in this work some theoretical and numerical issues on optimal design problems concerning evolution equation and more specifically the wave equation posed in 1-D and 2-D domain. We compute the gradient of the cost function and then use the level set approach to optimize the location of actuators in order to stabilize or control exactly a given system

Keywords: Optimal shape design, wave equation, level set, numerical schemes.

1. INTRODUCTION

The goal of this work is to present some theoretical and numerical remarks about optimal shape design problem associated to time dependent system, and more specifically to hyperbolic system. In order to simplify the presentation, we will consider here without loss of generality the isotropic and homogenous wave system. (see (Münch n.d.) for more details on the treatment and numerical simulations for non-isotropic elasto-dynamic systems). Let us present two "simple" problems posed in $\mathbb{R}^2$ we have in mind. The first one is illustrated on Figure 1. Let be given $y_d \in L^2(\mathbb{R}^2)$ and ω a subset of $\mathbb{R}^2$. Can we find a domain Ω containing ω such that the restriction to ω of y_Ω solution of the heat system (1)

$$\begin{cases} y'_\Omega(\boldsymbol{x},t) - \Delta y_\Omega(\boldsymbol{x},t) = f(\boldsymbol{x},t) & \text{in} \quad \Omega \times (0,T), \\ y_\Omega(\boldsymbol{x},0) = y^0(\boldsymbol{x}), & \text{in} \quad \Omega, \\ y_\Omega(\boldsymbol{x},t) = 0 & \text{on} \quad \partial\Omega \times (0,T), \end{cases} \tag{1}$$

is such that

$$y_\Omega(\boldsymbol{x},T)|_\omega = y_d(\boldsymbol{x}) \quad ? \tag{2}$$

In other words, we want to **control** exactly the solution on a part of the domain at time T **by the shape of the domain**. We point out that the domain Ω we are looking for is independent of the time. Besides, the domain ω may be equal to the whole domain Ω. Furthermore, by introducing a parameter $\varepsilon > 0$, we may replace this exact control problem by an approximate one: find Ω such that y_Ω solution of (1) verify

$$||y_{\Omega_\varepsilon}(\boldsymbol{x},T)|_\omega - y_d(\boldsymbol{x})||_2 \le \varepsilon. \tag{3}$$

By means of optimization technics, these problems are equivalent to study the infimum of the following functional

$$J(\Omega,T) = \int_\Omega (y_\Omega(\boldsymbol{x},T) - y_d(\boldsymbol{x}))^2 1_{(\boldsymbol{x}\in\omega)} dx. \tag{4}$$

In this example, the heat equation may be replaced by another time dependent systems. We may also design a second class of problem a bit more complex in the sense that the domain we are looking for now depends on the time. This second example is illustrated on Figure 2.

Let now $\Omega \subset \mathbb{R}^2$ be fixed and let us consider on Ω the following internal damped wave system:

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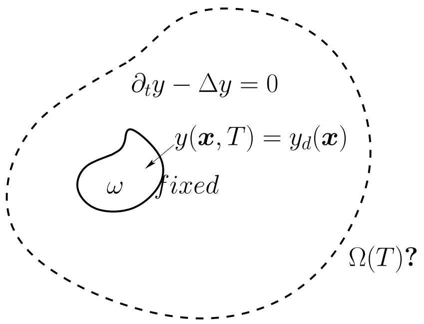

Fig. 1. $\Omega(T)$ independent of time t minimizing the functional ?

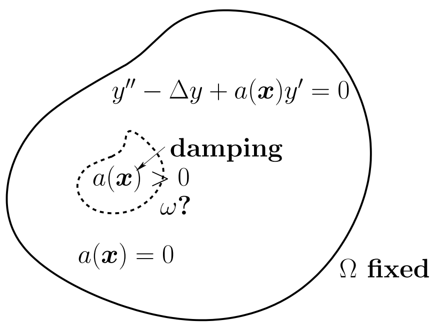

Fig. 2. $(\omega(t))_{0 \le t \le T}$ minimizing the energy E of the system?

$$\begin{cases} y''_{\omega(t)}(\boldsymbol{x},t) - \Delta y_{\omega(t)}(\boldsymbol{x},t) \\ \qquad +a(\boldsymbol{x})y'_{\omega(t)}(\boldsymbol{x},t) = f(\boldsymbol{x},t) \ \text{ in } \ \Omega \times (0,T), \\ y_{\omega(0)}(\boldsymbol{x},0) = y^0(\boldsymbol{x}) \ \text{ in } \ \Omega, \\ y'_{\omega(0)}(\boldsymbol{x},0) = y^1(\boldsymbol{x}) \ \text{ in } \ \Omega, \\ y_{\omega(t)}(\boldsymbol{x},t) = 0 \ \text{ on } \ \partial\Omega \times (0,T) \end{cases} \tag{5}$$

with for instance, $a(\boldsymbol{x}) = a1_{(\boldsymbol{x} \in \omega(t))}, a > 0, \forall 0 \le t \le T$. The energy associated to this system

$$E(\omega,T) = \frac{1}{2}\int_\Omega \left\{ |y'_\omega(\boldsymbol{x},T)|^2 + |\nabla y_\omega(\boldsymbol{x},T)|^2 \right\} dx \tag{6}$$

is decreasing due to the following inequality:

$$E'(\omega,t) = -\frac{1}{2}\int_\Omega a(x)|y'_\omega(\boldsymbol{x},t)|^2, \quad \forall t > 0. \tag{7}$$

Therefore, one may ask the following question: can we find a sequence of domain $(\omega(t))_t, 0 < t < T$ minimizing the energy E at time T. From a mechanical point of view, we would like to optimize the location of a moving actuator in order to damp the energy. In this case, we may add a constraint condition on the area of ω, the trivial solution being $\omega = \Omega, \forall t$. Despite an extensive literature on optimal design for time independent problem, these kind problem remains to be studied (see however (Zolésio and Truchi 1987) and the references therein) .

2. CONTINUITY RESULT

For Dirichlet boundary condition, we can obtain the following continuity result leading to the existence of minima of J and E defined in the previous section. Let us consider the following conservative wave equation defined on $\Omega \times [0,\infty)$, Ω being a bounded domain in $\mathbb{R}^2$ of class C^2:

$$\begin{cases} \Box y(\boldsymbol{x},t) = f(\boldsymbol{x},t) \ \text{ in } \ \Omega \times (0,T), \\ y(\boldsymbol{x},t) = 0 \quad \text{on} \quad \Gamma \equiv \partial\Omega \times (0,T), \\ y(\boldsymbol{x},t) = y_0(\boldsymbol{x}), \ \dfrac{\partial y}{\partial t}(\boldsymbol{x},t) = y_1(\boldsymbol{x}) \quad \text{in} \quad \Omega \times \{t=0\}, \end{cases} \tag{8}$$

where, for all $\boldsymbol{x} = (x_1, x_2) \in \Omega$,

$$\Box y(\boldsymbol{x},t) = \frac{\partial^2 y}{\partial t^2}(\boldsymbol{x},t) - \Delta y(\boldsymbol{x},t). \tag{9}$$

System (8) is well-posed in the space $H_0^1(\Omega) \times L^2(\Omega)$. Indeed, given $\{y_0, y_1\} \in H_0^1(\Omega) \times L^2(\Omega)$, and $f \in \mathcal{C}([0,\infty); L^2(\Omega))$ there exists a unique solution of (8) with (see (Lions and Magenes 1968))

$$y \in \mathcal{C}([0,\infty); H_0^1(\Omega)) \cap \mathcal{C}^1([0,\infty); L^2(\Omega)). \tag{10}$$

Let us reconsider the first example with this wave system: in order to show that the problem admits at least one solution, i.e. that the infimum of the energy is a minimum, we need a continuity of the solution y_Ω with respect to the domain Ω in a suitable topology, like those presented and obtained for elliptic system with Dirichlet ((Chambolle and Doveri 1997), (Pironneau 1983), (Sverak 1993), (Henrot 1994)). Following Sverak (Sverak 1993), in the case where the domain Ω is independent of the time, one may show the following result:

Theorem 1. Let $\mathcal{O}_l = \{\Omega$ open subset of D, with number of connected components of $D/\Omega \quad < l\}$.

- IF $\Omega_n \in O_l$ and D/Ω_n converge to D/Ω for the Haussdorff metric
- and IF $y^0_{\Omega_n} \to y^0_\Omega$ in $H^1_0(D), y^1_{\Omega_n} \to y^1_\Omega$ in $L^2(D)$

THEN $\Omega \in \mathcal{O}_l$ and $y_{\Omega_n}(.,t) \to y(.,t)$ in $H_0^1(\Omega) \forall t \in [0,T]$. ■

This kind of continuity result for the Haussdorff topology then leads to existence result of our problem. In the next section, we obtain the derivative of the functional J with respect to the domain and unknown Ω.

3. SHAPE DERIVATIVE WITH RESPECT TO THE DOMAIN

From a practical point of view, it is useful to have an expression of derivative of the cost functional with respect to the domain. We introduce a vector field $\boldsymbol{\theta} \in W^{1,\infty}(\overline{\Omega}, \mathbb{R}^2)$ and the transformation $T : \boldsymbol{x} \to \boldsymbol{x} + \eta\boldsymbol{\theta}(\boldsymbol{x}) \in \Omega_\eta$. The derivative of the functional J with respect to a variation of Ω in the direction $\boldsymbol{\theta}$ is defined as follow:

$$\frac{\partial J(\Omega)}{\partial \Omega}.\boldsymbol{\theta} = \lim_{\eta \to 0} \frac{J(\Omega + \eta\boldsymbol{\theta}(\Omega)) - J(\Omega)}{\eta}. \tag{11}$$

It can be shown that this limit exists and therefore depends only of the field $\boldsymbol{\theta}$ in an arbitrary neighborhood of $\partial\Omega$ (see for instance, (Cagnol and Zolesio 1999), (Murat and Simon 1976)), leading to the following expression:

Theorem 2. The derivative takes the following expression

$$\frac{\partial J(\Omega)}{\partial \Omega}.\boldsymbol{\theta} = \int_{\partial\Omega} \Big[(y_\Omega(\boldsymbol{x}, T) - y_d(\boldsymbol{x}))^2 1_{(\boldsymbol{x}\in\omega)} - \frac{\partial}{\partial n} \int_0^T \frac{\partial p_\Omega(\boldsymbol{x}, t)}{\partial n} y_\Omega(\boldsymbol{x}, t) dt \Big] \boldsymbol{\theta}.\boldsymbol{n} ds, \tag{12}$$

where $\boldsymbol{n}$ designs the normal derivative (oriented toward the exterior) and p_Ω solution of the following adjoint problem with Dirichlet boundary condition:

$$\begin{cases} \dfrac{\partial^2 p_\Omega(\boldsymbol{x}, t)}{\partial t^2} - \Delta p_\Omega(\boldsymbol{x}, t) = 0 \quad on \quad \Omega \times (0, T), \\ p_\Omega(\boldsymbol{x}, t) = 0 \quad on \quad \partial\Omega \times (0, T), \\ p_\Omega(\boldsymbol{x}, t) = 0 \quad on \quad \Omega \times \{t = T\}, \\ \dfrac{\partial p_\Omega(\boldsymbol{x}, t)}{\partial t} = 2(y_\Omega(\boldsymbol{x}, t) - y_d(\boldsymbol{x}))1_{(\boldsymbol{x}\in\omega)} \\ \qquad\qquad on \quad \Omega \times \{t = T\}. \end{cases} \tag{13}$$

■

Remark 1. The system (13) has a unique solution in $\mathcal{C}([0, T]; H_0^1(\Omega)) \cap \mathcal{C}^1([0, T]; L^2(\Omega))$. ■

Proof. We may derive a rather formal proof using the tools developed in (Cea 1986) and introducing the following Lagrangian defined for $(v, q) \in C^2([0, T]; H_0^1(\Omega))$ and

$$\begin{aligned} \mathcal{L}(\Omega, v, q, \lambda_1, \lambda_2, \lambda_3) = & \int_\Omega (v(\boldsymbol{x}, T) - y_d(\boldsymbol{x}))^2 1_{(\boldsymbol{x}\in\omega)} dx \\ & + \int_\Omega \int_0^T \left(\frac{\partial^2 v(\boldsymbol{x}, t)}{\partial t^2} - \Delta v(\boldsymbol{x}, t) - f(\boldsymbol{x}, t) \right) \times q(\boldsymbol{x}, t) dx dt \\ & + \int_{\partial\Omega} \int_0^T \lambda_1(\boldsymbol{x}, t) v(\boldsymbol{x}, t) ds dt \\ & + \int_\Omega \lambda_2(\boldsymbol{x})(v(\boldsymbol{x}, 0) - y_0(\boldsymbol{x})) dx \\ & + \int_\Omega \lambda_3(\boldsymbol{x})(\frac{\partial v(\boldsymbol{x}, 0)}{\partial t} - y_1(\boldsymbol{x})) dx. \end{aligned} \tag{14}$$

(see (Münch n.d.)) for the details.

Remark 2. When ω is strictly included in the domain Ω, the relation (12) simply becomes

$$\frac{\partial J(\Omega)}{\partial \Omega}.\boldsymbol{\theta} = -\int_{\partial\Omega} \frac{\partial}{\partial n} \int_0^T \frac{\partial p_\Omega(\boldsymbol{x}, t)}{\partial n} y_\Omega(\boldsymbol{x}, t) dt \; \boldsymbol{\theta}.\boldsymbol{n} ds \tag{15}$$

4. DETERMINATION OF THE OPTIMAL SHAPE DESIGN BY LEVEL SET METHOD

Thanks to the derivative of J, the simplest way to obtain a minimum is to use a gradient descent method. This can be done independently of the mesh of the domain using the well-known level set approach introduced in (Sethian 1996) which consists to give a description of the evolving interface - in our case the boundary $\partial\Omega$ - independent of the mesh. Let us consider the scalar function ϕ such that

$$\begin{cases} \phi(\boldsymbol{x}) \le 0 \quad \boldsymbol{x} \in \Omega, \\ \phi(\boldsymbol{x}) = 0 \quad \boldsymbol{x} \in \partial\Omega, \\ \phi(\boldsymbol{x}) \ge 0 \quad \boldsymbol{x} \in D/\Omega, \end{cases} \tag{16}$$

with D a fixed domain such that $\Omega \subset D$. Therefore, the evolving interface is characterized by

$$\partial\Omega = \{\boldsymbol{x}(\tau) \in D \text{ such that } \phi(\boldsymbol{x}(\tau), \tau) = 0\}, \tag{17}$$

where τ designs a pseudo-time variable, increasing with time, that may be the real time, a load factor or in our case, the iteration number of a given algorithm. Differentiation in τ of (17) then leads to

$$\frac{\partial \phi}{\partial \tau} + \nabla\phi(\boldsymbol{x}(\tau), \tau). \frac{dx(\tau)}{d\tau} = 0. \tag{18}$$

Denoting by F the speed in the outward normal direction, such that $\frac{d\boldsymbol{x}(\tau)}{d\tau}.\boldsymbol{n} = F$ where

$$\boldsymbol{n} = \nabla\phi / |\nabla\phi|, \tag{19}$$

we obtain the following evolution equation for ϕ:

$$\frac{\partial \phi}{\partial \tau} + F|\nabla\phi(\boldsymbol{x}(\tau), \tau)| = 0, \quad \text{given} \quad \phi(\boldsymbol{x}, \tau = 0). \tag{20}$$

Therefore the correct function F to take in order that ϕ converges to a function corresponding to a extremum of J is $-j(u_\Omega, p_\Omega)$ (see eq.32), leading to the algorithm defined on D

$$\frac{\partial \phi}{\partial \tau} - j(u_\Omega, p_\Omega)|\nabla\phi(\boldsymbol{x}(\tau), \tau)| = 0, \text{given} \quad \phi(\boldsymbol{x}, \tau = 0). \tag{21}$$

Remark 3. The former assertion is numerically observed in practice, although the question of the asymptotic behavior in the pseudo-time τ of the Hamilton-Jacobi equation (20) seems open. There exists some results but under very restrictive conditions. For instance, for F constant, see (Roquejoffre 2001) and the references therein. ■

From the computational point of view, this approach needs to extend the system (8) to the whole domain D. The simpler method to do so is to replace (8) by the following one:

$$\begin{cases} \frac{\partial^2 y}{\partial t^2}(\boldsymbol{x}, t) - 1_\Omega \Delta y(\boldsymbol{x}, t) = 1_\Omega f(\boldsymbol{x}, t) \ \text{ in } \ D \times (0, T), \\ y(\boldsymbol{x}, t) = 0 \quad \text{on} \quad \partial D \times (0, T), \\ y(\boldsymbol{x}, t) = y_0(\boldsymbol{x}) 1_\Omega \quad \text{in} \quad D \times \{t = 0\} \\ \frac{\partial y}{\partial t}(\boldsymbol{x}, t) = y_1(\boldsymbol{x}) 1_\Omega \quad \text{in} \quad D \times \{t = 0\}, \end{cases} \tag{22}$$

The numerical resolution of this kind of system, although usual, may be difficult due to the possible of spurious high frequencies modes. The uniform convergence of the domain may be lost. We discuss this point in the next section.

5. NUMERICAL EXAMPLE IN 1-D

In this section, we briefly address some numerical questions related to resolution of the former problem. Without loss of generalities, we consider the one dimension case. To approximate the solution y of (8), we introduce the following finite difference scheme

$$\begin{cases} \frac{y_j^{n+1} - 2y_j^n + y_j^{n-1}}{\Delta t^2} - \frac{y_{j+1}^n - 2y_j^n + y_{j-1}^n}{h^2} = f_j^n \\ \qquad \qquad \text{for } j \in [0, J], \ n \in [0, N], \\ y_j^n = 0 \ \text{ for } j = 0, j = J, n \in [0, N], \\ y_j^0 = (y_0)_j, \quad \frac{y_j^1 - y_j^{-1}}{2\Delta t} = (y_1)_j \quad \text{for} \quad j \in [0, J], \end{cases} \tag{23}$$

where as usual, y_j^n is an approximation of the solution y at the point $x_j = jh$, $h = 1/(J+1)$ and at time $t_n = n\Delta t$ with $\Delta t = T/N$. It is well-known that this scheme is convergent under the CFL condition : $\Delta t \leq h$ (Cohen 2002). It is also well known that the interaction of waves with a numerical mesh produces dispersion phenomena and spurious high frequencies. In particular, because of this nonphysical interaction of waves with the discrete medium, the velocity of propagation of numerical waves, the so called group velocity may converge to zero when the wavelength of solutions is of the order of the size of the mesh and the latter tends to zero. As consequence of fact, the time needed to uniformly observe the numerical waves from the boundary or from a subset of the medium in which they propagate may tend to infinity as the mesh becomes finer. Thus, the observation and control properties of the discrete model may eventually disappear. Actually, this strongly depends on the regularity of the initial condition (R. Glowinski and Lions 1990),(Münch n.d.). Very similar behavior appears in the context of stabilization. Numerical experiments shows that the exponential decay rate of the energy, predict by the theory at the continuous, is no longer true, the decay rate converging toward zero with the mesh size (Münch and Pazoto 2005). According to the closed links between optimal design and controllability, the same phenomena occurs here. For instance let us consider the following example: minimize the functional

$$J(b) = \int_0^b (y_b(x) - 1)^2 1_{(x \in [0,1/2])} dx \tag{24}$$

with $b \geq 1/2$, and y_b solution of

$$\begin{cases} \frac{\partial^2 y_b}{\partial t^2}(x, t) - \Delta y_b(x, t) = 0 \ \text{ in } \ [0, b) \times (0, T), \\ y_b(x, t) = 0 \quad \text{on} \quad \Gamma \equiv \partial\Omega \times (0, \infty), \\ y_b(x, t) = x \ 1_{(x \in [0,1/2])} \quad \text{in} \quad [0, b) \times \{t = 0\}, \\ \frac{\partial y_b}{\partial t}(x, t) = 0 \quad \text{in} \quad [0, b) \times \{t = 0\}. \end{cases} \tag{25}$$

The functional J admits a local minimum for a value b near $b = 1$ (see Figure 3). Let us write the Fourier series expansion of the solution. One can write

$$y^0(x) = \sum_{k=1}^{\infty} a_k \sin(\frac{k\pi x}{b}) \tag{26}$$

where

$$a_k = \frac{2\sin(1/2k\pi/b)b - k\pi\cos(1/2k\pi/b)}{k^2\pi^2} \tag{27}$$

leading to :

$$y(x, t) = \sum_{k=1}^{\infty} a_k \cos(\frac{k\pi T}{b}) \sin(\frac{k\pi x}{b}) \tag{28}$$

We highlight that the initial position in (25) is discontinuous. Thus, we will also consider the initial position

$$y_b(x, t) = x(x - 1/2) \ 1_{(x \in [0,1/2])}, \ \text{ in } \ \Omega \times \{t = 0\}, \tag{29}$$

the other datas being unchanged. Then, in this case, we have

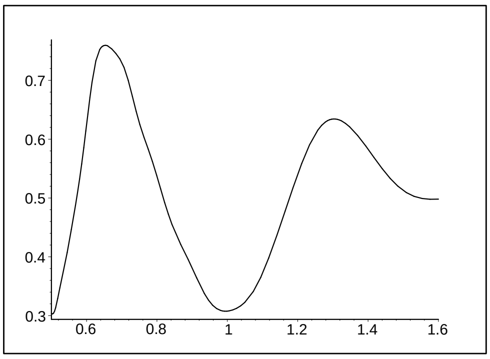

Fig. 3. Evolution of J in function of b:$y_b = x1_{(x\in[0,1/2])}$

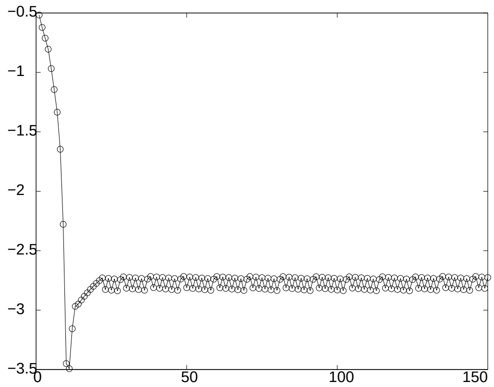

Fig. 4. Log10(Relative error) on the functional vs. iterations : usual scheme and $\Delta t = 0.9h$

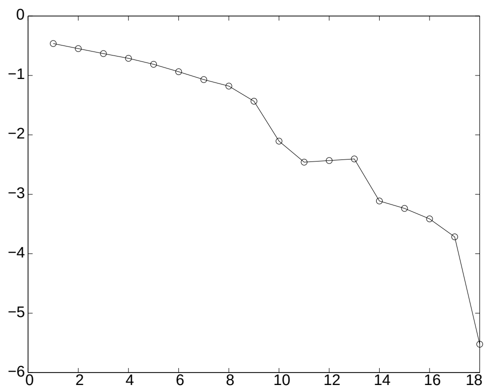

Fig. 5. Log10(Relative error) on the functional vs. iterations : usual scheme and $\Delta t = h$

$$a_k = \frac{-b(k\pi \sin(k\pi/b/2) + 4b\cos(k\pi/2/b) - 4b)}{k^3\pi^3} \tag{30}$$

and the energy is depicted on Figure 6. Then, let us consider the following algorithm, based on a gradient descent method (Allaire *et al.* 2004),(Mohammadi and Pironneau 2001)

$$\begin{cases} \text{initial prediction} \quad b^0; \\ b^n = b^{n-1} - \alpha\, j(b^{n-1}, y_{b^{n-1}}, p_{b^{n-1}}) \end{cases} \tag{31}$$

with j the integrand of the shape derivative $\frac{\partial J}{\partial \Omega}$ (see 12) such that

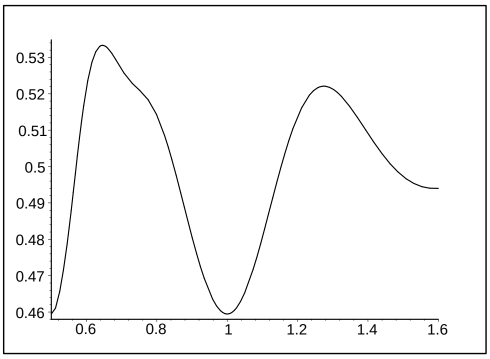

Fig. 6. Evolution of J in function of b:$y_b = x(1/2-x)1_{(x\in[0,1/2])}$

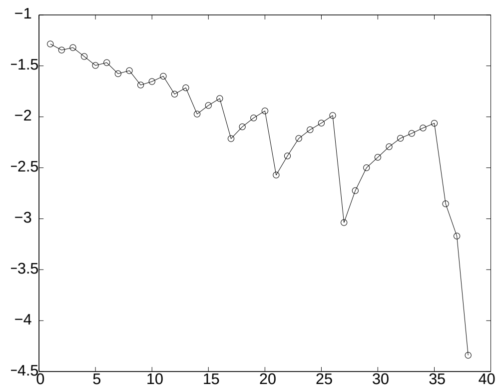

Fig. 7. Log10(Relative error) on the functional: usual scheme and $\Delta t = 0.9h$

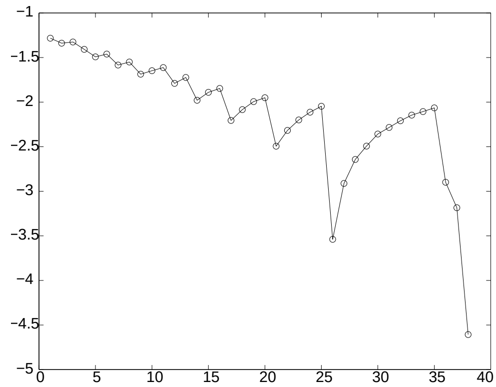

Fig. 8. Log10(Relative error) on the functional: usual scheme and $\Delta t = h$

$$\frac{\partial J(\Omega)}{\partial \Omega}.\boldsymbol{\theta} = \int_{\partial\Omega} j(\Omega, y_\Omega, p_\Omega)\boldsymbol{\theta}.\boldsymbol{n} ds \tag{32}$$

and $\alpha \in \mathbb{R}$ small enough in order that $(J(b^n))_n$ be a decreasing sequence with respect to the norm $||.||_{L^\infty(\mathbb{R})}$. Then, it appears that this algorithm combined with the scheme (23), with an initial value b_0 near 1, and $\Delta t = 9h/10$, does not converge to the value 1: it appears that the sequence (b^n) oscillates around the value 0.98 (see Figure 4). For the particular case $\Delta t = h$, the algorithm converge (see Figure 5). As mentioned above, this numerical pathologies is due to the clearly irregular initial condition y_0 which highlights high frequencies and produce wave packet of velocity of order h. As a consequence, the derivative of the functional is badly approximated leading to this false approximation of the solution. When the regular initial position is considered, the behavior is better (see Figures 7 and 8). In order to annihilate this pathologies, we proceed as in (Münch and Pazoto 2005) by introducing the following finite difference scheme

$$\begin{cases} \dfrac{y_j^{n+1}-2y_j^n+y_j^{n-1}}{\Delta t^2} - \dfrac{y_{j+1}^n-2y_j^n+y_{j-1}^n}{h^2} \\ \quad -\dfrac{y_{j+1}^{n+1}-2y_j^{n+1}+y_{j-1}^{n+1}}{2\Delta t} + \dfrac{y_{j+1}^{n-1}-2y_j^{n-1}+y_{j-1}^{n-1}}{2\Delta t} \\ \qquad = f_j^n \ \text{for} j\in[0,J],\ n\in[0,N], \\ y_j^n = 0 \ \text{ for } j=0, j=J, n\in[0,N], \\ y_j^0 = (y_0)_j, \quad \dfrac{y_j^1-y_j^{-1}}{2\Delta t} = (y_1)_j \quad \text{for } \ j\in[0,J], \end{cases} \tag{33}$$

which corresponds to the finite difference discretization of the following viscous equation

$$\frac{\partial^2 y}{\partial t^2}(x,t) - \frac{\partial^2 y(x,t)}{\partial x^2} - p(h)\frac{\partial^3 y}{\partial x^2 \partial t}(x,t) = f(x,t) \ \text{ in } \ \Omega\times(0,T), \tag{34}$$

with

$$p(h) = h^2. \tag{35}$$

This scheme is convergent under the same CFL condition: $\Delta t \leq h$. Numerical experiments displays that the gradient algorithm (31) combined with (33) converge therefore to a value arbitrarily closed from the exact value 1.

Remark 4. It is worth noticing however that the scheme (33) is an implicit one and therefore requires more computational time. It seems interesting to study whether the mass lumping technique without reducing consistency leads to numerical schemes still efficient to keep uniformly property with respect to the mesh size (see the book of Cohen (Cohen 2002), part II).

This methodology has been tested in length on 2-D domains coupled with the level set approach for the problem (5) (see figure 2). This method leads to an approximation of the position of an internal damping, constrained on his volume, in few iterations. The scheme (33) is also efficient in 2-D.

REFERENCES

Allaire, G., F. Jouve and A-M. Toader (2004). Structural optimization using sensitivity analysis and level-set methods, J. Comp. Phys. *J. Comp. Phys.* **194(1)**, 363–393.

Cagnol, J. and J.P. Zolesio (1999). Shappe derivative in the wave equation with Dirichlet boundary condition. *J. Diff. Eq.* **158**, 175–210.

Cea, J. (1986). Conception optimale ou identification de formes, calcul rapide de la dérivée directionnelle de la fonction cout. *Math. Model. Num. Anal.* **20(3)**, 371–402.

Chambolle, A. and F. Doveri (1997). Continuity of neumann linear elliptic problems on varying two-dimensional bounded open sets. *Commun. Partial. Diff. Equations* **22(5-6)**, 811–840.

Cohen, G. C. (2002). *Higher-order Numerical Methods for Transient Wave Equations.* Scientific Computation, Springer. New York.

Henrot, A. (1994). Continuity with respect to the domain for the laplacian : a survey. *Control and Cybernetics* **23(3)**, 427–443.

Lions, J.L. and E. Magenes (1968). *Problèmes aux limites non homogènes et applications, vol. 1.* Dunod. Paris.

Mohammadi, B. and O. Pironneau (2001). *Applied shape Optimization for Fluids.* Oxford Press.

Münch, A. (2005). An implicit and uniformly controllable scheme for the 1-D wave equation. *To appear in Math. Model. Numer. Anal..*

Münch, A. (n.d.). Optimal internal stabilization of a damped wave equation by a level set approach. *Preprint available at http://www-math.univ-fcomte.fr/pp_Annu/AMUNCH/.*

Münch, A. and A. Pazoto (2005). Uniform stabilization of a numerical approximation of the locally damped wave equation. *To appear in ESAIM:Cocv.*

Murat, F. and J. Simon (1976). Etudes de problèmes d'optimal design. *Lecture Notes in Computer Science, Springer Verlag* **41**, 54–62.

Pironneau, O. (1983). *Optimal shape design for elliptic equation.* Springer. Paris.

Glowinski, R., Li, C-H. and Lions, J-L. (1990). A numerical approach to the exact boundary controllability of the wave equation (i). dirichlet controls: Description of the numerical methods. *Japan. J. Appl. Math.* **7**, 1–76.

Roquejoffre, J. M. (2001). Convergence to steady states or periodic solutions of hamilton-jacobi equationn a class. *J. Math. Pure. Appl.* **80**, 85–104.

Sethian, J.A. (1996). *Level set methods: Evolving interfaces in geometry, fluid mechanics, computer vision, and materials science.* Cambridge University Press.

Sverak, V. (1993). On optimal shape design. *J. Math. Pure. Appl.* **72**, 537–551.

Zolésio, J.P. and C. Truchi (1987). Shape stabilization of wave equation. *Lecture Notes in Control and Information Sciences, J.P. Zolesio editor* **100**, 372–398.