ALLOWED GAIN ERRORS FOR ITERATIVE MODELLING AND CONTROL DESIGN

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Abstract: The automated design scheme called *exploratory modelling for controller optimization* (EMCO) is re-examined to obtain simple robust controllers for complex plant dynamics. The allowable gain error (AGE) function is revised to quantify a frequency dependent bound of model gain errors which allow for both robust servo performance and the required stability robustness margin to be maintained. AGE lends itself to iterative modelling and controller redesign and the resulting scheme is outlined. It is proven that control has synergistic interaction with system identification of the plant model.

Keywords: Internal model control, system identification, adaptive control, robust control

1. INTRODUCTION

Internal model control (IMC) is an attractive control design method to practitioners for various reasons, Morari and Zafirou (1989). It has a clear structure for studying the feedback mechanism and the effect of modelling errors can be taken into account to modify the controller transfer function for robust stability and robust performance. Figure 1 shows the block diagram of a two-degree of freedom IMC structure.

The input signal to the plant can be expressed as

$$u = d + Q(r - \Delta M u - M d) \tag{1}$$

where $\Delta M = P - M$ is the modelling error. The spectrum of input u to the plant in the closed-loop can be expressed as

$$\Phi_u = \frac{|Q|^2}{|1 + Q\Delta M|^2}\Phi_r + \frac{|1 - QM|^2}{|1 + Q\Delta M|^2}\Phi_d \tag{2}$$

This clearly shows that the input excitation to the plant is modified by modelling errors $\Delta M \neq 0$.

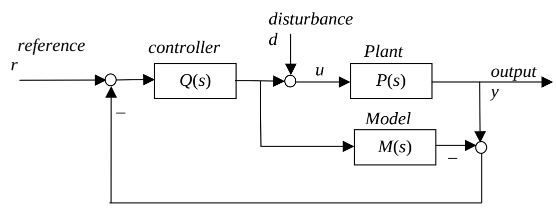

Fig. 1. Block diagram of an IMC control scheme.

If the $\Phi_r(\omega)$ and $\Phi_d(\omega)$ are constant spectra of two white noise signals, then an increase in input excitation will occur at those frequencies where $\Gamma = 1 + Q\Delta M$ is small and where the roll off of Q will not attenuate this excitation, usually within the desired closed loop bandwidth. It will be shown that feedback structure provides self-excitation for remodelling if the reduced stability is caused by modelling error within the bandwidth. The Γ will be related to the generalized stability margin in Section 3. Supportive interaction between modelling and feedback control is called *synergy of identification and control*, Anderson and Kosut (1991); Veres and Wall (2000); Hjalmarsson et al. (1996); Veres (2001).

To achieve a required servo-performance or disturbance attenuation, the simplest model should be found so that its associated controller tolerates the model's gain errors along all frequencies. The problem is that this idea is difficult to realize in practice as illustrated in Fig. 2. It is difficult to find a simple model structure, a parametric model and a controller optimized for that model at the same time for the following reasons.

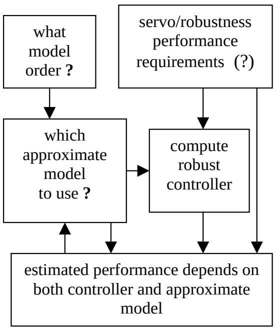

Fig. 2. The problem of modelling for control is complex because the controller depends on the model and the estimated performance depends on both the uncertain model and the controller.

(a) The significance of modelling errors depends on the feedback controller to be used
(b) The controller is designed on the basis of the uncertain model.

Hence there is a cycle of interaction between a simple model and the controller.

In this paper the problem of finding a simple model for a prescribed robust servo performance (or for robust disturbance attenuation) is solved by an iterative remodelling and controller redesign scheme. The procedure is guided by a repeatedly evaluated *allowed gain error* (AGE) function over the frequency axis.

2. ASSUMPTIONS AND PERFORMANCE MEASURES

For clarity of ideas, only single-input single-output (SISO) plants will be considered. It is assumed that the dynamics of the real plant in Fig. 1 can be well approximated by an unknown linear model of possibly very high complexity (for instance in active vibration control, active flow and process control etc.). In many of these applications empirical modelling for control, based on limited amount of input-output data, becomes a non-trivial task. Instead of higher order modelling, this paper takes the direct approach of exploring the possibilities with a limited complexity model and controller. The model-controller action within

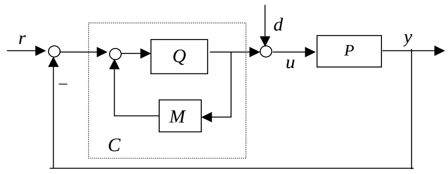

Fig. 3. Reformulation of the IMC structure.

IMC is reorganized in Figure 3 into the form of a classical control loop where the controller is

$$C(s) = \frac{Q(s)}{1 - Q(s)M(s)} \qquad (3)$$

Consider continuous time models and controllers with closed-loop equations

$$\begin{aligned} y(t) &= P(s)u(t) \\ u(t) &= C(s)[r(t) - y(t)] + d(t) \end{aligned} \qquad (4)$$

Introducing the signals

$$z = \begin{bmatrix} y \\ u \end{bmatrix}, \quad w = \begin{bmatrix} r \\ d \end{bmatrix} \qquad (5)$$

the closed-loop system can be written as

$$z = \begin{bmatrix} PCS & PS \\ CS & S \end{bmatrix} w = T(P,C)w \qquad (6)$$

where $S = (1 + PC)^{-1}$ is the sensitivity function.

The norm $b_{P,C} = \|T(P,C)\|_\infty^{-1}$ is the *generalized stability margin* and it measures the maximum allowed normalized H_∞-norm coprime factor dynamic perturbations (McFarlane and Glover (1990)) of $P(s)$, which still preserve stability with the same controller $C(s)$. As $T(P,C)$ contains the sensitivity and complementary sensitivity functions, $\|T(P,C)\|_\infty$ also provides a rough measure of control performance, without frequency weighting.

Required *servo control performance* can be formulated in the general form $\|S(P,C)W\|_\infty \leq 1$ where W is a suitable weighting function (proper rational transfer function). To ensure a closed loop bandwidth of ω_b it is required that $|S(P(j\omega), C(j\omega))| \leq 1/\sqrt(2)$, $\omega \leq \omega_b$. Hence a $W(s)$ that satisfies $|W(j\omega)| \geq \sqrt(2)$ will do. If zero steady state error is required then $W(s) \to \infty$ as $s \to \infty$ is needed.(See e.g. Doyle et al. (1992)).

Robust performance for a nominal plant model M can be expressed by a worst-case performance criterion under frequency dependent plant uncertainty $|M(j\omega) - P(j\omega)| \leq \delta(\omega), \omega \geq 0$ (for short denoted by $|M - P| \leq \delta$) defined by

$$M_{rp}(M, C, \delta) = \\ = \sup\{\|S(P, C)W\|_\infty \mid |M - P| \leq \delta)\} \leq 1 \quad (7)$$

M_{rp} is the measure of robust performance, dependent on the nominal plant M, controller C and uncertainty δ. The $\delta(\omega) > 0$, $\omega \geq 0$, is a frequency dependent bound of the plant uncertainty.

The stability margin $b_{M,C} = \|T(M, C)\|_\infty^{-1}$ has a geometric interpretation on the Riemann sphere (Vinnicombe (1993)). The chordal distance (Vinnicombe (1993)) between two projected points on the Riemann sphere is

$$\kappa(M_1(j\omega), M_2(j\omega)) = \\ = \frac{|M_1(j\omega) - M_2(j\omega)|}{(1 + |M_1(j\omega)|^2)^{1/2}(1 + |M_1(j\omega)|^2)^{1/2}} \quad (8)$$

Later $b_{M,Q}$ is written for $b_{M,C}$ as Q is used in IMC. The following well known lemma by is useful for the geometric interpretation of stability robustness.

Lemma 1. The generalized stability margin can be expressed as:

$$b_{M,Q} = \inf_{\omega \in [0,\infty)} \kappa(M(j\omega), -C(j\omega)^{-1}) \quad (9)$$

This lemma means that the projected curve of $-C(j\omega)^{-1}$ has to be kept at a distance b from $M(j\omega)$ to achieve a stability margin $b > 0$.

3. JOINT DEPENDANCE OF PERFORMANCE AND STABILITY ROBUSTNESS

This section examines how the frequency dependent modelling error $\Delta M(\omega) = P(\omega) - M(j\omega)$ affects the servo performance measured by $\|SW\|_\infty$ and stability robustness in terms of the generalized stability margin $b_{P,Q}$.

The sensitivity function can also be written as

$$S = \frac{1}{1 + PC} = \frac{1}{1 + PQ/(1 - QM)} = \frac{1 - QM}{1 + \Delta MQ}$$

The chordal distance function, the minimum of which is the generalized stability margin, can be expressed as

$$\kappa(P, -C^{-1}) = \frac{|P + C^{-1}|}{(1 + |P|^2)^{1/2}(1 + |C^{-1}|^2)^{1/2}} = \\ \frac{1 + Q\Delta M}{(1 + |P|^2)^{1/2}(|Q|^2 + |1 - QM|^2)^{1/2}}$$

There are two joint terms in the final expressions of κ and S: the $1 + Q\Delta M$ term and the $1 - QM$ term, which will be denoted by Γ and γ respectively. Then the final expressions are:

$$S = \frac{\gamma}{\Gamma}, \quad \kappa = \frac{\Gamma}{(|Q|^2 + |\gamma|^2)^{1/2}(1 + |P|^2)^{1/2}} \quad (10)$$

Here only the $\Gamma = \Gamma(\Delta M) = 1 + Q\Delta M$ is dependent on the modelling error. Note also that the plant amplitude gain is bounded for a stable plant so that $1 + |P|^2 \leq B_p$ with some $B_p > 0$ and the second factor in the denominator cannot grow very large. Concerning the γ note that in IMC the Q is obtained as a solution to a model matching problem $\min \|(1 - MQ)W\|_\infty < 1$ and $|Q|^2(j\omega) + |1 - QM(j\omega)|^2$, $\omega > 0$ is uniformly bounded from below by a $B_l > 0$ and from above by a $B_u > 0$. The latter follows from the continuity of $Q(s)$ and that $Q(s) \to 0$ as $s \to \infty$ by the roll off definition of Q.

Lemma 2. For the IMC scheme the following statements hold.

(a) The generalized stability margin $b_{P,Q}$ is bounded as:

$$\inf_\omega \frac{|\Gamma(\omega)|}{\sqrt{B_p B_u}} \leq b_{P,Q} \leq \frac{|\Gamma(\omega)|}{\sqrt{B_l}}, \ \forall \omega$$

if $B_p = 1 + \|P\|_\infty^2$ and $B_l \leq |Q|^2(j\omega) + |1 - QM(j\omega)|^2 \leq B_u, \omega > 0$.

(b) The servo performance requirement can be expressed as

$$|SW(j\omega)| = \left| \frac{\gamma(\omega)}{\Gamma(\omega)} W(j\omega) \right| \leq 1, \forall \omega \quad (11)$$

Q normally takes on larger values within the closed-loop bandwidth to perform the control action and rolls off at higher frequencies to cancel the effect of uncertain plant dynamics. By (a) the stability will be lowered at frequencies ω where $\Gamma(\omega)$ is dangerously low due to $\Delta M(j\omega)$ approximately cancelling $Q^{-1}(j\omega)$. Exactly at these frequencies the plant input is large as outlined in the introduction and pointed out using (2). Large input amplitude at a frequency ω allows for more accurate identification of the response of the plant at that frequency. This proves that, under IMC, closed-loop plant input favours frequency response estimation at those frequencies which are responsible for lowering the stability margin within the closed loop bandwidth. In turn more accurate frequency response estimation will reduce $\Delta M(\omega)$ relative to Q^{-1} and hence increases the stability margin. This explains the synergistic interaction between closed-loop identification and stability robustness of IMC.

4. ALLOWED PLANT GAIN ERRORS - AGE

The frequency-dependent maximum allowed modelling error of a nominal plant model M for a given controller C is quantified to keep a given stability margin b and required servo performance given by W. Let $\bar{P}(\omega)$ be an estimated upper bound function of the plant gain, obtainable from frequency response testing of the plant.

Definition 3. The frequency dependent allowed gain error, i.e. AGE, is defined by

$$\beta_W^b(\omega|Q) = \min\{B_\Delta(j\omega), B_\delta(j\omega)\}$$

where

$$B_\Delta(\omega) = \frac{1 - b\sqrt{1+|\bar{P}(\omega)|^2}\sqrt{|Q(j\omega)|^2 + |\gamma(j\omega)|^2}}{|Q(j\omega)|}$$

and

$$B_\delta(\omega) = \frac{1 - |\gamma(j\omega)||W(j\omega)|}{|Q(j\omega)|}$$

To achieve a robust stability margin b and the servo performance (as defined by $|W|$) for an uncertain plant $P = M + \Delta M$, it is a necessary condition that the AGE bound $\beta_W^b(\omega|Q)$ must bound the actual error ΔM of the plant model. In general this may not ensure stability of the closed loop under all plant deviations within the AGE. If, however, the nominal plant M is stable, then this is the case as the following result shows. Define the plant frequency response error function by $\Delta M(\omega) = |P(j\omega) - M(j\omega)|, \omega \geq 0$.

Theorem 4. Assume that the plant P is stable and the AGE $\beta_W^b(\omega|Q) > 0,\ \omega \in [0,\infty)$, has been computed for a stable proper model M and an associated IMC controller Q. If

$$|\Delta M(\omega)| < \beta_W^b(\omega|Q),\ \omega \in [0,\infty) \tag{12}$$

then the closed-loop with controller Q and the plant P will be stable with generalized stability margin b and the servo performance will be achieved as required by W.

The relevance of this results is that it relates frequency domain identification with performance and stability robustness of the controller directly, and the test is the satisfaction of an inequality (12). The proof is in the Appendix where it is also clarified that AGE is the tightest such bound if only the magnitude $|\Delta M(\omega)|$ of gain errors is considered without any phase errors.

5. EXPLORATORY MODELLING FOR CONTROL

This section summarizes the scheme of *exploratory modelling and controller optimization* (EMCO). Assume that frequency response error bounds are measurable on the plant, i.e. a nominal response $\hat{P}(j\omega),\ \omega \geq 0$ is obtained with an error bound function $0 < \delta(j\omega),\ \omega \geq 0$ so that

$$|\hat{P}(j\omega) - P(j\omega)| \leq \delta(\omega),\ \omega \geq 0 \tag{13}$$

Then the $\bar{P}$, used as in the AGE, can be set as $\bar{P}(\omega) = |\hat{P}(\omega)| + \delta(\omega)$. (Note that $\hat{P}$ and M are not the same, the former is nonparametric frequency response measurement and the latter is the response of a parametric model.)

The essence of EMCO is to find a model structure and model parameters, i.e. model M, such that (12) is satisfied for the given stability robustness margin $b > 0$ and servo performance $W(j\omega), \omega > 0$ requirements. For a given model structure ν , the model parameter vector θ is to be determined by nonlinear optimization of the cost function

$$L_\nu(\theta) = \sup_\omega\{|\hat{P}(j\omega) - M_\nu^\theta(j\omega)| + \delta(\omega) - \beta_W^b(\omega|Q^\theta)\} \tag{14}$$

where Q^θ is an IMC controller optimized for the model M_ν^θ with regard to the weighting function W and stability margin b.

Fig. 4) four design considerations. The $W(j\omega)$ is

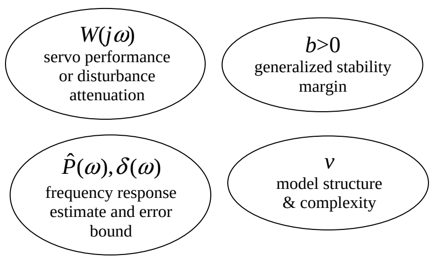

Fig. 4. The design factors that are to be manipulated.

defined to set a weight for the sensitivity function S in $|SW| \leq 1$ and hence to set requirements on the steady state error or on disturbance attenuation. With a suitable W the closed loop bandwidth can be extended well beyond the open loop bandwidth. The price to pay for this is the accurate identification of the plant dynamics within the required closed loop bandwidth. How accurate the plant model needs to be cannot be quantified until the controller $Q(j\omega)$ is also known in association with the nominal model $M_\nu^\theta(j\omega)$. This dependance of the required model quality on the controller itself makes modelling for control particularly unfriendly. Nevertheless the EMCO scheme solves this problem by performing robust control design for an initial model and computing the associated AGE. Using this AGE, the inaccuracies of the model are to be corrected at frequencies where the AGE increases above zero.

An iterative procedure can be defined to "close the loop of identification and control".

The performance requirements may not be possible to achieve for two reasons: (1) Because of inherent limitations of linear feedback control, due to large phase lags, right half plane zeros, time delays, etc. (2) Because of the model structure ν is not rich enough to afford a suitable approximate model and associated robust controller. The first problem is inevitable if linear feedback is to be used, the second problem can be reduced by extending the model order further.

The final goal of performance as given by W and b.

(1) As W and b may not be both achievable with linear feedback on the given plant, some allowances must be made for this in the iterative procedure. This W can be set as a target for the closed loop bandwidth and should only be given up gradually if the iterative investigation cannot obtain it. The purpose of the stability margin $b > 0$ is to provide some protection against sudden changes in plant dynamics. It makes sense to start tuning first for performance and to enhance stability robustness at a later stage.An initially low value of $b \leq 0.05$ is reasonable to start with.

(2) High bandwidth required by W may need accurate models within the bandwidth and that makes sufficient extension of the model complexity inevitable. What usually happens is that the AGE $L_\nu(\theta)$ may exceed 0 for a high bandwidth W, while it can be below 0 for a low bandwidth W with given Q. Hence the model complexity of ν has to be increased.

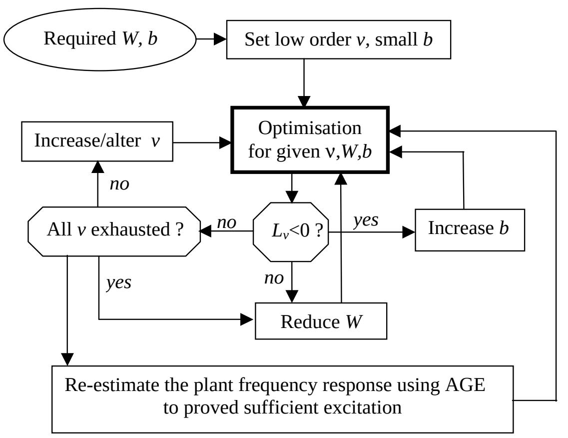

Fig. 5. Block diagram for a family of procedures based on EMCO.

(3) For a fixed model order the parameter space is explored for all the possibilities of finding a model, and robust controller associated with the model inaccuracy of that model, that minimizes $L_\nu(\theta)$. Hence the name exploratory modelling for control.

The model for control can be far from accurate as an open loop model, the closed-loop can distort what is important in the model.

(4) If the model and controller complexity reaches its upper limit and the optimized $L_\nu(\theta)$ still exceeds 0 for some very low $b > 0$ then the feedback control problem has its fundamental limitations due to the plant dynamics. In this case the bandwidth required by W has to be gradually decreased to the level when $L_\nu(\theta)$ sinks below 0.

(5) If the closed loop bandwidth is achieved for a W and using a model structure ν then an exploration procedure can continue for raised values of $b > 0$.

The block diagram in Figure 5 shows the basic features of the EMCO scheme, modifications can be introduced depending on the application requirements.

6. EXAMPLE

This sections illustrates the results of an EMCO based iterative scheme applied to the design of feedback control for a headrest (Fig. 6), for which

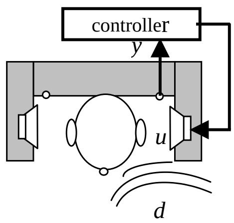

Fig. 6. Schematics of headrest noise control: y=microphone signal, u=speaker signal, d=noise

a frequency response was measured Rafaely and Elliott (1999). The response of the high order model is shown with the dotted lines in the Fig. 7. The results show that the EMCO design method is a promising tool to find simple controllers for a complicated dynamics. The points to make are: (1) a much lower order controller was found (figures 7-9) than in Rafaely and Elliott (1999); Rafaely et al. (1999) and (2) it was experienced that high order models, apparently matching the open loop response measurements, are not necessarily good for control design (3) the best achieved attenuation performance in this particular example was not that great, due to the large phase lag of the plant, not to our inability to find a good controller.

2nd, 4th, 6th and 8th order controller search was carried out. A 4th order model structure of the form

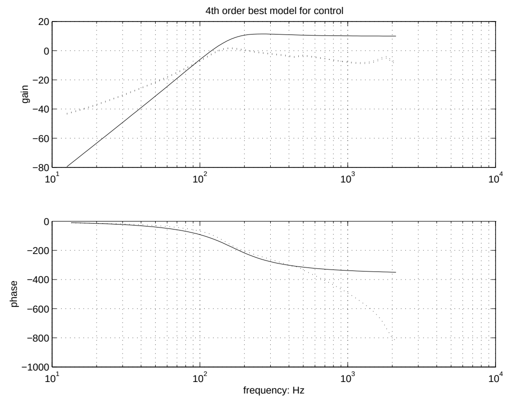

Fig. 7. 4th order model for control (solid line) and the measured response of the headrest (dotted line).

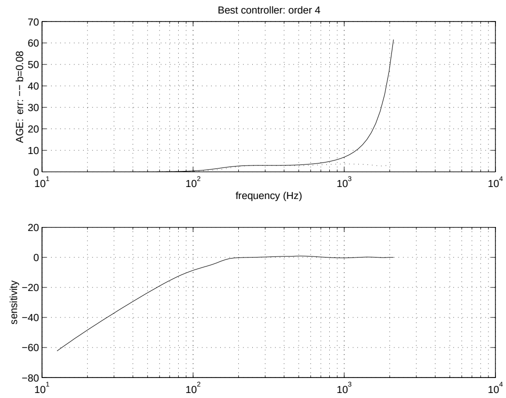

Fig. 8. The AGE (solid line on top plot) and actual dynamic error of the model (dotted line). The AGE> *error bound* condition is satisfied. The simulated complementary sensitivity function (lower plot).

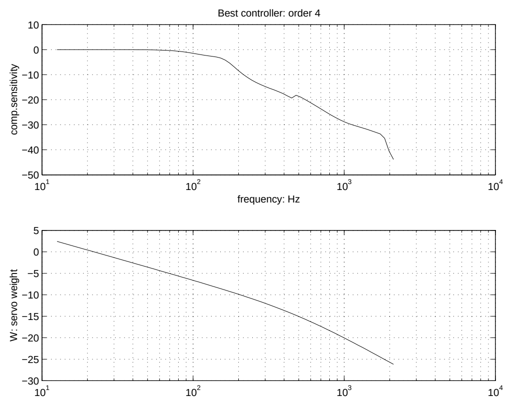

Fig. 9. tThe complementary sensitivity and the weighting function.

$$\frac{c^2 s^4}{a^4(s^2 + 2bs + c)^2} \tag{15}$$

was found the best with associated IMC controller

$$\frac{a^4(s^2 + 2bs + c)^2}{c^2 s^4} \frac{d^8}{(s+f)^8} \tag{16}$$

Figures 7-9 display a relatively successful controller design. The phase response improved but the gain response shows some significant deviation. Despite of that this was the best optimized model for control in terms of minimizing $L_v(\theta, f)$. The AGE bound (12) is satisfied and reasonably good noise attenuation was achieved as shown by the sensitivity function. For the 4th order controller in Figures 7-9 the noise attenuation was limited to the 0-150Hz region, still it is interesting that it was possible to find a simple 4th order controller for such a complex plant.

7. CONCLUSIONS

The paper defined the frequency dependent AGE function and the associated procedure of exploratory modelling for control that "closes the loop of identification and control". Future research will be concerned with extension to the multi input multi output case.

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