APPLICATION OF UNCERTAIN VARIABLES TO STABILIZATION AND PARAMETRIC OPTIMIZATION OF UNCERTAIN DYNAMIC SYSTEMS

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Abstract: A class of dynamic discrete systems (control systems) with unknown parameters is considered. The unknown parameters are assumed to be values of uncertain variables described by an expert in the form of certainty distributions. The method of an estimation (evaluation) of the certainty index that the system is stable is presented and stabilization problems based on such an estimation are formulated. The analogous approach for the system with uncertain and random parameters is described. The method of a parametric optimization considered as a specific decision problem is proposed. Simple examples illustrate the presented approach.

Keywords: uncertainty, uncertain dynamic systems, stability analysis, robust stability, stabilization, robustness.

1. INTRODUCTION

There exists a great variety of formal descriptions of uncertainties and uncertain systems (see e.g. (Klir and Folger, 1988; Yager, 2002)). The idea of uncertain variables, introduced and developed in recent years, is specially oriented for analysis and decision problems in a class of uncertain systems described by classical models or relational knowledge representations with unknown parameters characterized by an expert (Bubnicki, 2001a, b, 2002, 2004). It has been shown how to apply the uncertain variables in stabilization and optimization problems for a class of uncertain control systems (Bubnicki, 2003a, b). The purpose of this paper is to present new problems and results in this area:
1. The application of so called *C-uncertain variables* which permits to use an expert's knowledge in a better way. 2. A new approach to the stabilization problem. 3. An extension of the considerations for the systems containing uncertain and random parameters in one mathematical model. 4. A new method of a parametric optimization based on the uncertain variables.

In recent years, a considerable amount of works have been devoted to different problems of uncertain control systems, including problems of stability and stabilization, and an idea of robust control (e.g. (Amato, *et al.*, 1998; Zhang and Mizukami, 1999; Bubnicki, 2000; Krstic and Hua, 1998; Qu, 1998) and the references therein). Roughly speaking, considerations in these works are based on non-probabilistic descriptions of an uncertainty in the form of a given set of unknown parameters and/or a given set of nonlinearities. A new idea described in (Bubnicki, 2003a, b) and developed in this paper consists in using the description of unknown parameters given by an expert in the form of so called certainty distributions characterizing his/her opinion on different approximate values of these parameters. The approach is not related to any particular stability conditions or any particular formulations of a quality index, but shows how to use the known stability conditions and the forms of quality indexes to the stability estimation, stabilization and parametric optimization based on the uncertain variables. The methods of the stability estimation and stabilization for two cases are described in Sec. 3 and 4, and the method of a parametric optimization, strictly related to the previous considerations based on the uncertain variables, is shortly presented in Sec. 5.

2. PRELIMINARIES

A. Uncertain variables. Details concerning uncertain variables may be found in the book (Bubnicki, 2004). The *uncertain variable* $\bar{x}$ is defined by a set of values *X* (vector space) and a certainty distribution $h(x) = v(\bar{x} \cong x)$ given by an expert, where $v \in [0,1]$ denotes the certainty index that $\bar{x}$ is approximately equal to *x*. In this paper we use so called *C-uncertain variables* and the certainty index $v_c(\bar{x} \tilde{\in} D)$ that $\bar{x}$ approximately belongs to a set $D \subset X$.

Definition 1. *C*-uncertain variable is defined by *X*, $h(x)$ given by an expert and

$$v_c(\bar{x} \tilde{\in} D) = \frac{1}{2}[\max_{x \in D} h(x) + 1 - \max_{x \in X-D} h(x)]. \qquad (1)$$

B. Stability estimation. Consider a nonlinear time-varying system described by

$$s_{n+1} = A(s_n, c_n, x, e)s_n \qquad (2)$$

where $s_n \in S$ is the state vector, $c_n \in C$ is the vector of time-varying parameters, $x \in X$ is the vector of unknown parameters which are characterized by an expert, $e \in E$ is the vector of parameters which may be chosen by a designer; $S = R^k$, *C*, *X* and *E* are real number vector spaces, the matrix $A = [a_{ij}(s_n, c_n, x, e)] \in R^{k \times k}$. Assume that for every $c \in C$, $x \in X$ and $e \in E$ the equation $s = A(s, c, x, e)s$ has a unique solution $s_e = \bar{0}$ (the vector with zero components). The uncertainty concerning c_n is formulated as follows

$$\forall n \ge 0 \quad c_n \in D_c \qquad (3)$$

where D_c is a given set in *C*.

Definition 2. The uncertain system (2), (3) (or the equilibrium state s_e) is globally asymptotically stable (GAS) iff for every sequence c_n satisfying (3) s_n converges to $\bar{0}$ for any s_0 .

Assume now that *x* is a value of an *C*-uncertain variable $\bar{x}$ described by the certainty distribution $h(x)$ given by an expert. Let $M(x,e)$ and $G(x,e)$ denote properties concerning *x* and *e* such that $M(x,e)$ is a sufficient and $G(x,e)$ is a necessary condition of the global asymptotic stability for the system (2), (3), i.e.

$M(x,e) \rightarrow$ the system (2), (3) is GAS,
the system (2), (3) is GAS $\rightarrow G(x,e)$.

Then

$$v_{cm}(e) \le v_{cs}(e) \le v_{cg}(e) \qquad (4)$$

where $v_{cs}(e)$ denotes the certainty index that the system is GAS, $v_{cm}(e)$ and $v_{cg}(e)$ are the certainty indexes that the sufficient and necessary conditions are satisfied, respectively.

3. STABILITY ESTIMATION AND STABILIZATION OF THE SYSTEM WITH UNCERTAIN PARAMETERS

According to (1)

$$v_{cm}(e) = \frac{1}{2}[\max_{D_m} h(x) + 1 - \max_{X-D_m} h(x)], \qquad (5)$$

$$v_{cg}(e) = \frac{1}{2}[\max_{D_g} h(x) + 1 - \max_{X-D_g} h(x)] \qquad (6)$$

where $D_m = \{x \in X : M(x,e)\}$, $D_g = \{x \in X : G(x,e)\}$. Thus, for the known stability conditions $M(x,e)$ and $G(x,e)$ it is possible to estimate the certainty index $v_{cs}(e)$ by the determination of the lower and upper bounds (5) and (6), respectively. Exactly speaking, $v_{cm}(e), v_{cg}(e)$ and $v_{cs}(e)$ denote the certainty indexes that for the given *e* the corresponding properties are “approximately satisfied” or satisfied for an approximate value of *x*. In general, $D_m \subseteq D_g$ and $D_g - D_m$ may be called “a grey zone” which is a result of an additional uncertainty caused by the fact that $M(x,e) \ne G(x,e)$. The stabilization consists here in a proper choosing of the stabilizing parameter *e* by a designer who in this way may have an influence on the values $v_{cm}(e)$ and $v_{cg}(e)$. Let us introduce the index of the grey zone $\delta(e) = v_g(e) - v_m(e)$ and take into account that usually there is a constraint $e \in D_e \subset E$ where D_e is determined by a requirement concerning a quality of the system. The stabilization problem may be formulated in the following ways:

1. Choose *e* maximizing $v_{cm}(e)$ subject to the constraint $e \in D_e$.
2. Choose *e* maximizing $v_{cg}(e)$ subject to the

constraint $e \in D_e$.

3. Choose e maximizing $v_{cg}(e)$ subject to the constraints $e \in D_e$ and $v_{cm}(e) \ge \bar{v}$ where $0 < \bar{v} < 1$ is given.

4. Choose e maximizing $v_{cg}(e)$ subject to the constraints $e \in D_e$ and $\delta(e) \le \bar{\delta}$ where $0 < \bar{\delta} < 1$ is given.

In the cases 3. and 4. the grey zone is included into the optimization problem in two different ways. Let us consider a special case where x and e are one-dimensional positive parameters and the conditions $M(x,e)$, $G(x,e)$ are reduced to inequalities $xe \le b_m$, $xe \le b_g$, respectively ($b_g \ge b_m$). In a typical case x denotes an unknown amplification factor of a control plant and e denotes an amplification factor of a controller in a closed-loop control system. Assume that $h(x) = 0$ for $x \le \alpha$ or $x \ge \beta$ ($\alpha, \beta > 0$), $h(x) = 1$ for $x = z$ and $h(x)$ is an increasing (a decreasing) function for $\alpha \le x \le z$ ($z \le x \le \beta$). It is easy to show that (5) is then reduced to

$$v_{cm}(e) = \begin{cases} 0 & \text{for} \quad e \ge \frac{b_m}{\alpha} \\ \frac{1}{2} h(\frac{b_m}{e}) & \text{for} \quad \frac{b_m}{z} \le e \le \frac{b_m}{\alpha} \\ 1 - \frac{1}{2} h(\frac{b_m}{e}) & \text{for} \quad e \le \frac{b_m}{z} . \end{cases} \tag{7}$$

The function $v_{cg}(e)$ has an analogous form with b_g instead of b_m. Introduce the constraint $e \ge \bar{e}$ and denote the solutions of the problems 1. and 2. (maximization of v_{cm} and v_{cg}) by e_m^* and e_g^*, respectively.

Theorem 1. *Under the assumptions introduced above:*

a. For $\frac{b_m}{\beta} \le \bar{e} \le \frac{b_g}{\beta}$,

$e_m^* = \bar{e}$, e_g^* *is any value from* $[\bar{e}, \frac{b_g}{\beta}]$.

b. For $\frac{b_g}{\beta} \le \bar{e} \le \frac{b_m}{\alpha}$, $e_m^* = e_g^* = \bar{e}$.

c. For $\frac{b_m}{\alpha} \le \bar{e} \le \frac{b_g}{\alpha}$

e_m^* *is any value from* $[\frac{b_m}{\alpha}, \bar{e}]$, $e_g^* = \bar{e}$.

Proof. According to the assumptions concerning $h(x)$, it follows from (7) and from the analogous formula for $v_{cg}(e)$ that $v_{cm}(e)$ is a decreasing function of e for $\frac{b_m}{\beta} \le e \le \frac{b_m}{\alpha}$ and $v_{cg}(e)$ is a decreasing function of e for $\frac{b_g}{\beta} \le e \le \frac{b_g}{\alpha}$. Then for $\frac{b_g}{\beta} \le e \le \frac{b_m}{\alpha}$ the both functions are decreasing what proves the case b. of the Theorem. The cases a. and c. follow from the fact that $v_{cg}(e) = 1$ for $\frac{b_m}{\beta} \le e \le \frac{b_g}{\beta}$ and $v_{cm}(e) = 0$ for $\frac{b_m}{\alpha} \le e \le \frac{b_g}{\alpha}$. □

The method presented above may be applied to different stability conditions $M(x,e)$ and $G(x,e)$ known from the literature. The particular forms of the functions $v_{cm}(e)$, $v_{cg}(e)$ and $\delta(e)$ may be determined for particular stability conditions, and consequently particular forms of the sets D_m and D_g used in (5) and (6). It is worth noting that for the estimation of v_{cs} not only a sufficient but also a necessary stability condition should be formulated. To illustrate the presented approach we may use the following conditions presented in (Bubnicki, 2000, 2003a), based on the principle of contraction mapping (see e.g. (Bubnicki, 1968)):

Theorem 2. *If there exists a norm* $\| \cdot \|$ *such that*

$$\forall c \in D_c, \ \forall s \in S, \quad \| A(s,c,x,e) \| < 1$$

then the system (2), (3) is GAS.

Theorem 3. *If*

$$D_c = \{ c \in C : \forall s \in S \quad \underline{A}(x,e) \le A(s,c,x,e) \le \bar{A}(x,e) \tag{8}$$

where the inequality in (8) denotes the inequalities for the entries, all entries of $\underline{A} + \bar{A}$ *are nonnegative and*

$$\| \bar{A}(x,e) \| < 1 \tag{9}$$

where $\| \cdot \|$ *is one of the norms*

$$\| A \|_1 = \max_{1 \le i \le k} \sum_{j=1}^{k} | a_{ij} | , \quad \| A \|_\infty = \max_{1 \le j \le k} \sum_{i=1}^{k} | a_{ij} | , \tag{10}$$

then the system (2), (3) is GAS.

Theorem 4. *Assume that all entries of* $\underline{A}$ *are nonnegative. If the system (2), (3) is GAS then*

$$\exists j \ \sum_{i=1}^{k} \underline{a}_{ij}(x,e) < 1 \quad \text{and} \quad \exists i \ \sum_{j=1}^{k} \underline{a}_{ij}(x,e) < 1 .$$

Example 1. Consider the system (2) where $k = 2$,

$$A(s_n, c_n, x, e) = \begin{bmatrix} a_{11}(s_n, c_n) + xe & a_{12}(s_n, c_n) \\ a_{21}(s_n, c_n) & a_{22}(s_n, c_n) + xe \end{bmatrix}$$

with the uncertainty (8), i.e. nonlinearities and the sequence c_n are such that $\forall c \in D_c$, $\forall s \in S$ $\underline{a}_{ij} \le a_{ij}(s,c) \le \overline{a}_{ij}$ and $\underline{a}_{ij} \ge 0$. Applying the condition (9) with the norm $\|\cdot\|_\infty$ yields $xe < b_m$ where $b_m = 1 - \max(\overline{a}_{11} + \overline{a}_{21},\ \overline{a}_{12} + \overline{a}_{22})$. Applying Theorem 4 yields $xe < b_g$ where $b_g = 1 - \min(\underline{a}_{11} + \underline{a}_{21},\ \underline{a}_{12} + \underline{a}_{22})$. Assume that x is a value of *C*-uncertain variable $\overline{x}$ described by a triangular $h(x)$ presented in Fig. 1. In this case, according to (7)

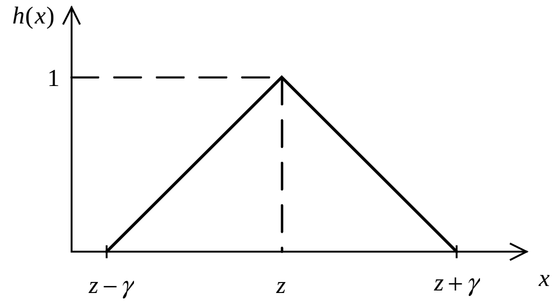

Fig. 1. Certainty distribution.

$$v_{cm}(e) = \begin{cases} 0 & \text{for} \quad e \ge \dfrac{b_m}{z-\gamma} \\ \dfrac{b_m}{2\gamma e} - \dfrac{z-\gamma}{2\gamma} & \text{for} \quad \dfrac{b_m}{z+\gamma} \le e \le \dfrac{b_m}{z-\gamma} \\ 1 & \text{for} \quad e \le \dfrac{b_m}{z+\gamma}, \end{cases} \tag{11}$$

and $v_{cg}(e)$ has the same form with b_g in the place of b_m. The functions $v_{cm}(e)$, $v_{cg}(e)$ and $\delta(e)$ are illustrated in Fig. 2, for $b_m = 0.2$, $b_g = 0.5$, $z = 0.4$.

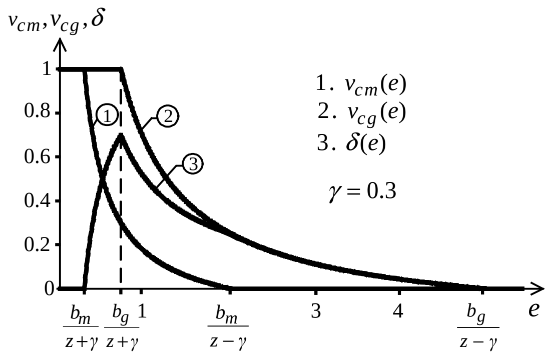

Fig. 2. Illustration of the results.

The solutions of the problems 1. and 2. are such as in Theorem 1, with $\alpha = z - \gamma$, $\beta = z + \gamma$. In case a. $v_{cg} = 1$, $v_{cm} = v_{cm}(\overline{e})$. In case b. $v_{cg} = v_{cg}(\overline{e})$, $v_{cm} = v_{cm}(\overline{e})$ and according to (11) $\delta = (b_g - b_m)(2\gamma\overline{e})^{-1}$. In case c. $v_{cg} = v_{cg}(\overline{e})$, $v_{cm} = 0$. For the numerical data presented above, choosing $e_m^* = e_g^* = \overline{e}$, we obtain the following estimation of the certainty index v_s that the system is GAS: $0.17 \le v_s \le 0.67$ for $\overline{e} = 1$, $0.25 \le v_s \le 0.86$ for $\overline{e} = 0.8$, $0.66 \le v_s \le 1$ for $\overline{e} = 0.4$. The solutions of the problems 3. and 4. are the same as in the problems 1. and 2., under the conditions $\overline{v} \le v_{cg}(e)$ and $\delta(\overline{e}) \le \overline{\delta}$, respectively.

4. STABILIZATION OF A SYSTEM WITH UNCERTAIN AND RANDOM PARAMETERS

The problem and method presented in Sec. 3 may be extended to a system containing two kinds of unknown parameters in its description: *uncertain parameters* described by certainty distributions and *random parameters*. Let us consider a system described by

$$s_{n+1} = A(s_n, c_n, x, w, e)s_n$$

where $x \in X$ is a value of an uncertain variable $\overline{x}$ characterized by an expert in the form of the certainty distribution $h(x)$ and $w \in W$ is a value of a continuous random variable $\tilde{w}$ described by a probability density $f(w)$. In general, w is a vector and W is a vector space. Now the stability conditions $M(x,w,e)$, $G(x,w,e)$ and the certainty indexes (5), (6) $v_{cm}(w,e)$, $v_{cg}(w,e)$ depend on w. Consequently, for the stability estimation and the stabilization, expected values of the certainty indexes may be used, i.e.

$$\mathrm{E}[v_{cm}(\tilde{w}, e)] \le \mathrm{E}[v_{cs}(\tilde{w}, e)] \le \mathrm{E}[v_{cg}(\tilde{w}, e)]. \tag{12}$$

Thus, the expected certainty index that the system is GAS may be estimated by the lower and upper bounds $E_m(e)$ and $E_g(e)$ where

$$E_m(e) = \int_W v_{cm}(w,e) f(w) dw, \quad E_g(e) = \int_W v_{cg}(w,e) f(w) dw.$$

The stabilization problems may be formulated in the way analogous to that in Sec. 3, with $E_m(e)$, $E_g(e)$ and $E_\delta \triangleq \mathrm{E}[\delta(\tilde{w}, e)] = E_g(e) - E_m(e)$ in the place of $v_{cm}(e)$, $v_{cg}(e)$ and $\delta(e)$, respectively. In (Bubnicki, 2004) another version concerning uncertain and random parameters in a decision problem has been described: w has been a random parameter in the certainty distribution given by an expert. The application of this description of the uncertainty to our considerations concerning the stability means that the system is described by (2) but the certainty distribution for $\bar{x}$ has the form $h(x, w)$ where w is a value of a random variable $\tilde{w}$ described by $f(w)$. The consequence is the same as in the first version, i.e. according to (5) and (6) with $h(x, w)$, the certainty indexes $v_{cm}(w, e)$ and $v_{cg}(w, e)$ depend on w and the stability may be estimated by (12). It is worth noting that the presented versions have different practical (empirical) interpretations.

Example 2. Consider the system described in Example 1 and assume that the parameter $z \triangleq w$ in the certainty distribution (Fig. 1) is a value of $\tilde{w}$ with an exponential probability density: $f(w) = \lambda e^{-\lambda w}$ for $w \ge 0$ and $f(w) = 0$ for $w \le 0$. The formula (11) may be rewritten in the form

$$v_{cm}(z) = \begin{cases} 0 & \text{for} \quad z \ge \frac{b_m}{e} + \gamma \\ \frac{b_m}{2\gamma e} - \frac{z-\gamma}{2\gamma} & \text{for} \quad \frac{b_m}{e} - \gamma \le z \le \frac{b_m}{e} + \gamma \\ 1 & \text{for} \quad z \le \frac{b_m}{e} - \gamma . \end{cases} \qquad (13)$$

Using (13) we may obtain $E_m(e) = E_1(e) + E_2(e)$ where

$$E_1(e) = \begin{cases} 1 - \exp[-\lambda(\frac{b_m}{e} - \gamma)] & \text{for} \quad e \le \frac{b_m}{\gamma} \\ 0 & \text{for} \quad e \ge \frac{b_m}{\gamma}, \end{cases}$$

$$E_2(e) = \begin{cases} (\frac{b_m}{2\gamma e} + \frac{1}{2})\{\exp[-\lambda(\frac{b_m}{e} - \gamma)] - \exp[-\lambda(\frac{b_m}{e} + \gamma)]\} \\ \quad + \frac{1}{2\gamma}\{\exp[-\lambda(\frac{b_m}{e} + \gamma)](\frac{b_m}{e} + \gamma + \frac{1}{\lambda}) \\ \quad - \exp[-\lambda(\frac{b_m}{e} - \gamma)](\frac{b_m}{e} - \gamma + \frac{1}{\lambda})\} \\ \qquad \text{for} \quad e \le \frac{b_m}{\gamma} \\ (\frac{b_m}{2\gamma e} + \frac{1}{2})\{1 - \exp[-\lambda(\frac{b_m}{e} + \gamma)]\} \\ \quad + \frac{1}{2\gamma}\{\exp[-\lambda(\frac{b_m}{e} + \gamma)](\frac{b_m}{e} + \gamma + \frac{1}{\lambda}) - \frac{1}{\lambda}\} \\ \qquad \text{for} \quad e \ge \frac{b_m}{\gamma} . \end{cases}$$

The formula for $E_g(e)$ has the same form with b_g instead of b_m. The functions $E_m(e)$, $E_g(e)$ and E_δ for the numerical data the same as in Example 1 and $\lambda = 2.5$ are illustrated in Fig. 3.

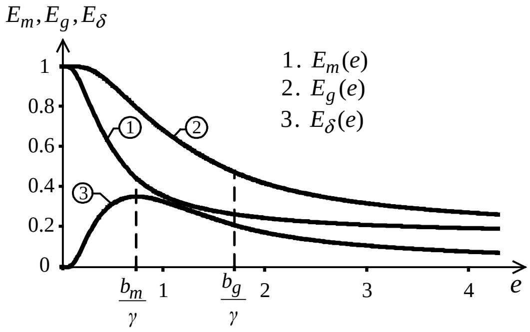

Fig. 3. Stability estimation.

5. PARAMETRIC OPTIMIZATION

The uncertain variables may be applied to the evaluation of a quality of the system under consideration and to a parametric optimization based on a general approach to decision problems of uncertain systems using the uncertain variables (Bubnicki, 2004). Let us introduce a quality index for the system (2) in the form

$$Q_N(x, e) = \sum_{n=0}^{N} \varphi(s_n)$$

for the given s_0 and N, where $\varphi(s_n)$ denotes a local performance index, e.g. $\varphi(s_n) = s_n^T s_n$ (quadratic form). Assume that x is a value of an C-uncertain variable $\bar{x}$ described by $h(x)$. Then, for the requirement $Q_N(x, e) \le \alpha$ where $\alpha > 0$ is a given number, and according to (1)

$$v_c[Q_N(\bar{x}, e) \,\tilde{\le}\, \alpha] = v_c\{Q_N(\bar{x}, e) \,\tilde{\in}\, [0, \alpha]\}$$
$$\triangleq v_c(e, \alpha) = \frac{1}{2}[\max_{x \in D_x(e)} h(x) + 1 - \max_{x \in X - D_x(e)} h(x)] \qquad (14)$$

where $D_x(e) = \{x \in X : Q_N(x, e) \le \alpha\}$.

The optimal decision problem is formulated as follows: For the given $Q_N(x, e)$, α and $h(x)$ one should find the optimal parameter $\bar{e}(\alpha)$ maximizing the certainty index (14), i.e. the certainty index that the requirement is satisfied for an approximate value of $\bar{x}$:

$$\bar{e}(\alpha) = \arg\max_e v_c(e, \alpha) . \qquad (15)$$

It is easy to note that for the given e, $v_c(e, \alpha)$ is an increasing (in general, non-decreasing) function of α. Consequently, for the given desirable value $\bar{v}_c$ it is possible to determine the strongest requirement, i.e. the minimum possible value $\alpha = \overline{\alpha}$, which should be determined by solving the equation

$$\overline{v}_c = v_c[\overline{e}(\alpha), \alpha] \tag{16}$$

with respect to α. Finally, a designer should determine the optimal value $e^* = \overline{e}(\overline{\alpha})$. Thus, the procedure of the parametric optimization presented here contains the determination of v_c (14), the maximization (15), the solution of the equation (16) and the determination of the final value e^*.

Example 3. To illustrate the presented method let us consider very simple example of one-dimensional feed-back control system containing a plant with the input u_n and the output y_n, described by the equation $y_{n+1} - Ty_n = xu_n$, and a controller described by $u_n = e\varepsilon_n$. For $\varepsilon_n = -y_n$ the control system is described by the equation $\varepsilon_{n+1} = (T - xe)\varepsilon_n$. Then, for $|T - xe| < 1$ and $\varepsilon_0 = 1$

$$Q_\infty(x, e) = \sum_{n=0}^{\infty} \varepsilon_n^2 = [1 - (T - xe)^2]^{-1}$$

and the requirement $Q_\infty \le \alpha$ is reduced to

$$(T - \sqrt{1-\alpha^{-1}})e^{-1} \le x \le (T + \sqrt{1-\alpha^{-1}})e^{-1}.$$

Assume that $h(x)$ has the form presented in Fig. 1, with $z = \gamma = 0.5$. Using (14) we obtain

$$v_c(x, e) = \begin{cases} (T + \sqrt{1-\alpha^{-1}})\, e^{-1} & \text{for } e \ge 2T \\ 0 & \text{for } e \le 1 - (T - \sqrt{1 - e^{-1}}) \\ 1 - (T - \sqrt{1-\alpha^{-1}})\, e^{-1} & \text{otherwise}. \end{cases}$$

It is easy to see that $\overline{e}(\alpha) = \overline{e} = 2T$ (it does not depend on α) and $v_c(\overline{e}, \alpha) = 0.5(1 + T^{-1}\sqrt{1-\alpha^{-1}})$. From the equation $v_c(\overline{e}, \alpha) = \overline{v}_c$ we obtain $\overline{\alpha} = [1 - T^2(2\overline{v}_c - 1)^2]^{-1}$. For the numerical data $T = 1$ and $\overline{v}_c = 0.9$, the results are as follows: $\overline{e} = e^* = 2$, $\overline{\alpha} = 2.8$.

6. CONCLUSIONS

The uncertain variables are proved to be a convenient tool for stability estimation, stabilization and a parametric optimization in a class of uncertain dynamic systems with unknown parameters characterized by an expert. In the case of *C*-uncertain variables the considerations are more complicated but the expert's knowledge is used in a better way. The methods described for discrete systems may be applied to continuous systems in an analogous way. The presented approach may be extended to complex control systems considered as specific cases of uncertain systems with a distributed knowledge (Bubnicki, 2004).

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