SWITCHED STATE JUMP OBSERVERS FOR SWITCHED SYSTEMS

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Abstract: In this paper, observers are synthesized for switched linear systems, resulting in switched observers including state jumps. The synthesis problem is formulated as a linear matrix inequality problem. By using multiple Lyapunov functions, a switched state jump observer is designed for a broader class of switched systems than earlier proposed in the literature.

Keywords: Switched systems; Hybrid systems; Observer design; Linear matrix inequalities (LMI); Lyapunov stability.

1. INTRODUCTION

In this paper, we are interested in the problem of estimating the states of a *switched system* in the case when they are not all measured and a switched observer including state jumps is synthesized. Conditions for the observer will be derived, guaranteeing that the estimation error will be upper bounded.

Synthesizing an observer, two different types of switched system assumptions are made which affects the precision regarding the estimation error. Some results assume that the active mode of the switched system is known, and the mode of the observer can be changed correspondingly, see e.g. (Alessandri and Coletta, 2001; Feron, 1996). With this assumption, it is shown that the estimation error eventually becomes zero. Other results do not make this assumption, see e.g. (Juloski *et al.*, 2002). However, to guarantee zero estimation error convergence, it is required that the vector fields of the switched system is equal at the switching boundaries, an assumption that is rarely met. Without this requirement, it can be shown that the estimation error at least is bounded.

Most of the existing results for the state estimation of switched systems use a common quadratic Lyapunov function, see for instance (Alessandri and Coletta, 2001; Juloski *et al.*, 2002). By using a common quadratic Lyapunov function, stability is guaranteed regardless of the mode switches in the system (and observer). However, the existing results are conservative since the estimation error might be stable without the existence of a common Lyapunov function.

The approach in this paper is in line with the method suggested in (Juloski *et al.*, 2002), where no assumptions made on the active mode of the switched system. However, to relax the conservatism using a common quadratic Lyapunov function, we will introduce multiple quadratic Lyapunov functions, one for each observer mode. As in (Juloski *et al.*, 2002), an upper bound of the estimation error (which depends on the system state) can then be guaranteed. A feature of the result in this paper is to properly update the estimated states when changing observer mode, resulting in abruptly changes (state jumps) in the estimated states. The update of the estimated states will be based only on the known information of the current estimated states and the output signal from the switched system. The synthesis problem how to design the observer gains, or showing stability for existing observer gains, is formulated as a linear matrix inequality problem.

The outline of this paper is: we start by defining the switched linear system model in the next section, followed by the description of the switched observer with state jumps. In Section 4, the observer synthesis problem is formulated, followed by a section explaining how to solve the problem. Finally, the method is applied to an example.

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2. SWITCHED LINEAR SYSTEM

The switched linear systems considered in this paper are described by the equations

$$\dot{x} = A_{q(t)}x + Bu, \quad y = Cx, \qquad (1)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector, $y \in \Re^p$ is the measurement (output) vector and $q(t)$ is an index function $q : [0\ \infty) \to I_N = \{1, \ldots, N\}$ deciding which one of the linear vector fields that is active at a certain time instant. Each of the indexes corresponds to a different model description and is referred to as a *mode* of the switched linear system.

The change of value of the index function occurs at defined switch sets $S_{i,j}$, which is described by linear hyper planes according to [2]

$$S_{i,j} = \{x \in \Re^n \,|\, s_{i,j}x = 0\}, \quad (i, j) \in I_s, \qquad (2)$$

where I_s is a set of tuples indicating which mode changes that might occur in the switched system.

We will assume that there are only a finite number of mode changes in finite time. This does not exclude sliding motions, since if sliding motions occur in the switched system, new modes corresponding to the sliding modes are additionally introduced. The dynamics associated with the sliding mode is given by a (unique) vector field specified for instance by Fillipov's convex combination (Filippov, 1988). Then, a switched system with an equivalent dynamics is obtained, where there is a finite number of switches of the modes in finite time. The observer is designed for this equivalent switched system dynamics.

3. SWITCHED OBSERVER WITH STATE JUMPS

An observer for the switched system is defined as follows:

$$\dot{\hat{x}} = A_{r(t)}\hat{x} + Bu + K_{r(t)}(y - \hat{y}), \quad \hat{y} = C\hat{x}, \qquad (3)$$

where $\hat{x} \in \Re^n$ is the estimate of the state vector x and $K_r \in \Re^{n\times p}$, $r \in I_N$, are the observer gains. The index function $r : [0\ \infty) \to I_N = \{1, \ldots, N\}$ decides which one of the observer modes that is active at a certain time instant. To mimic the switching of the switched system (1) occurring at the hyperplanes defined in (2), the change of value of the index function r occurs at the switch sets defined correspondingly

$$S_{i,j} = \{\hat{x} \in \Re^n \,|\, s_{i,j}\hat{x} = 0\}, \quad (i, j) \in I_s. \qquad (4)$$

[2] There is nothing in the result later on that require the sets $S_{i,j}$ to be linear hyper planes, but they can be arbitrarily specified if desirable.

For ordinary linear systems, i.e. for systems with only one mode, it is well known that a design of the observer gain K such that the eigenvalues of $(A - KC)$ lies strictly in the left complex half-plane (which is possible if the system is detectable, see (Levine, 1996)) implies that the estimated states $\hat{x}$ converges to the states x. Therefore it seems reasonable to design all observer gains K_i such that the eigenvalues of $(A_i - K_iC)$ lies strictly in the left complex half-plane. If the active mode of the switched system is known, it is only to activate the corresponding mode of the observer. However, there is no guarantee that the estimation error will converge using this approach, even if the estimation error converges for each mode.

What is further needed in the observer design is to properly update the estimated states of the observer, at times when the observer changes mode occur at the switch sets (4). If observer mode i is active and $\hat{x}(t)$ reaches $S_{i,j}$ at some time, the estimate $\hat{x}$ will abruptly be changed (jump) to $\hat{x}^+$, where $\hat{x}^+$ indicates the updated value of $\hat{x}$. More specifically, the estimated state jumps will be updated according to

$$\hat{x}^+ = T_1\hat{x} + T_2y, \quad x \in S_{i,j},$$

which only depends on the observer states $\hat{x}$ and the measured value y. In the next section, we will show how to calculate T_1 and T_2, guaranteeing that the error between the estimated states and the states of the switched system is bounded (which depends on the system state).

4. OBSERVER SYNTHESIS

The estimation error dynamics obeys the equation

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = (A_r - K_rC_r)\tilde{x} + [A_q - A_r]x.$$

Let us introduce multiple Lyapunov functions, one for each observer mode i,

$$V_i(\tilde{x}) = \tilde{x}^TP_i\tilde{x}, \quad i \in I_N,$$

as an abstract measure of the energy or (scaled) distance of $\tilde{x}$ from the origin, where each $P_i \in \Re^{n\times n}$ is a symmetric positive definite matrix. The time derivative for the observer mode i, when the system state evolves according to mode j, becomes

$$\begin{aligned}\dot{V}_i(\tilde{x}) = &\ \tilde{x}^T([A_i - K_iC]^TP_i + P_i[A_i - K_iC])\tilde{x} \\ &+ \tilde{x}^TP_i(A_j - A_i)x + x^T(A_j - A_i)^TP_i\tilde{x}.\end{aligned} \qquad (5)$$

Before the results of this paper, we need to define a number of matrices, needed for a proper updating of the estimated states at the switching instants, which is done next.

A real symmetric matrix can, according to the *spectral (or principal axis) theorem*, see (Strang, 1988), be factored into

$$P_i = V_i \Lambda_i V_i^T,$$

with the orthonormal eigenvectors of P_i in $V_i \in \Re^{n\times n}$ and $\Lambda_i \in \Re^{n\times n}$ is the diagonal matrix consisting of the (positive) eigenvalues of P_i. Later on, we need to factorize P_i according to

$$P_i = R_i^T R_i, \tag{6}$$

where $R_i \in \Re^{n\times n}$ is a symmetric positive definite matrix. There are many possibilities to achieve this, see (Strang, 1988), for instance

$$R_i = V_i \sqrt{\Lambda}_i V_i^T, \tag{7}$$

where $\sqrt{\Lambda_i}$ is a diagonal matrix consisting of the square root of the (positive) eigenvalues of P_i.

We are now ready for the main theorem. If desirable, we can associate regions $x^T Q_i x \geq 0$ to the switched system (1) where mode i is possible, see (Pettersson and Lennartson, 2002). If not desirable, the $\mu_{i,j}$'s in the theorem is put to zero. The advantage of specifying regions where mode i is possible is to improve the bound given in the theorem. This is one form of relaxation which is similar to the one in (Juloski *et al.*, 2002).

Theorem 1. If there exist a solution to
($\epsilon \geq 0$, $\alpha > 0$, $\mu_{i,j} \geq 0$, $\nu_{i,j} \geq 0$)

1. $\alpha I \leq P_i \leq \beta I, \qquad i \in I_N$
2. $\Gamma_{i,j} = \begin{bmatrix} \Gamma_{i,j}^{11} & \Gamma_{i,j}^{12} \\ (\Gamma_{i,j}^{12})^T & \Gamma_{i,j}^{22} \end{bmatrix} \leq 0, \qquad (i,j) \in I_s$
3. $P_j = P_i + d_{i,j}^T C + C^T d_{i,j}, \qquad (i,j) \in I_s$

where

$$\begin{aligned} \Gamma_{i,j}^{11} &= (A_i - K_i C)^T P_i + P_i (A_i - K_i C) + I + \nu_{i,j} I \\ \Gamma_{i,j}^{12} &= P_i (A_j - A_i) \\ \Gamma_{i,j}^{22} &= \mu_{i,j} Q_j - \epsilon^2 \nu_{i,j} I \end{aligned}$$

and the states of the hybrid observer is updated according to [3]

$$\begin{aligned} &\hat{x}^+ = (I - R_i^{-1}(CR_i^{-1})^\dagger C)\hat{x} + R_i^{-1}(CR_i^{-1})^\dagger y \\ &\forall \hat{x} \in S_{i,j}, \quad (i,j) \in I_s, \end{aligned} \tag{8}$$

then if for some $T_0 > 0$

$$\sup_{t>T_0} ||x(t)|| \leq x_{max}, \tag{9}$$

we have

$$\lim_{t\to\infty} \sup ||\tilde{x}(t)|| \leq \sqrt{\frac{\overline{\nu}}{1+\overline{\nu}}} \sqrt{\frac{\beta}{\alpha}} \epsilon x_{max}, \tag{10}$$

where $\overline{\nu}$ is the largest $\nu_{i,j}$, $(i,j) \in I_s$.

[3] $(*)^\dagger$ is the *pseudoinverse* of $(*)$, see (Strang, 1988).

Proof: We will prove that the overall energy function $V(\tilde{x}(t))$ eventually is upper bounded by a constant. To do this, we need to show that the energy decreases at the switching instants when changing observer mode and that the energy in every mode eventually is upper bounded by a constant. We begin by the first part and have to verify that

$$(x - \hat{x}^+)^T P_j (x - \hat{x}^+) \leq (x - \hat{x})^T P_i (x - \hat{x}). \tag{11}$$

Let $\hat{x}^+$ be an arbitrary estimated state satisfying $y = C\hat{x}^+$. Since also $y = Cx$, we have

$$C(x - \hat{x}^+) = y - y = 0,$$

implying that $(x-\hat{x}^+)^T(d_{i,j}^T C + C^T d_{i,j})(x-\hat{x}^+) = 0$. Due to the relation in Condition 3, it means that (11) becomes

$$(x - \hat{x}^+)^T P_i (x - \hat{x}^+) \leq (x - \hat{x})^T P_i (x - \hat{x}), \tag{12}$$

and it remains to choose $\hat{x}^+ \in S_{i,j}$ such that this inequality is satisfied.

Since P_i can be factorized as $P_i = R_i^T R_i$ according to (6), where R_i is the symmetric positive definite matrix defined in (7), condition (12) is equivalent to show that

$$||R_i(x - \hat{x}^+)|| \leq ||R_i(x - \hat{x})||, \tag{13}$$

is fulfilled where $\hat{x}^+ \in S_{i,j}$.

We are now interested to find the updated value $\hat{x}^+$, lying on the hyper plane $y = C\hat{x}^+$, that minimizes the distance $||R_i(\hat{x}^+ - \hat{x})||$. This optimization problem can formally be defined as

$$\begin{aligned} &\min_{\hat{x}^+} ||R_i(\hat{x}^+ - \hat{x})|| \\ &\text{subject to: } C\hat{x}^+ = y \end{aligned} \tag{14}$$

which is geometrically illustrated in Figure 1.

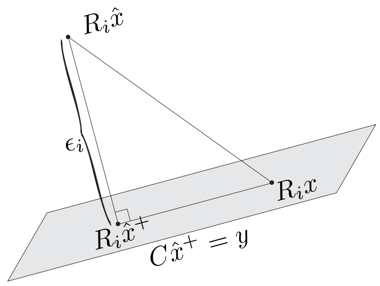

Fig. 1. The projection of $R_i\hat{x}$ onto the plane $C\hat{x}^+ = y$, resulting in the point $R_i\hat{x}^+$.

By introducing $\epsilon_i = R_i(\hat{x}^+ - \hat{x})$, we have $R_i\hat{x}^+ = \epsilon_i + R_i\hat{x}$, leading to the optimization problem

$$\begin{aligned} &\min ||\epsilon_i|| \\ &\text{subject to: } CR_i^{-1}\epsilon_i = y - C\hat{x} \end{aligned}$$

The solution to this problem, the minimum length least squares solution to $y - C\hat{x}$, is

$$\epsilon_i = (CR_i^{-1})^{\dagger}(y - C\hat{x}).$$

Hence, $R_i\hat{x}^+ = R_i\hat{x} + (CR_i^{-1})^{\dagger}(y - C\hat{x})$, which is equivalent to (8) after a multiplication of R_i^{-1} from the left.

It remains to show that the condition in (13) is satisfied for the state jump update (8). By construction, the vectors $R_i(\hat{x}^+ - \hat{x})$ and $R_i(x - \hat{x}^+)$ are orthogonal; otherwise ϵ_i would not be optimal. Hence, by Pythagoras' law

$$\begin{aligned}||R_i(x-\hat{x})||^2 &= ||R_i(x-\hat{x}^+) + R_i(\hat{x}^+ - \hat{x})||^2 = \\ &= ||R_i(x-\hat{x}^+)||^2 + \underbrace{2(x-\hat{x}^+)^T R_i^T R_i(\hat{x}^+ - \hat{x})}_{0} + \\ &+ ||R_i(\hat{x}^+ - \hat{x})||^2 \geq ||R_i(x-\hat{x}^+)||^2,\end{aligned}$$

where the inequality is true since $||R_i(\hat{x}^+ - \hat{x})|| \geq 0$. Hence, we have shown that (13), and consequently (11), is satisfied, ending the first part of the proof.

We now need to proof that the energy in every mode eventually is upper bounded by a constant. By adding and subtracting $\nu_{i,j}\tilde{x}^T\tilde{x}$, $-\nu_{i,j}\epsilon^2 x^T x$, I and $\mu_{i,j}Q_i$ (where $\mu_{i,j} \geq 0$), $\dot{V}_i$ in (5) becomes

$$\begin{aligned}\dot{V}_i(\tilde{x}) &= [\tilde{x}^T x^T]\Gamma_{i,j}[\tilde{x}^T x^T]^T - \mu_{i,j}Q_i \\ &\quad -\tilde{x}^T\tilde{x} - \nu_{i,j}\tilde{x}^T\tilde{x} + \nu_{i,j}\epsilon^2 x^T x \\ &\leq -(1+\nu_{i,j})\tilde{x}^T\tilde{x} + \nu_{i,j}\epsilon^2 x^T x \\ &\leq -\frac{(1+\nu_{i,j})}{\beta}V_i(\tilde{x}) + \nu_{i,j}\epsilon^2 x^T x \\ &\leq -\frac{(1+\nu_{i,j})}{\beta}V_i(\tilde{x}) + \nu_{i,j}\epsilon^2 x_{max}^2,\end{aligned}$$

where the first, second and third inequality is due to Condition 2 and the fact that $-\mu_{i,j}Q_i \leq 0$ (since $\mu_{i,j} \geq 0$ and $x^T Q_i x \geq 0$ in regions where mode i of the switched system (1) is possible), Condition 1 and (9) respectively. This differential inequality implies that

$$\begin{aligned}V_i(\tilde{x}(t)) &\leq e^{-\frac{(1+\nu_{i,j})}{\beta}(t-t_0)}V(\tilde{x}(t_0)) \\ &+ \frac{\nu_{i,j}}{1+\nu_{i,j}}\beta\epsilon^2 x_{max}^2(1 - e^{-\frac{(1+\nu_{i,j})}{\beta}(t-t_0)}) \\ &\leq e^{-\frac{(1+\underline{\nu})}{\beta}(t-t_0)}V(\tilde{x}(t_0)) \\ &+ \frac{\overline{\nu}}{1+\overline{\nu}}\beta\epsilon^2 x_{max}^2(1 - e^{-\frac{(1+\overline{\nu})}{\beta}(t-t_0)}),\end{aligned}$$

where $t_0 \geq T_0$, and $\underline{\nu}$ and $\overline{\nu}$ is the smallest respectively the largest value of $\nu_{i,j}$, $(i,j) \in I_s$. Consequently, the overall energy $V(\tilde{x}(t))$ decreases at the switching instants and is upper bounded by a constant. Due to Condition 1, we then have

$$\begin{aligned}||\tilde{x}(t)|| &\leq \Big(e^{-\frac{(1+\underline{\nu})}{\beta}(t-t_0)}V(\tilde{x}(t_0))/\alpha \\ &+ \frac{\overline{\nu}}{1+\overline{\nu}}\frac{\beta}{\alpha}\epsilon^2 x_{max}^2(1 - e^{-\frac{(1+\overline{\nu})}{\beta}(t-t_0)})\Big)^{\frac{1}{2}}.\end{aligned}$$

Hence, when $t \to \infty$ the exponential functions converge to zero implying that (10) is satisfied, ending the proof. ∎

Remark The convergence to the limit is exponential. Consequently, there is a finite time for which $||\tilde{x}(t)||$ converges to all limits greater than the right-hand side of (10). For instance, it can be shown that

$$\begin{aligned}&\sup_{t>T}||\tilde{x}(t)|| \leq \sqrt{\frac{\beta}{\alpha}}\epsilon x_{max}, \\ &\text{for } T \leq T_0 + \max(0, \beta\ln\frac{\beta||\tilde{x}(T_0)||^2}{\alpha\epsilon^2 x_{max}^2}).\end{aligned}$$ □

A sufficient condition for the existence of a solution to the inequalities in the theorem is that Condition 2 is replaced by $\Gamma_{i,j}^{11} < 0$ with $\nu_{i,j} = 0$. This is the formulation of the estimation problem assuming that the system mode is known. In this case, the estimation error obeys

$$||\tilde{x}(t)|| \leq \sqrt{\frac{\beta}{\alpha}}e^{-\frac{1}{2\beta}t}||\tilde{x}_0||,$$

implying that $||\tilde{x}(t)||$ goes to zero as time goes to infinity regardless of the value of $x(t)$. When we do not know the mode, as in the theorem, we cannot say that the estimation error goes to zero but is upper bounded according to (10), which depends on the largest value of $||x(t)||$. This bound is usually very conservative, indicated by the example later on, since it is obtained having the worst possible combination of observer mode and system mode.

Except the properly updates according to (8), the theorem uses multiple Lyapunov functions, which increases the possibility to find the unknown variables satisfying the conditions in the theorem. Using a common Lyapunov function (corresponds to $d_{i,j} = 0$ in the theorem) to prove convergence, the energy decrease condition (12) is trivially satisfied by letting $\hat{x}^+ = \hat{x}$, i.e. no updates of the estimated states are necessary. However, also in that case, the updates of the estimated states according to (8) will improve the real convergence rate and should be used also in case when a common Lyapunov function is searched for.

5. SOLUTION USING LINEAR MATRIX INEQUALITIES

Theorem 1 has to be valid whether the observer gains K_i are decided *a priori* or not. The unknowns in Theorem 1 will be found by iteratively fixing ϵ to a value and search for the smallest β satisfying the conditions to find a low bound on the right-hand side of (10). If there is no solution for the fixed value of ϵ, the value is increased. Furthermore, without loss of generality, α is scaled to 1 to prevent the P_i's to be positive semi-definite.

For *a priori* decided observer gains K_i and a fixed value of ϵ, Theorem 1 is directly a linear matrix inequality (LMI) problem in the unknown variables P_i, $d_{i,j}$, and scalars $\mu_{i,j}$ and $\nu_{i,j}$. LMI problems are convex optimization problems that can be solved efficiently by existing numerical software, for instance (Gahinet *et al.*, 1995) which is the one used in this paper. Finding the solution with the smallest β satisfying the conditions, requires only more iterations in the optimization procedure.

If the observer gains K_i are unknown *a priori*, they have to be included as well in the optimization problem. However, they have to be constrained in some way to prevent them from being too large. Without observer gain constraints, if the system is observable, the eigenvalues of each of the observer dynamics in (3) can be placed arbitrarily far away on the negative real axis in the complex plane, leading to arbitrarily fast (exponential) convergence of the switched linear system (with or without switching). Hence, it is necessary to introduce constraints to prevent the (norm of the) observer gains K_i from being infinitely large. Furthermore, it is reasonable to have the constraints to restrict the observer gain matrices from being too large, which would make the observer dynamics sensitive to measurement noise.

To be able to formulate the observer synthesis problem as an LMI problem, we will indirectly introduce constraints on the observer gains. By introducing new unknown variables $W_i \in \Re^{n\times p}$ according to $W_i = P_i K_i$ in Theorem 1, the observer gains can, for a solution P_i and W_i satisfying the conditions, be calculated as $K_i = P_i^{-1} W_i$ where P_i^{-1} exists since P_i is positive definite. Hence, restricting the W_i's according to $W_i^T W_i \leq \lambda_i^2 I_{p\times p}$, where λ_i is a design parameter, implies that

$$W_i^T W_i = \geq K_i^T K_i \lambda_{min}(P_i P_i) \geq \alpha^2 K_i^T K_i$$

where $\lambda_{min}(P_i P_i)$ denotes the smallest eigenvalue of $P_i P_i$. The last inequality is due to the first condition in Theorem 1. Hence, the restriction $W_i^T W_i \leq \lambda_i I_{p\times p}$ implies that $K_i^T K_i \leq \lambda_i^2/\alpha^2 I_{p\times p}$ (where we have scaled α to 1, mentioned in the beginning of this section), and the observer gains are consequently bounded. The condition $W_i^T W_i \leq \lambda_i^2 I_{p\times p}$ is not directly an LMI condition, due to the product $W_i^T W_i$ of unknown variables W_i. However, the condition is equivalent to

$$\begin{bmatrix} \lambda_i^2 I_{p\times p} & W_i^T \\ W_i & I_{n\times n} \end{bmatrix} \geq 0, \quad i \in I_N$$

which is an LMI condition, see (Boyd *et al.*, 1994). Theorem 1 can now be formulated as an LMI problem, synthesizing also the unknown observer gains K_i.

6. EXAMPLE

We now illustrate the observer synthesis in this paper in case of two modes of the (autonomous) switched linear system (1) given by

$$A_1 = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ -2.4 \end{bmatrix}^T,$$

$$s_{1,2} = [1.56 \;\; 1], \quad s_{2,1} = [1 \;\; -1.56].$$

We assume that the design of the the observer gains is not known *a priori* but is a part of the synthesis problem. We will study the solution in case when $\lambda = \lambda_1 = \lambda_2 = 5$.

Solving the corresponding LMI problem of Theorem 1, results in a solution

$$K_1 = \begin{bmatrix} 1.77 - 1.09 \end{bmatrix}^T, \; K_2 = \begin{bmatrix} -3.21 - 3.50 \end{bmatrix}^T,$$

with $\alpha = 1$, $\beta = 5.47$, $\epsilon = 6.77$, $\overline{\nu} = 1.82$. According to the theorem, we therefore have the bound $||\tilde{x}(t)|| \leq 12.72 x_{max}$. It should be noted that there does not exist a solution to Theorem 1 with a common P; hence, the suggested observer synthesis in this paper is less conservative than existing results using a common quadratic Lyapunov function.

Figure 2 shows a trajectory simulation x of the switched linear system, in the case when $x_{max} = 1$, together with the estimated states $\hat{x}$ updating the estimator states according to (8) at the switching instants. As can be seen from the figure, the estimated states converge to the switched linear states exactly. Hence, the convergence is in this example better than the upper bound in (10).

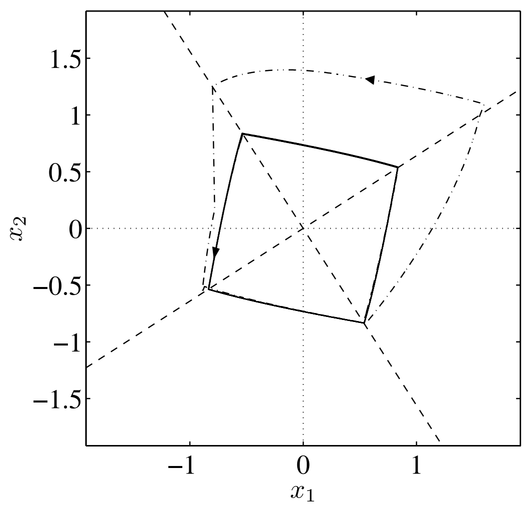

Fig. 2. The estimated states $\hat{x}$ (dash-dotted) converges to the switched linear system states x (solid line) using the projection.

To compare, a trajectory simulation of the estimated states $\hat{x}$ when not updating the estimator states according to (8) at the switching instants is shown in Figure 3. In this case, it can be seen that the estimated states converges to a limit cycle. Hence, it is advantageously to update the estimator states at the switching instants.

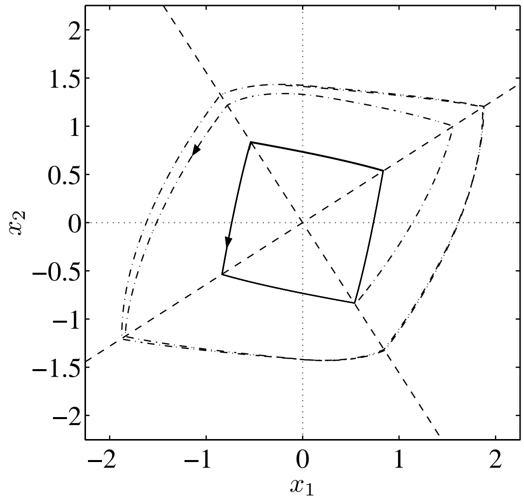

Fig. 3. The estimated states $\hat{x}$ (dash-dotted) converges not to the switched linear system states x (solid line) since projection is not used.

The energy decrease for the two different simulations in Figures 2 and 3 are shown in Figure 4. The energy is equal to approximately 0.2 time units where the energy not using the projection luckily decreases slightly more than using the projection. However, at time approximately at 0.7 time units, the energy decreases almost to zero using the projection (meaning that the estimation error almost becomes zero) while it increases not using the projection. Without the projection, the energy, and hence the estimation error, becomes cyclic, as can be seen from Figures 3 and 4 (dash-dotted line).

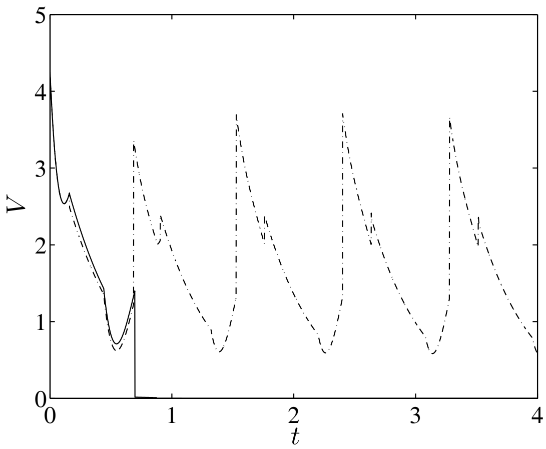

Fig. 4. The energy decrease for the two different simulations in Figures 2 (solid line) and 3 (dash-dotted line). The energy decreases at the switching instants using the projection but increases if not using the projection.

7. CONCLUSIONS

In this paper, it has been shown how to estimate the states of a switched linear systems by designing a switched observer including state jumps. By using multiple Lyapunov functions, one for each mode, and properly update the estimated states when the mode changes occur, an observer is synthesized for a broader class of switched systems than earlier proposed in the literature. The observer synthesis is cast as an linear matrix inequality (LMI) problem, which is a convex optimization problem that can be solved efficiently by existing numerical software.

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