ADAPTIVE ROBUST CONTROL OF NONHOLONOMIC WHEELED MOBILE ROBOTS

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Abstract: This paper develops methodologies for an adaptive robust path tracking control of a nonholonomic Wheeled Mobile Robot (WMR) with nonlinear driving characteristics and unknown dynamic parameters. To solve the problem of position/orientation tracking control of WMR, a novel robust kinematics control law is developed to steer the vehicle to asymptotically follow the desired trajectories. To compensate for dynamic effects associated with the dynamic models, an adaptive backstepping path tracking control law with robustness is designed to ensure asymptotic path tracking for the vehicle with unknown dynamic parameters and changeable time-varying payload. Simulation results are included to illustrate feasibility and effectiveness of the proposed control laws.

Keywords: adaptive control, robust control, mobile robot, nonlinear characteristics, path tracking

1. INTRODUCTION

Wheeled mobile robots have been mainly excited by a wide variety of practical mobile robots applications due to their versatile abilities to work in various working domains. They have already been used in the fields of planetary exploration, materials transportation, military tasks, manufacturing servicing, hazardous environment, and mine excavation. To achieve the aforementioned tasks, the WMR requires intelligent sensing of the environment, intelligent trajectory planning, and high precision control. This desired autonomous or intelligent behavior has motivated an intensive research over the past decades.

To achieve high-precision path tracking control for the WMRs, many sophisticated control approaches have been proposed by several researchers. These methods can be divided into two research paradigms. The first one started form Bloch *et al.* (1992), uses the discontinuous feedback whereas the second one, which was first investigated by Samson (1991), employs the time-varying continuous feedback. The existing tracking control methods for the WMRs include (i.) sliding mode control by Yang (1999); (ii.) nonlinear control by Kanayama, (1990); Samson, (1995); Jiang, (1997) (iii.) fuzzy control by Ollero, (1994); (iv.) neural network control by Boquete (1999); and (v.) adaptive backstepping control (Dixon, 2001; Fukao, 2000). In 1999, Yang *et al.* used sliding mode control for tracking control, which is complicated and computationally expensive. The generated velocity command with respect to time is not a smooth curve in Yang (1999). Ollero *et al.* introduced the fuzzy tracking control approaches in 1994. However, it is very difficult to formulate the fuzzy rules, which are usually obtained from the trial-and-error procedure. In 1999, computational intensive neural networks were adopted by Boquete *et al.* (1999), but the proposed algorithm requires on-line learning in order to make the robot perform properly. Moreover, the literature on the robustness and the control in presence of uncertainties in the dynamical model of such systems seems to be rare. Recently, nonlinear control for this class of system has been studied more and more extensively. Kanayama *et. al* (1990) , Samson *et al.* (1995)and Jiang *et. al* (1997) have introduced several nonlinear control approaches for tracking control of mobile robots. In addition, when the uncertain model parameters in the real mobile robots occur, adaptive control policies have been proposed in (Dixon, 2001; Fukao, 2000).

Most of the above approaches have been devoted to developing path tracking control laws without considering nonlinear characteristics, such as backlash and dead-zone. However, few have paid attention to designing robust control methods for a nonholonomic wheeled mobile robot (WMR) with such bounded, nonlinear driving characteristics in Oelen (2000). Nonlinear characteristics, such as backlash and dead-zone, can be extensively found in gear trains and driving mechanism for mobile robots.

These characteristics usually have detrimental effects on path tracking accuracy. For example, it causes the control system to have nonzero steady-state errors. Another type of robust tracking control policy deals with the parameter uncertainties of dynamic models for mobile robots (Fukao, 2000). These parameter uncertainties usually come from changeable payload or aging components, and these aforementioned robust controller will exhibit desired tracking control performance if the upper bounds of the uncertain parameters are known in advance.

To tackle with nonlinear driving characteristics, Oelen and Amerongen (2000) pioneered on the work by considering both linear and angular velocity dead-zone bands occurring in kinematics models of two degrees of freedom mobile robots. However, this control method in Oelen and Amerongen (2000) cannot be directly derived using the well-known Lyapunov stability theory. By combing aforementioned adaptive and robust control methodologies, this paper attempts to use Lyapunov stability theory to develop a novel, asymptotical, nonlinear adaptive robust path tracking control law for achieving path tracking goal and canceling the undesirable effects resulting from nonlinear effects and unknown dynamic parameters. These novel techniques can be expected to be useful and effective in controlling more general wheeled mobile robots with nonholonomic constraints and nonlinear driving characteristics.

The remaining parts of the paper are organized as follows. Section II presents a novel path tracking law and Section III proposes a robust tracking controller for dealing with nonlinear characteristics from the kinematic models. Section IV designs an adaptive robust controller for overcoming unknown parameters associated with the vehicle' dynamic models. In Section V simulation results are presented. Section VI concludes this paper

2. KINEMATICS PATH TRACKING CONTROL LAW

2.1 Problem Formulation

It has been well known that, under the assumption of pure rolling, a kinematics model of the WMR shown in Fig. 1, is described by

$$\dot{q} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \cdot V \qquad (1)$$

where $q(t) = [x\ y\ \theta\ ]^T \in R^3$ denote the posture of the WMR in the Cartesian frame, and $V = [\upsilon\ \ \omega]^T$ is a control input vector containing the linear velocity, υ, and the angular velocity ω.

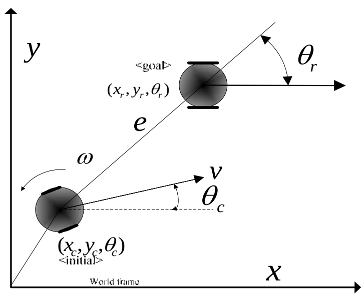

Fig.1 Position and orientation of the WMR

The aim of the path tracking design problem is to find a path tracking control law so as to keep the pose trajectories of the WMR asymptotically follow the desired posture trajectories given by

$$\dot{q}_r = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \cdot V_r \qquad (2)$$

where $q_r(t) = [x_r\ y_r\ \theta_r\ ]^T$. To formulate the problem, let $\tilde{x}(t), \tilde{y}(t), \tilde{\theta}(t) \in R^1$ be the errors between the actual and desired postures of the WMRs in the Cartesian frame, i.e.,

$$\tilde{x} = x_r - x\ ,\ \tilde{y} = y_r - y\ ,\ \tilde{\theta} = \theta_r - \theta$$

In what follows will develop a novel kinematics control law such that $\lim_{t\to 0}[\tilde{x}(t), \tilde{y}(t), \tilde{\theta}(t)]^T \to [0,0,0]^T$.

2.2 Novel Asymptotical Path Tracking Design

To achieve the path-tracking goal in asymptotical, we use a well-known globally invertible transformation given by (Dixon, 2001)

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} \qquad (3)$$

where $e_1(t), e_2(t), e_3(t)$ represent the tangential error, the normal (lateral) error and the orientation error, respectively. By differentiating (3) we obtain the error dynamics as follows,

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 - \upsilon + \upsilon_r \cos e_3 \\ -\omega e_1 + \upsilon_r \sin e_3 \\ \omega_r - \omega \end{bmatrix} \qquad (4)$$

Note that, from (3), if all the transformed errors $e_1(t), e_2(t), e_3(t) \in L_\infty$, are continuous and bounded, then the original errors $\tilde{x}(t), \tilde{y}(t), \tilde{\theta}(t) \in L_\infty$ are bounded and continuous, it yields to show that

$$\lim_{t\to\infty} e_1(t) \to 0, \lim_{t\to\infty} e_2(t) \to 0, \lim_{t\to\infty} e_3(t) \to 0 \Leftrightarrow \qquad (5)$$

$$\lim_{t\to\infty} \tilde{x}(t) \to 0, \lim_{t\to\infty} \tilde{y}(t) \to 0, \lim_{t\to\infty} \tilde{\theta}(t) \to 0 \qquad (6)$$

The basic idea of designing the proposed control law is based on two facts: (i) the lateral error $e_2(t)$ can not be directly controlled using the control vector $V = [\upsilon\ \ \omega]^T$;(ii) if the lateral error $e_2(t)$ exits, then the orientation error $e_3(t)$ will not approach zero. According to these two observations, one defines the following auxiliary variable $\bar{e}_3(t)$ as

$$\bar{e}_3(t) = e_3 + \alpha e_2 \quad (7)$$

where $\alpha \in R$. In the sequel derives a novel control law to achieve kinematics path tracking. Taking the time derivative of $\bar{e}_3(t)$, it yields:

$$\dot{\bar{e}}_3(t) = \dot{e}_3 + \alpha\dot{e}_2 = \omega_r - \omega + \alpha(-\omega e_1 + \upsilon_r \sin e_3) \quad (8)$$

To stabilize the system (4), we propose the following control laws for υ and ω

$$\begin{aligned} &\upsilon = k_1 e_1 + \upsilon_r \cos e_3 + \alpha\omega \sin e_3, k_1 > 0 \\ &\omega = \frac{1}{1+\alpha e_1}(k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) + \upsilon_r e_2 + \alpha \upsilon_r \sin e_3 + \omega_r), k_2 > 0 \end{aligned} \quad (9)$$

By substituting (9) into (4), we obtain the sequel equation:

$$\dot{e}_1 = -k_1 e_1 + \omega e_2 - \alpha\omega \sin e_3$$

and substituting (9) into (7) yields

$$\dot{\bar{e}}_3 = -k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2$$

Hence, the closed loop error system is summarized as belows

$$\begin{aligned} &\dot{e}_1 = -k_1 e_1 + \omega e_2 - \alpha\omega \sin e_3 \\ &\dot{e}_2 = -\omega e_1 + \upsilon_r \sin e_3 \\ &\dot{\bar{e}}_3 = -k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2 \end{aligned} \quad (10)$$

To show the asymptotical stability of the closed-loop error system, one finds the following Lyapunov function candidate

$$V_1 = \frac{1}{2}(e_1{}^2 + e_2{}^2) + (1 - \cos e_3) \quad (11)$$

which leads to its time derivative expressed by

$$\dot{V}_1 = e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3 \sin e_3 \quad (12)$$

Since

$$\begin{aligned} &\dot{\bar{e}}_3 = \dot{e}_3 + \alpha\dot{e}_2 = -k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2 \quad \Rightarrow \\ &\dot{e}_3 = -k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2 + \alpha\bar{\omega} e_1 - \alpha\upsilon_r \sin e_3 \end{aligned}$$

We substitute $e_1(t), e_2(t), e_3(t)$ and their derivatives into (12) and then obtain

$$\begin{aligned} \dot{V}_1 &= e_1(\omega e_2 - k_1 e_1 - \alpha\omega \sin e_3) + e_2(-\omega e_1 + \upsilon_r \sin e_3) \\ &\quad + (-k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2 + \alpha\omega e_1 - \alpha\upsilon_r \sin e_3)\sin e_3 \\ &= -k_1 e_1^2 - \alpha e_1 \omega \sin e_3 + e_2 \upsilon_r \sin e_3 - k_2 |e_3 \sin e_3| \\ &\quad - \upsilon_r e_2 \sin e_3 + \alpha\omega e_1 \sin e_3 - \alpha\upsilon_r \sin^2 e_3 \\ &= -k_1 e_1^2 - k_2 |e_3 \sin e_3| - \alpha\upsilon_r \sin^2 e_3 \le 0, \ if \ \alpha\upsilon_r \ge 0 \end{aligned} \quad (13)$$

Based on the Barbalat lemma, $e_1(t)$ and $e_3(t)$ approach zero when time goes to infinity, i.e.,

$$\lim_{t\to\infty} e_1 \to 0 \quad \lim_{t\to\infty} e_3 \to 0$$

Next, we show that $e_2(t)$ approaches to zero when time goes to infinity. Since $\lim_{t\to\infty} \dot{e}_3 = 0$, thus

$$\begin{aligned} \lim_{t\to\infty} \dot{e}_3 &= \lim_{t\to\infty}(-k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) - \upsilon_r e_2 \\ &\quad + \alpha\omega e_1 - \alpha\upsilon_r \sin e_3) = -\lim_{t\to\infty} \upsilon_r e_2 = 0 \end{aligned}$$

If $\lim_{t\to\infty} \upsilon_r(t) \neq 0$ for $t \ge 0$, $\lim_{t\to\infty} e_2(t) = 0$ can be obtained. Similarly, if $\lim_{t\to\infty} \omega(t) = \omega_r(t) \neq 0$, then $\lim_{t\to\infty} e_2(t) = 0$. Note that all the stable equilibrium points, $[0 \quad 0 \quad 2n\pi]^T \ n \in I$, have the orientation errors, $2n\pi$, thereby implying that they have the same physical orientation. Hence, we intend to claim that the proposed path tracking controller makes the wheeled mobile robot always follow the desired trajectories if $\lim_{t\to\infty} \upsilon_r(t) \neq 0$.

<u>*Theorem 1*</u> Assume that υ_r, $\dot{\upsilon}_r$, ω_r and $\dot{\omega}_r$ are continuous and bounded on the time interval $[0, \infty)$. Then all the trajectories of the system (4) with the proposed control law of Eq.(9) are locally uniformly bounded. Furthermore, if $\lim_{t\to\infty} \upsilon_r(t) \neq 0$ or $\lim_{t\to\infty} \omega_r \neq 0$, then $\lim_{t\to\infty} e_1(t) = 0$, $\lim_{t\to\infty} e_2(t) = 0$ and $\lim_{t\to\infty} e_3(t) = 0$.

<u>Remark 1</u> For conservative choice of α in (9), it is constrained by the following relation

$$0 \le |\alpha| < 1/|e_1|_{\max} \quad (14)$$

Due to the fact that $e_1 \in L_\infty$ and $|\alpha\, e_1| < 1$, $1 + \alpha e_1 \neq 0$ for any value of $e_1(t)$.

<u>Remark 2</u> If $\alpha = 0$, the control law (9) becomes

$$\begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = \begin{bmatrix} \upsilon_r \cos e_3 + k_1 e_1 \\ \omega_r + \upsilon_r e_2 + k_2 e_3 \operatorname{sgn}(e_3 \sin e_3) \end{bmatrix}, k_1 > 0, k_2 > 0 \quad (15)$$

3. ROBUST PATH TRACKING FOR ELIMINATING NONLINEAR DRIVING CHARACTERISTICS

In order to achieve accurate path tracking control, the aim of this section is to develops a robust path tracking controller utilizing a vision-based system accompany together with the dead-reckoning system to eliminate nonlinear characteristics which caused by mechanical backlash in gear trains, dead-zone bands of the motor drivers. To facilitate the design process, we model the effects of nonlinear characteristics on the system behavior by considering the perturbed control error signal denoted by $\delta V = [\delta\upsilon \quad \delta\omega]^T$, $\|\delta V\|_\infty \le c < \infty$, where $\delta\upsilon$ denotes the difference between actual linear velocity and desired linear velocity, and $\delta\omega$ is the difference between actual angular velocity and desired angular velocity, and $\|\delta V\|_\infty = \max\{|\delta\upsilon|, |\delta\omega|\}$. It can be easily note that the nonlinear characteristics will cause the system to have nonzero steady-state path tracking errors. Therefore, the closed-loop error system with the perturbed control errors becomes

$$\begin{aligned} &\dot{e}_1 = (\omega + \delta\omega)e_2 - (\upsilon + \delta\upsilon) + \upsilon_r \cos e_3 \\ &\dot{e}_2 = -(\omega + \delta\omega)e_1 + \upsilon_r \sin e_3 \\ &\dot{e}_3 = \omega_r - (\omega + \delta\omega) \end{aligned} \quad (16)$$

Replacing $\dot{e}_3$ with $\dot{\bar{e}}_3$ in (16) yields

$$\begin{bmatrix}\dot e_1\\ \dot e_2\\ \dot e_3\end{bmatrix}=\begin{bmatrix}\upsilon_r\cos e_3\\ \upsilon_r\sin e_3\\ \omega_r\end{bmatrix}+\begin{bmatrix}-1 & e_2\\ 0 & -e_1\\ 0 & -1\end{bmatrix}[V+\delta V],\|\delta u\|_\infty\le c \quad (17)$$

In the sequel Lyapunov redesign is used to find a robust path tracking control law so as to reduce or even eliminate the effects of nonlinear characteristics on steady-state tracking errors. The discontinuous robust control law designed via Lyapunov redesign will completely cancel out the nonzero steady-state errors. To state the proposed robust control method, we decompose the control vector into two parts, i.e., $V=\phi+\psi$, where ψ is an additional feedback control such that the overall control V asymptotically stabilizes the system (4) in the presence of nonlinear driving characteristics. Here the control ϕ is chosen as in the form of (9),

$$\phi=\begin{bmatrix}\upsilon\\ \omega\end{bmatrix}=\begin{bmatrix}k_1e_1+\upsilon_r\cos e_3+\alpha\omega\sin e_3\\ \dfrac{1}{1+\alpha e_1}(k_2e_3\,\mathrm{sgn}(e_3\sin e_3)+\upsilon_re_2+\alpha\upsilon_r\sin e_3+\omega_r)\end{bmatrix}$$

which leads to the following closed-loop error system with the matched uncertainties

$$\begin{bmatrix}\dot e_1\\ \dot e_2\\ \dot e_3\end{bmatrix}=\begin{bmatrix}-k_1e_1+\omega e_2-\alpha\omega\sin e_3\\ -\omega e_1+\upsilon_r\sin e_3\\ -k_2e_3\,\mathrm{sgn}(e_3\sin e_3)-\upsilon_re_2+\alpha\omega e_1-\alpha\upsilon_r\sin e_3\end{bmatrix}+\begin{bmatrix}-1 & e_2\\ 0 & -e_1\\ 0 & -1\end{bmatrix}[\psi+\delta V] \quad (18)$$

Next, the additional control ψ is designed such that the time derivative of the Lyapunov function (11) is negative semi-definite if $\alpha\upsilon_r\ge 0$. Thus,

$$\begin{aligned}\dot V_1&=e_1\dot e_1+e_2\dot e_2+\dot e_3\sin e_3\\ &=e_1(-k_1e_1+\omega e_2-\alpha\omega\sin e_3)+e_2(-\omega e_1+\upsilon_r\sin e_3)\\ &\quad+\sin e_3(-k_2e_3\,\mathrm{sgn}(e_3\sin e_3)-\upsilon_re_2+\alpha\omega e_1-\alpha\upsilon_r\sin e_3)\\ &\quad+\eta^T[\psi+\delta\mu]\\ &=-k_1e_1^2-k_2|e_3\sin e_3|-\alpha\upsilon_r\sin^2e_3+\eta^T[\psi+\delta\mu]\le 0\end{aligned} \quad (19)$$

where η^T is given by

$$\eta^T=[e_1\quad e_2\quad \sin e_3]\begin{bmatrix}-1 & e_2\\ 0 & -e_1\\ 0 & -1\end{bmatrix}=[-e_1\quad -\sin e_3] \quad (20)$$

and η^T satisfies the inequality

$$\eta^T[\psi+\delta V]\le 0 \quad (21)$$

To accomplish the inequality (21), we choose

$$\psi=-k_3\,\mathrm{sgn}(\eta)=k_3\begin{bmatrix}\mathrm{sgn}(e_1)\\ \mathrm{sgn}(\sin e_3)\end{bmatrix} \quad (22)$$

such that , if $k_3\ge c$, and $\|\delta V\|_\infty\le c<\infty$, then

$$\begin{aligned}\eta^T[\psi+\delta V]&=\eta^T\psi+\eta^T\delta V\le\eta^T\psi+\|\eta\|_1\|\delta V\|_\infty\\ &\le -k_3\|\eta\|_1+\|\eta\|_1c\le-(k_3-c)\|\eta\|_1\le 0,\end{aligned} \quad (23)$$

The following theorem summarizes the previous result.

Theorem 2 Assume that, $\upsilon_r,\dot\upsilon_r,\omega_r,\dot\omega_r\in L_\infty$, are continuous and bounded on the time interval $[0,\infty)$. Then all the trajectories of the system (16) with the upper bound of the perturbed control errors, $\|\delta V\|_\infty\le c$, and the overall control yields

$$V=\begin{bmatrix}k_1e_1+\upsilon_r\cos e_3+\alpha\omega\sin e_3+k_3\,\mathrm{sgn}(e_1)\\ \dfrac{1}{1+\alpha e_1}(k_2e_3\,\mathrm{sgn}(e_3\sin e_3)+\upsilon_re_2+\alpha\upsilon_r\sin e_3+\omega_r)+k_3\,\mathrm{sgn}(\sin e_3)\end{bmatrix}$$
$$,k_1>0,k_2>0,\ k_3\ge\|\delta V\|_\infty \quad (24)$$

are globally uniformly bounded. Furthermore, if $\lim_{t\to\infty}\upsilon_r\ne 0$ or $\lim_{t\to\infty}\omega_r\ne 0$, then all the error signals $e_1(t),e_2(t)$, and $e_3(t)$ asymptotically converge to zeros.

Remark 3 The robust control law given by (24) is discontinuous due to the discontinuous signum function, thus resulting in the well-known chattering problem. This problem can be circumvented if the signum function is approximated by a continuous saturation function, i.e.,

$$V=\begin{bmatrix}k_1e_1+\upsilon_r\cos e_3+\alpha\omega\sin e_3+k_3sat(e_1/\varepsilon)\\ \dfrac{1}{1+\alpha e_1}(k_2e_3\,\mathrm{sgn}(e_3\sin e_3)+\upsilon_re_2+\alpha\upsilon_r\sin e_3+\omega_r)+k_3sat(\sin e_3/\varepsilon)\end{bmatrix}$$
$$,0<\varepsilon\ll 1 \quad (25)$$

$$where\quad sat(z/\varepsilon)=\begin{cases}z/\varepsilon, if\ |z|<\varepsilon\\ 1, if\ |z|\ge\varepsilon\end{cases}.$$

However, such an approximation will cause the overall close loop system to have nonzero steady-state tracking errors.

4. ADAPTIVE BACKSTEPPING TRACKING CONTROL

The objective of adaptive control for the WMRs with the dynamic model is to find an adaptive control with a set of parameter adaptation rules in order to achieve the aforementioned path tracking. The integral backstepping approach will be employed to yield such an adaptive control goal. In doing so, let the WMRs under consideration have the following task-space dynamic equation

$$M\dot V+F(V)=B\tau,\qquad M=\begin{bmatrix}m_0 & 0\\ 0 & I_0\end{bmatrix}$$

$$F(V)=\begin{bmatrix}F_{s1} & 0\\ 0 & F_{s2}\end{bmatrix}\begin{bmatrix}\mathrm{sgn}(\upsilon)\\ \mathrm{sgn}(\omega)\end{bmatrix},\ B=\frac{1}{r_0}\begin{bmatrix}1 & 1\\ \frac{l_0}{2} & -\frac{l_0}{2}\end{bmatrix}$$

where $V\in\Re^2$ denotes the control vector similarly defined in (1), $M\in\Re^{2\times2}$ represents the constant mass and inertia matrix, $F(V)\in\Re^2$ represents the friction effects containing dynamic and static frictions, $\tau(t)\in\Re^2$ denotes the torque input vector, and $B\in\Re^{2\times2}$ is the input matrix that governs torque transmission. This above dynamic model can be expressed in the camera space by pre-multiply $T_0=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the equation yields

$$\bar M\dot{\bar u}+\bar N=\bar B\tau\ ,\ \bar M(t)=T_0^TMT_0 \quad (26)$$

$$\bar{N}(t) = -T_0^T\left(MT_0\dot{\Pi} + F\left(T_0\left(-\bar{u}+\Pi\right)\right)\right), \bar{B}(t) = -T_0^T B$$

To facilitate the subsequent adaptive controller design, we define an auxiliary signal defined in terms of the camera-space orientation, the velocity and the desired trajectory as follows

$$\bar{u} = -\bar{V} + \bar{\Pi} \tag{27}$$

where $\bar{V} \in \Re^2$ is defined in Eq.(1), and the auxiliary vector $\bar{\Pi}\left(e(t), \upsilon_r(t)\right) \in \Re^2$ is given by

$$\bar{\Pi} = \begin{bmatrix} \upsilon_r \cos e_3 \\ \omega_r \end{bmatrix} \tag{28}$$

Now we rewrite (4) by substituting the auxiliary signal $\bar{u}(t)$, and then obtain the following expression

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 + u_1 \\ -\omega e_1 + \upsilon_r \sin e_3 \\ u_2 \end{bmatrix} \tag{29}$$

Since the backstepping technique will be used in this section to find the wanted torque input vector, we also define an auxiliary backstepping error signal, denoted by $\eta(t) \in \Re^2$ as follows;

$$\eta = \begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix}^T = u_d - u \tag{30}$$

where $u_d(t) \in \Re^2$ denotes the desired control input vector. Then we obtain the following open-loop error system with the control law (9)

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 + u_{d1} - \eta_1 \\ -\omega e_1 + \upsilon_r \sin e_3 \\ u_{d2} - \eta_2 \end{bmatrix} \tag{31}$$

$$\begin{bmatrix} u_{d1} \\ u_{d2} \end{bmatrix} = \begin{bmatrix} -k_1 e_1 \\ -\upsilon_r e_2 - k_2 \phi\left(e_2, e_3\right)\sin e_3 \end{bmatrix}.$$

To obtain the torque input vector, we let the model (26) be linearly parameterized as follows

$$Y_0 \vartheta_0 = \bar{M}\dot{\bar{u}} + \bar{N} \tag{32}$$

wher $\vartheta_0 \in \Re^p$ contains unknown constant mechanical parameters (i.e., inertia, mass, friction effects), and $Y_0 \in \Re^{2\times p}$ denotes a known regression matrix.

Similarly, by replacing $u(t)$ with $u_d(t)$, one obtains

$$Y_{d0}\vartheta_0 = \bar{M}\dot{\bar{u}}_d + \bar{N} \tag{33}$$

The torque input vector is proposed as $\tau = B^{-1}\tau_0$,

$$\tau_0 = \begin{bmatrix} \tau_{01} \\ \tau_{02} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_1}\left(\left(Y_{d0}\hat{\vartheta}_0\right)_1 + k_{\eta 1}\eta_1 - e_1\right) \\ -\frac{1}{T_2}\left(\left(Y_{d0}\hat{\vartheta}_0\right)_2 + k_{\eta 2}\eta_2 - \sin e_3\right) \end{bmatrix}, k_{\eta 1} > 0, k_{\eta 2} > 0 \tag{34}$$

where $k_{\eta 1}, k_{\eta 2} \in \Re^1$ are positive constant control gains, $\left(Y_{d0}\hat{\vartheta}_0\right)_i$ represents the ith entry of the vector $Y_{d0}\hat{\vartheta}_0$ for i=1,2. To accomplish the derivation of adaptive control, we propose a parameter adaptation rule for estimating $\hat{\vartheta}_0$ as follows;

$$\dot{\hat{\vartheta}}_0 = \Gamma_0 Y_{d0}^T \eta \tag{35}$$

where $\Gamma_0 = C_0 + C_1 e^{-t} \in \Re^{p\times p}$ is a positive definite diagonal gain matrix where C_0 and C_1 are two positive definite diagonal gain matrices. Then we develop close-loop error system for the auxiliary back-stepping error signal $\eta(t)$. Premultiplying the signal $\eta(t)$ by $\bar{M}(t)$, taking the time derivative and using (30) and (33), one has

$$\bar{M}\dot{\eta} = \bar{M}\dot{\bar{u}}_d - \bar{M}\dot{\bar{u}} \tag{36}$$

Furthermore, let the parameter estimate error signal be denoted as $\tilde{\vartheta}_0 = \vartheta_0 - \hat{\vartheta}_0$. Hence, (36) can be rewritten as

$$\bar{M}\dot{\eta} = \bar{M}\dot{\bar{u}}_d - \bar{M}\dot{\bar{u}} = Y_{d0}\vartheta_0 - \bar{B}\tau \tag{37}$$

$$\bar{M}\dot{\eta} = Y_{d0}\vartheta_0 - \left(-T_0 B\right)\cdot B^{-1}\tau_0 = Y_{d0}\tilde{\vartheta}_0 + \begin{bmatrix} -k_{\eta 1}\eta_1 + e_1 \\ -k_{\eta 2}\eta_2 + \sin e_3 \end{bmatrix} \tag{38}$$

To show local bounded tracking property of the proposed adaptive controller, we choose the following Lyapunov function candidate V_2

$$V_2 = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \left(1 - \cos e_3\right) + \frac{1}{2}\eta^T \bar{M}\eta + \frac{1}{2}\tilde{\vartheta}_0^T \Gamma_0^{-1}\tilde{\vartheta}_0 \tag{39}$$

Taking the time derivative of (39) by using (31) and go on the previous proof process, we obtain

$$\dot{V}_2 = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 \sin e_3 + \eta^T \bar{M}\dot{\eta} - \tilde{\vartheta}_0^T \Gamma_0^{-1}\dot{\hat{\vartheta}}_0 - \frac{1}{2}\tilde{\vartheta}_0^T \Gamma_0^{-1}\dot{\Gamma}_0\Gamma_0^{-1}\tilde{\vartheta}_0$$

$$= -k_1 e_1^2 - k_2\phi\left(e_2, e_3\right)\sin^2 e_3 - k_{\eta 1}\eta_1^2 - k_{\eta 2}\eta_2^2 - \frac{1}{2}\tilde{\vartheta}_0^T \Gamma_0^{-1}\dot{\Gamma}_0\Gamma_0^{-1}\tilde{\vartheta}_0 \le 0$$

Therefore, $\dot{V}_2$ is always negative semi-definite. It follows that all the signals e_1, e_2, e_3, η and $\hat{\vartheta}$ are easily proven locally uniformly bounded. With the same arguments in the previous section, we claim that the proposed adaptive global path tracking controller makes the wheeled mobile robot always follow the desired trajectories with bounded errors if $\lim_{t\to\infty}\upsilon_r \ne 0$ or $\lim_{t\to\infty}\omega_r \ne 0$.

Theorem 3 Assume that υ_r , $\dot{\upsilon}_r$, ω_r and $\dot{\omega}_r$ are continuous and bounded on the interval $[0,\infty)$. Then all the trajectories of the closed-loop error system composed of the torque control vector (34) and the parameter adaptation rules (35) are uniformly bounded.

5. THE RESULTS OF SIMULATION

The aim of the simulations is to examine the performance of the proposed adaptive robust control (34) and (35) for a mobile robot whose dynamic model is given by

$$\begin{bmatrix} m_0 & 0 \\ 0 & I_0 \end{bmatrix}\begin{bmatrix} \dot{\upsilon} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} F_{s1} & 0 \\ 0 & F_{s1} \end{bmatrix}\begin{bmatrix} \mathrm{sgn}(\upsilon) \\ \mathrm{sgn}(\omega) \end{bmatrix} = \frac{1}{r_0}\begin{bmatrix} 1 & 1 \\ l_0/2 & -l_0/2 \end{bmatrix}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

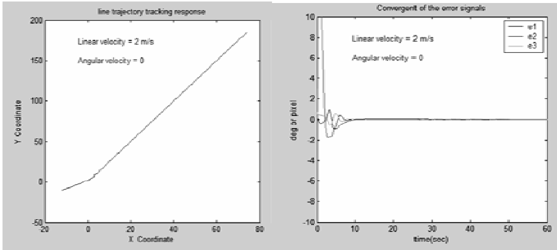

Fig. 2 (a) the results of tracking response with the reference line trajectory in x-y plot (b) the results of error signals with the reference line trajectory

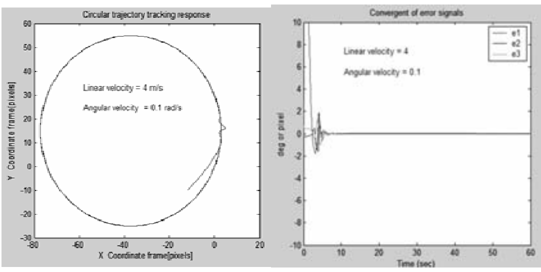

Fig. 3 (a) the results of tracking response with the reference circular trajectory in x-y plot (b) the results of error signals with the reference circular trajectory

where their nominal values are specified by r_0 =2.7056cm, l_0 =15cm, F_{s1} =0.3Nm, F_{s2} =0.3Nm, m_0 =2.6kg, I_0 =12 kg-m^2 . All the simulations were implemented using SIMULINK. To make the closed-loop system exhibit desired asymptotical path tracking responses, we choose $O_{i1}, O_{i2} = 0$ [Pixels], $\alpha_1 = 1$ [Pixels], $\alpha_2 = 1$ [Pixels], $O_{o1}, O_{o2} = 0$ [Pixels] and $\theta_0 = 0$ [rad], $k_1 = 26,\ k_2 = 2,\ k_{\eta 1} = k_{\eta 2} = 20$.

To perform different tracking simulations, we selected the desired initial position and orientation of the robot as (x_{ini} , y_{ini} , θ_{ini})=(5, 0, 1), and the actual initial condition (-50, -50, 1). Fig. 2 shows the simulated line path tracking trajectory, where the desire linear velocity and angular velocity are given by $v_r = 2$ m/s and $\omega_r = 0$ rad/s. Fig. 3 shows the simulated circular path tracking trajectory, where the desire linear velocity and angular velocity are given by and $v_r = 4$ m/s and $\omega_r = 0.2$ rad/s. The results in Fig. 2-3 indicated that the proposed control methods are shown effective for steering WMR to follow the desired reference trajectories.

6. CONCLUSIONS

This paper has developed methodologies for adaptive robust trajectory tracking control of a wheeled mobile robot with nonlinear driving characteristics and unknown dynamic parameters. The novel kinematics path tracking control law has been proposed in order to achieve asymptotical path tracking, and it has been successfully extended to the adaptive nonlinear tracking law for the robot associated with the dynamic model with unknown parameters. Through simulations, the proposed control laws have been successfully used to steer the WMR to follow the desired reference line and circle trajectories. An interesting topic for future research is to implement such control laws in a mobile robot and to physically verify the efficacy of the proposed control laws.

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