NONLINEAR CONTROL OF A LINEAR MOTION SYSTEM

Wouter Aangenent * **René van de Molengraft** **
Maarten Steinbuch ***

* *W.H.T.M.Aangenent@tue.nl*
** *M.J.G.v.d.Molengraft@tue.nl*
*** *M.Steinbuch@tue.nl*

Abstract: It is well-known that Bode's gain/phase relation imposes limitations on the performance of a linear system. To circumvent these limitations, two examples from literature on nonlinear controllers, more precisely, a PID with nonlinear gain and a SPAN (split-path nonlinear) filter, are implemented on a motion system in order to improve the step response in comparison with a conventional linear PID controller. Simulations and experiments are presented and it is shown that these nonlinear control strategies can outperform a linear PID controller.

Keywords: nonlinear control, nonlinear gain, SPAN filter, fundamental limitations

1. INTRODUCTION

There has been considerable interest in literature on the topic of fundamental limitations in feedback systems with known plant dynamics. This has its origin in the seminal work of Bode (Bode, 1945), more recent work is contained in (Seron *et al.*, 1997). Most of this work is restricted to linear feedback loops. Part of the limitations is inherently linked to the plant and thus hold irrespective of how the input is generated, be it via linear, nonlinear or time-varying feedback. Other limitations are a consequence of the plant acting in combination with linear time invariant (LTI) feedback control. This naturally rises the question if these latter limitations can be ameliorated by using nonlinear or time-varying in stead of LTI feedback.

Being aware of the difficulties and bad effects normally associated with the presence of nonlinearities, it may appear a step backwards to intentionally introduce nonlinearities into an otherwise essentially linear system. A problem that arises is to be able to predict the systems response for various inputs. In many cases however, there may be good reasons to intentionally introduce nonlinear elements in the feedback loop. Indeed, in literature several examples have been presented showing that nonlinear control can, in certain circumstances, outperform linear time invariant feedback control for known plants. In (Feuer *et al.*, 1997), it is shown that a simple PI controller whose integrator is switched on and off depending on the size of the error, performs better than its linear time invariant counterpart. Also, based on experience, several nonlinear 'tricks' are used in industry to obtain better performance of an LTI feedback system (Heertjes and Steinbuch, 2004). A more systematic strategy for nonlinear control of an LTI plant is reset control. Reset control action resembles a number of popular nonlinear control strategies such as relay control (Tsypkin, 1984), sliding mode control (Decarlo, 1988) and switching control (Branicky, 1988).

The motivation for these and other types of nonlinear control for linear systems is the fact that linear controllers have the inherent disadvantage that their gain and phase characteristics are re-

lated (Bode, 1945). Specifically, the need to optimize the open-loop high frequency gain often competes with required high levels of low frequency loop gains and phase margin bounds. To illustrate this point, consider a typical fourth order electromechanical motion system modeled as two double integrators connected by a spring and damper. The open-loop frequency response is required to have sufficient bandwidth and large low frequency gain to obtain a fast response and good settling behavior or tracking. On the other hand, at high frequencies, it needs to be small to suppress residual vibration and sensor noise. This performance trade off is defined by Bode's gain/phase relation, which limits how fast the open-loop gain can cross unity gain while maintaining closed-loop stability, whereas in the ideal case these characteristics should be designed independently of one another. This paper reviews two examples of nonlinear control of linear systems and presents simulation and experimental results obtained on a dual rotary 4th order motion system. Since a common test of servo performance is the step response, the goal of this paper is to show that the introduction of nonlinear elements in an essential linear motion system can improve the step response with respect to the combination of overshoot and settling time when compared to standard linear feedback. To the authors knowledge, no experimental results of this kind obtained on motion systems have been presented in literature yet. In section 2, a nonlinear gain and a SPAN (split-path nonlinear) filter are discussed, and simulation and experimental results are presented. Finally, conclusions and some ideas for future research are given.

2. NONLINEAR CONTROL EXAMPLES

In this section, simulations of nonlinear control strategies are presented, i.e. nonlinear gains (Kalman, 1955), and a SPAN filter (Foster *et al.*, 1966). The system under consideration is a dual rotary 4th order motion system, the bode diagram of which is shown with the dashed line in Fig. 1. The identified fourth order model of this system is depicted in the same Fig. with the solid line. The control setup is depicted in Fig. 2. In this figure, NL denotes the nonlinear control element, C denotes the linear controller and P denotes the linear system dynamics. In order to guarantee zero settling error, a linear controller consisting of a proportional part, a lead/lag filter and an integrator is applied to the system. The bode diagram of this controller and the resulting open loop system are depicted in figures 3 and 4. The bandwidth is not chosen high since we are only interested in the difference in performance between the linear and the nonlinear controllers.

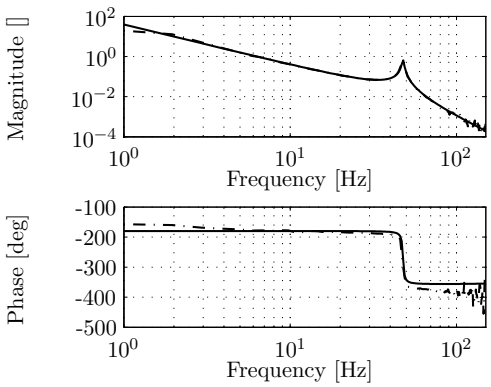

Fig. 1. Bode diagram of the system (dashed) and of the model (solid).

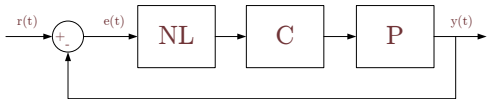

Fig. 2. Control scheme.

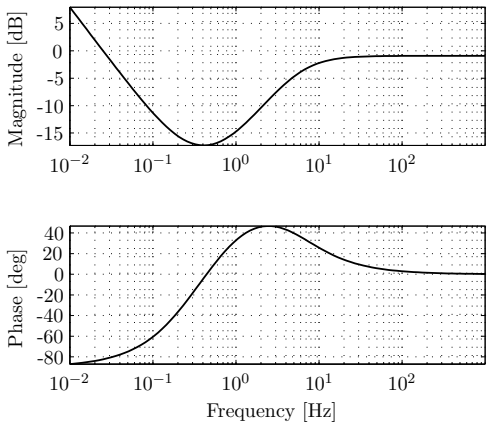

Fig. 3. Bode diagram linear controller.

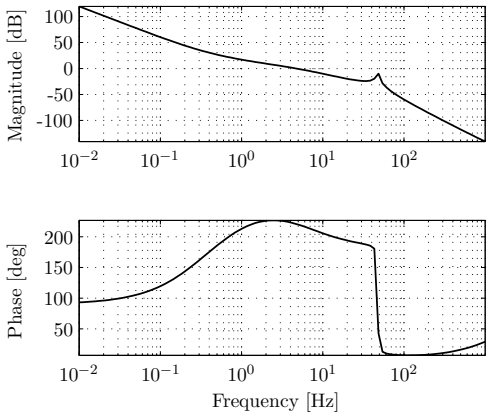

Fig. 4. Open loop of the controlled system.

2.1 Nonlinear gain element

A nonlinear gain element is an element where a multiplication composes the nonlinearity. The gain element may be presented anywhere in the loop and can be a function of any loop variable, which in this case is the error. The proportional gain, $K_{\text{lin}} = 0.9$, is substituted by a nonlinear non-

smooth gain element. The objective of this nonlinear gain is to diminish the overshoot without increasing the settling-time (here defined as the smallest time for which the response comes and stays in a band of 95% of its final value) of the transient response to a step input. A gain element consisting of three parts is depicted in Fig. 5. Since a nonlinear gain is a single-valued nonlinearity, its describing function is only a function of the amplitude of its input, and not of the frequency. The describing function of this nonlinear gain is shown in Fig. 6. The three parts of this gain are denoted by A, B, and C in these figures. Since our purpose is to obtain an overshoot as small as possible, combined with a small settling time, the following choice has been made for the gains in part A, B, and C. The gain corresponding with part A determines the initial response on a changing setpoint, it is used when the error is relatively large. In order to obtain a small rise time, this gain should be chosen relatively large to have a similar initial response as the linear system, $K_A = K_{\text{lin}}$. To obtain a small overshoot, a relatively small gain should be used when the systems response is approaching the setpoint, therefore $K_B = \frac{1}{100} K_{\text{lin}}$, and finally, a small settling-time can be obtained by choosing a relatively large value for the small error gain, $K_C = 2.5 K_{\text{lin}}$. The values of the error at which switching occurs, also influences the final response. In this case, we would like to have a fast response, so a relatively high bandwidth, within a settling band of 10% of the step value of 5 radians. Therefore, part C has its boundaries at $e = -0.5$ rad. and $e = 0.5$ rad. To diminish the overshoot, a large band has been chosen in which the second stage gain is active, so B is in the range $\pm[0.5, 4]$.

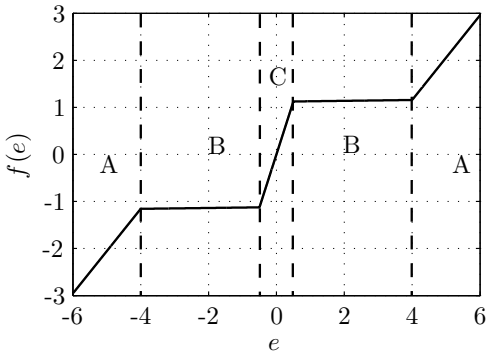

Fig. 5. Nonlinear gain element consisting of three parts.

Stability / limit cycling. Since non-smooth nonlinear gain elements are used, care has to be taken that the system remains stable and that no unwanted phenomena like limit cycling occur. A nonlinear element can cause this behavior as is shown by the following. The closed loop transfer function of the control system is given by

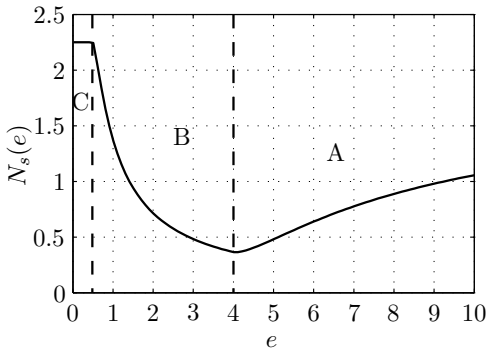

Fig. 6. Describing function of the nonlinear gain element.

$$T(i\omega) = \frac{N_s(e)C(i\omega)G(i\omega)}{1 + N_s(e)C(i\omega)G(i\omega)}, \tag{1}$$

where $N_s(e)$ is the describing function of the nonlinear gain. The characteristic equation

$$1 + N_s(e)C(i\omega)G(i\omega) = 0 \tag{2}$$

indicates stability. Therefore, the condition which governs the existence of a limit cycle is given by

$$N_s(e)C(i\omega)G(i\omega) = -1 \Rightarrow C(i\omega)G(i\omega) = -\frac{1}{N_s(e)} \tag{3}$$

The part of the curve $-1/N_s(e)$ encompassed by the $C(i\omega)G(i\omega)$ Nyquist curve indicates the amplitudes of e for which the system is unstable and vice versa. The intersection of both curves defines the possible oscillation on which the system behavior will stay. In Fig. 7, the Nyquist curve of $C(i\omega)P(i\omega)$ together with $-1/N_s(e)$ is depicted for the proposed nonlinear gain. Since $-1/N_s(e)$ is always outside the Nyquist curve of $C(i\omega)P(i\omega)$, the closed-loop system is stable.

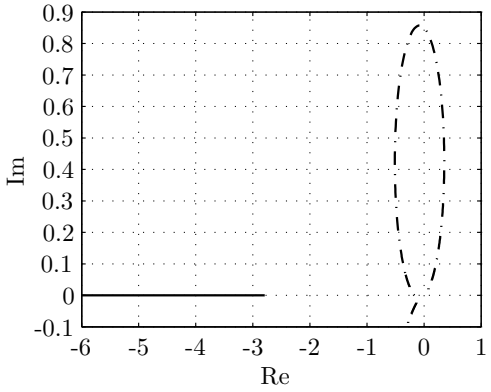

Fig. 7. Nyquist diagram of the open-loop (dashed) and $-1/N_s(e)$ (solid).

Simulation and experimental results. The simulated step response of the linear system with constant gain, and the system incorporating the gain consisting of three parts is depicted in Fig. 8. The results are as expected, the initial response is fast, then the response slows down resulting in a much smaller overshoot than was the case

with the linear controller, and when the response is close to the setpoint, the high gain, K_C, enforces fast settling behavior. In Fig. 9, the measured step response is depicted. As can be seen, the qualitative behavior is the same, however, due to friction and other un-modeled dynamics, the quantitative behavior differs from the simulation. The main difference is the smaller overshoot when the linear controller is applied due to the friction that is present in the setup.

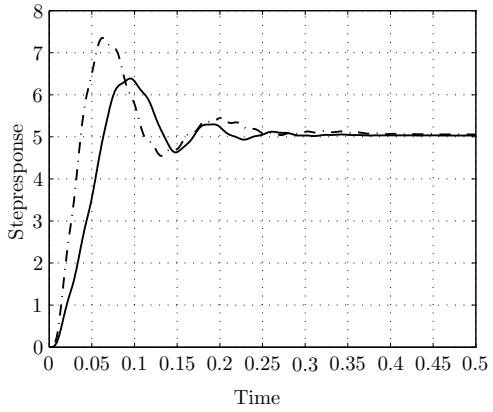

Fig. 8. Simulated stepresponse of the linear system (dashed) and with the nonlinear gain (solid).

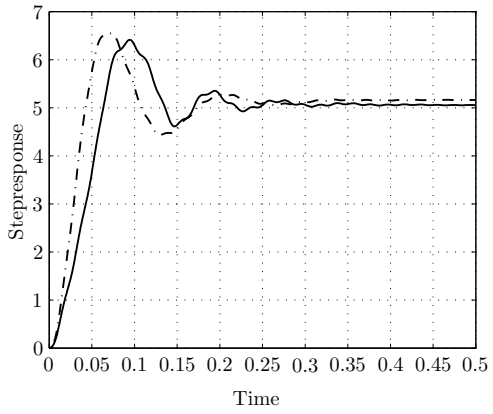

Fig. 9. Measured stepresponse of the linear system (dashed) and with the nonlinear gain (solid).

If the gain for small-amplitude errors is enlarged to $K_C = 12.5K_{\text{lin}}$, limit cycling occurs. In Fig. 10, again the Nyquist curve of $C(i\omega)G(i\omega)$ together with $-1/N_s(e)$, with $N_s(e)$ the new describing function, is depicted. It is obvious that there is a crossing, hence a limit cycle will exist. Since the gain is a single-valued nonlinearity, the crossing takes place on the real axis, and therefore it is straightforward to estimate the frequency and amplitude of the oscillation. The value of the describing function at the crossing point is equal to the gain margin of the open loop system, which is in this case 5.82. The frequency at this point is 44.8 Hz. An estimate can be made of the amplitude form the describing function, namely the value of the amplitude for which the describing

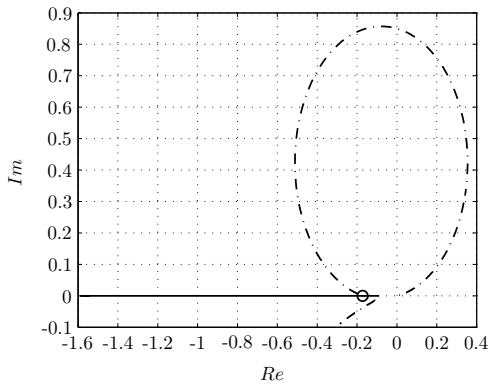

Fig. 10. Nyquist diagram of the open-loop (dashed) and of the describing function (solid).

function has value $N_s(e) = 5.82$, is approximately $e = 1.16$. The simulated step response of the linear system with constant gain and of the system incorporating the nonlinear gain are depicted in Fig. 11. Indeed, limit cycling occurs, and the estimated values of the amplitude and frequency of the oscillation correspond very well. In Fig. 12, the measured step response of the linear system with constant gain, and the system incorporating the nonlinear gain are depicted. Again the qualitative behavior is the same, the differences up to 0.4 seconds can be explained by friction, and the additional frequency which is visible in the limit cycle is caused by un-modeled dynamics such as cogging.

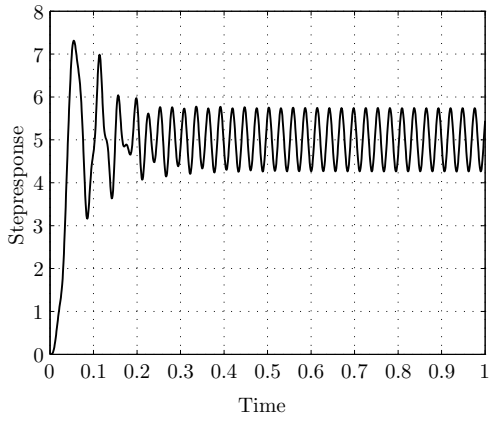

Fig. 11. Simulated stepresponse of the system with increased K_C (solid).

2.2 SPAN filter

The SPAN (split-path nonlinear) filter is an attempt to obtain a filter which has independent gain and phase characteristics. In Fig. 13, a block scheme of a SPAN filter is depicted. This filter processes the input in two paths and multiplies the output of the two branches. The path containing the *sign* element controls the sign of the signal and destroys all magnitude information,

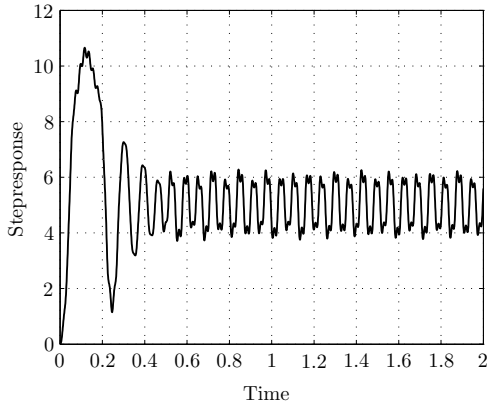

Fig. 12. Measured stepresponse of the system with the increased nonlinear gain (solid).

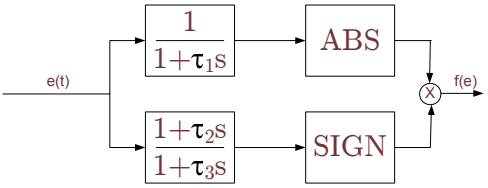

Fig. 13. Block diagram of a SPAN filter.

while the *absolute value* element destroys all sign information, and therefore controls the magnitude information. The phase shift of the absolute value path is reflected in the output. With this filter, the sign and the magnitude can be independently chosen.

The SPAN filter can be used as a phase lead filter that does not cause magnitude amplification. In the control scheme in Fig. 2, the SPAN filter takes the place of the nonlinear element and the integrator and the lead filter are still used. It is now possible to increase the cut-off frequency of the integrator while keeping the closed loop stable by applying a lead filter in the sign path of the SPAN filter. In the absolute value path, a low-pass filter is used to attenuate higher frequencies. In Fig. 14, the describing function of the SPAN filter is depicted. This describing function is independent of the amplitude of the input but depends only on the frequency. As can be seen, within the describing function theory, this filter is able to obtain phase lead while attenuating the magnitude, something which is not possible with any linear filter.

Stability. With the SPAN filter in the loop, the cut-off frequency of the integrator can be increased without destabilizing the closed-loop system. To see this, in Fig. 15 the Bode diagram of the open-loop with increased cut-off frequency of the integrator but without the SPAN filter is depicted by the dashed line. It is obvious that to stabilize the closed loop, extra phase needs to be created around the bandwidth. With a linear compensator, it is not possible to accomplish this

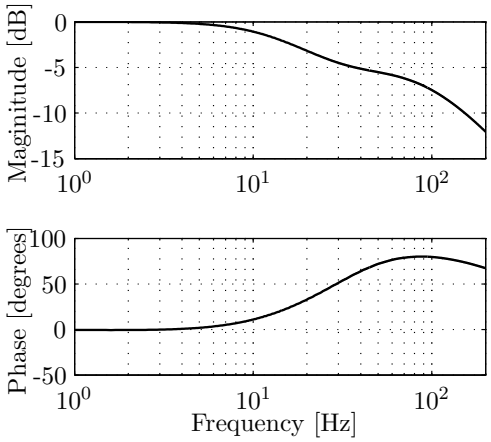

Fig. 14. Frequency dependent describing function of the SPAN filter.

without magnitude amplification, while the SPAN filter is, as can be seen in the same figure where the solid line depicts the open-loop including the SPAN filter. The Nyquist plot of the open-loop

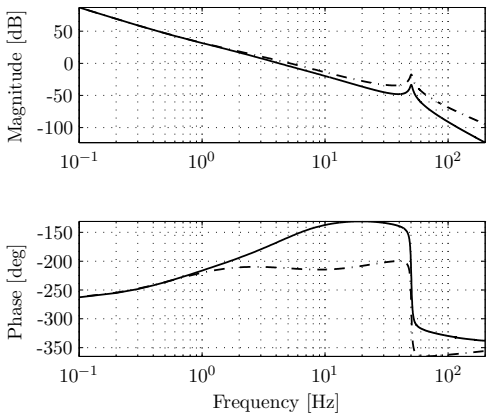

Fig. 15. Bode diagram of the open-loop without (dashed) and with (solid) the SPAN filter.

with and without the SPAN filter is depicted in Fig. 16, and as can be seen, the SPAN filter stabilizes the closed-loop. In the design of the filter,

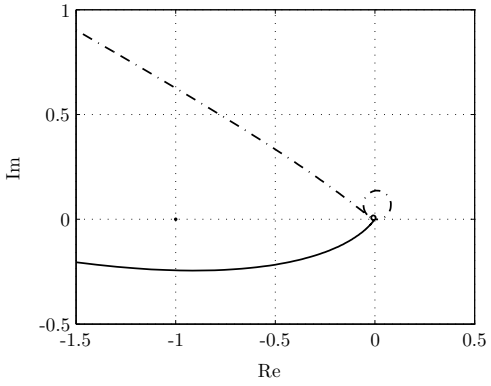

Fig. 16. Nyquist plot of the open-loop with (solid) and without (dashed) the SPAN filter.

the cut-off frequency of the integrator is increased to 18.5 Hz, the cut-off frequency of the low-pass filter is set to 11.14 Hz, the zero of the lead filter

is placed at 2.12 Hz, and the pole is located at 38.19 Hz. The gain of the SPAN filter was tuned to obtain a step response without overshoot and a reasonable settling time and is set to 0.15.

Simulation and experimental results. In figures 17 and 18, the simulated and measured step responses of the linear controlled system and of the system with the SPAN filter are shown. This filter is able to obtain approximately the same settling time, while avoiding overshoot completely. This is a response that cannot be obtained using a linear controller. A drawback of the SPAN filter is the tedious tuning. Since it is a nonlinear filter, superposition does not hold and, therefore, the tuning procedure for every parameter is based on trial-and-error. A big advantage of the SPAN filter is the fact that its performance is independent of the amplitude of the input.

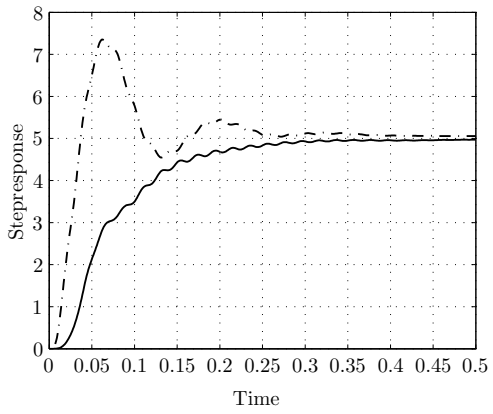

Fig. 17. Simulated stepresponse of the linear system (dashed) and with the SPAN filter (solid).

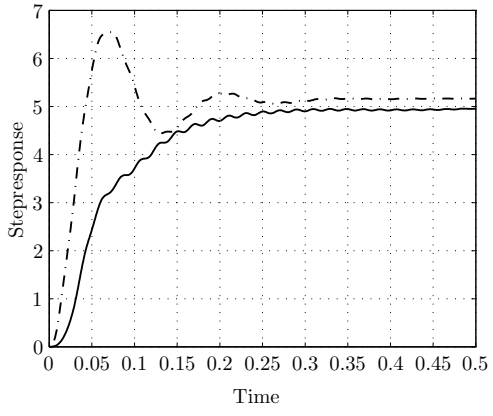

Fig. 18. Measured stepresponse of the linear system (dashed) and with the SPAN filter (solid).

3. CONCLUSION AND FUTURE RESEARCH

In this paper, two nonlinear control strategies are discussed for a linear time-invariant plant, i.e. a nonlinear gain and a SPAN filter. Simulations and experiments performed on a fourth order motion system show that a controller with nonlinear elements can improve performance with respect to overshoot and settling time of a step response when compared to an LTI controller. The SPAN filter is even able to settle in approximately the same time as a linear controller, but without any overshoot. It would be useful to know if such nonlinear strategies are also able to increase performance when other reference signals are used or when disturbances are present on the system.
Future research aims to give an answer to these and other issues regarding nonlinear control of linear systems, and to develop a systematic approach to nonlinear controller synthesis for linear plants in order to outperform LTI controllers. A promising link with respect to the synthesis of nonlinear controllers for linear systems is the control theory of gain scheduled and linear parameter-varying (LPV) plants (Rugh and Shamma, 1999).

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