UNBIASED THERMOCOUPLE SENSOR CHARACTERISATION IN VARIABLE FLOW ENVIRONMENTS

Peter C.F. Hung*, Dr Seán McLoone#, Prof. George Irwin*, Dr Robert Kee*

*Virtual Engineering Centre**
Queen's University Belfast
Belfast, Northern Ireland
BT9 5HN

#Department of Electronic Engineering
National University of Ireland, Maynooth
Maynooth, Co. Kildare
Ireland

E-mail: p.hung@qub.ac.uk

Abstract: A novel two-thermocouple sensor characterisation method for use in variable velocity flow environments is described. A difference equation method, recently developed by the authors for constant velocity flow applications, is extended to accommodate variable velocity flows using polynomial parameter fitting on a sliding data window. In particular, by using a novel difference equation formulation the invariance of time-constant ratio with respect to flow velocity is exploited to produce an efficient unbiased and consistent time-constant estimator. Monte-Carlo simulation studies show that the new algorithm outperforms alternatives in the literature without the restrictive requirement of *a priori* knowledge of thermocouple time constant ratios.

Keywords – Least Squares, Sensor, System Identification

1. INTRODUCTION

Dynamic measurement of exhaust gas temperature (EGT) gives valuable insights into car engine performance, particularly during transient operation. As manufacturers strive to design more fuel efficient, low emission cars EGT is likely to become a desirable input in next generation electronic engine management systems (EEMS) (Kee, *et al.*, 1999). However, performing accurate, reliable and cost-effective measurement of rapidly changing gas temperature is a challenging problem.

Fast response temperature measurement can be performed using sensors based on techniques such as Coherent Anti-Stokes Spectroscopy (CARS), Laser-Induced Fluorescence, and Infrared Pyrometry, but these are expensive, difficult to calibrate and maintain and therefore not practical for wide scale deployment. The use of a thermocouple as an instrument for temperature measurement is common because of its simplicity, robustness, relatively low cost, ease of manufacture and installation. Unfortunately, their design, a compromise between robustness and speed of response, poses major problems when measuring high frequency temperature fluctuations. From thermodynamic considerations it can be shown that the bandwidth of a thermocouple (w_B) is dependent on its diameter according to the equation

$$w_B = \kappa d^{m-2} v^m \tag{1}$$

where κ and m are constants, d is the diameter of the thermocouple wire and v is the velocity of the gas. Thus, we require large diameters for the harsh environments presented by engine exhausts, but small diameters to follow the rapid temperature fluctuations. One solution to this problem is to employ robust large diameter thermocouples and then utilise software techniques to reconstruct the true temperature from the attenuated and phase shifted measurements. Before such reconstruction can take place, however, a model of the thermocouple must be determined, a process referred to as sensor characterisation.

If certain criteria regarding the mechanical construction and placement of thermocouples are met (Forney and Fralick, 1994) they can be adequately modelled as having first order dynamics with time constant τ and unity gain.

$$T_g(t) = T_m(t) + \tau \dot{T}_m(t) \,. \qquad (2)$$

Here $T_g(t)$ is the true gas temperature and $T_m(t)$ is the measured temperature.

In theory, using this model $T_g(t)$ can be reconstructed from the measured temperature and its derivative. In practice this approach is infeasible as measurements are generally corrupted by noise, making $\dot{T}_m(t)$ difficult to compute accurately. Furthermore, τ, a function of the bandwidth w_B $(\tau = 2\pi w_B^{-1})$, varies with gas flow velocity (1) and is generally unknown *a priori*.

Thus, a single thermocouple provides insufficient information to determine sensor characteristics *in situ*. Data fusion can be used to help with sensor characterisation and it is quite common to use two thermocouples for this purpose. There are numerous publications on the problem of time constant estimation and subsequent temperature reconstruction on the basis of measurements taken from two or more thermocouples with different time-constants. (e.g. Kee, *et al.*, 1999; Tagawa and Ohta, 1997; Forney and Fralick, 1994). These two-thermocouple probe (TTP) methods rely on the restrictive assumption that the ratio of the thermocouple time constants α ($\alpha < 1$ by definition) is known *a priori*. They are also subject to singularities and sensitive to noise.

Hung, *et al.* (2003 and 2004) developed difference equation methods for TTP characterisation that do not require any *a priori* assumptions about the time constant ratios. However, to date the methods have mainly focused on single-valued time constant estimation, where the gas flow velocity is kept relatively constant. Unfortunately, this is not always applicable in practical situations such as engine exhausts where gas velocity varies continuously.

To tackle the issue of sensor characterisation under varying gas flow conditions, this paper proposes a novel *difference equation* algorithm that exploits the invariance of time constant ratio with respect to gas flow velocity (Hung, *et al.*, 2004). This reduces the problem to one where only a single time-varying parameter needs to be tracked and allows efficient sliding data window and polynomial parameter fitting techniques to be used to obtain unbiased time-constant estimates. The proposed algorithm gives improved performance in terms of time constant estimation accuracy and noise tolerance compared to existing sliding window methods.

The remainder of the paper is organised as follows. Difference equation based sensor characterisation formulations and associated unbiased parameter estimation algorithms are described in Section 2. The new sliding window formulation incorporating polynomial parameter fitting is then introduced in Section 3. Simulation results demonstrating the operation and performance of the new algorithm are given in Section 4 and finally Section 5 provides some conclusions.

2. DIFFERENCE EQUATION SENSOR CHARACTERISATION

2.1 Difference Equation Thermocouple model

The equivalent discrete time representation for the thermocouple model (2) is:

$$T_m(k) = aT_m(k-1) + bT_g(k-1) \,, \qquad (3)$$

where a and b are difference equation ARX parameters and k is the sample instant. Assuming ZOHs and a sampling interval τ_s, the parameters of the discrete and continuous time thermocouple models are related by

$$a = \exp(-\tau_s/\tau), \quad b = 1 - a \,. \qquad (4)$$

2.2 Three-parameter ARX model (Gamma model)

The discrete parameters a and b cannot be identified using (3) alone because T_g is unknown. However, using two thermocouples that are subject to the same environmental conditions (i.e. the same gas temperature T_g and gas velocity v), T_g can be eliminated to produce the following 3-parameter ARX representation:

$$\begin{aligned} T_{m2}(k) = {} & \gamma_1 T_{m2}(k-1) + \gamma_2 T_{m1}(k) \\ & + \gamma_3 T_{m1}(k-1) \end{aligned} \qquad (5)$$

where

$$\gamma_1 = a_2, \gamma_2 = \frac{1-a_2}{1-a_1}, \gamma_3 = -\frac{a_1(1-a_2)}{1-a_1} \cdot \qquad (6)$$

Here subscripts 1 and 2 are used to distinguish between signals from different thermocouples. The 3 gamma parameters are introduced to convert the non-linear 2-parameter (a_1, a_2) model into a linear-in-the-parameter ARX formulation (Fig. 1) that can be solved using linear least-squares techniques.

$$T_{m1}(k) \longrightarrow \boxed{\frac{\gamma_2 + \gamma_3 z^{-1}}{1 - \gamma_1 z^{-1}}} \longrightarrow T_{m2}(k)$$

Fig. 1. Equivalent ARX model for TTP sensor characterisation.

Analysis of the *gamma* model formulation shows that conventional least-squares identification leads to biased parameter estimates because noise is present on both the model input and output signals. Hung, *et al.* (2003) showed that unbiased parameter estimates can be achieved using total least squares (TLS), provided the variance of the noise on both thermocouple measurements is the same. However, Monte-Carlo simulation studies showed that TLS was only effective for low noise levels as the variance of the estimates grew rapidly with noise level. This is partly a feature of TLS, which is known to yield higher variances than conventional least squares (Van Huffel and Vandewalle, 1991), and partly due to the extra degree of freedom introduced by having a 3-parameter model for a system with only two unknowns.

2.2 Two-parameter ARX model (Beta model)

The 3-parameter model can be reduced to a linear two-parameter formulation by defining a new parameter $\beta \triangleq b_2/b_1$ and expressing (5) in terms of β and b_2 only. This gives

$$\Delta T_{m2}^{k} = \beta \Delta T_{m1}^{k} + b_2 \Delta T_{m12}^{k-1}, \tag{7}$$

where ΔT_{m1}^{k} , ΔT_{m2}^{k} , and ΔT_{m12}^{k-1} are composite variables defined as

$$\begin{aligned} \Delta T_{m1}^{k} &= T_{m1}(k) - T_{m1}(k-1) \\ \Delta T_{m2}^{k} &= T_{m2}(k) - T_{m2}(k-1) \\ \Delta T_{m12}^{k-1} &= T_{m1}(k-1) - T_{m2}(k-1) \end{aligned} \quad . \tag{8}$$

For an *M*-sample data set (7) can be expressed in vector-matrix form as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\theta}\,, \tag{9}$$

with $\boldsymbol{Y} = \Delta\boldsymbol{T}_{m2}^{k}$, $\boldsymbol{X} = [\Delta\boldsymbol{T}_{m1}^{k}\ \Delta\boldsymbol{T}_{m12}^{k-1}]$, and $\boldsymbol{\theta} = [\beta\ b_2]^T$. Here $\Delta\boldsymbol{T}_{m1}^{k}$, $\Delta\boldsymbol{T}_{m2}^{k}$ and $\Delta\boldsymbol{T}_{m12}^{k-1}$ are vectors containing *M*-1 samples of the corresponding composite signals ΔT_{m1}^{k}, ΔT_{m2}^{k}, and ΔT_{m12}^{k-1}.

Due to the form of the composite input and output signals, the noise terms in the $\boldsymbol{X}$ and $\boldsymbol{Y}$ data blocks are no longer independent with the result that conventional least-squares and TLS both generate biased parameter estimates even when the measurement noise on the thermocouples is independent. Generalised total least squares (GTLS) on the other hand, which employs generalised singular value decomposition (GSVD), can produce unbiased parameter estimates under these conditions provided the noise covariance matrix, $\boldsymbol{C}$ of the augmented data matrix $[\boldsymbol{X}\ \boldsymbol{Y}]$ is known to within an arbitrary scalar, that is $\boldsymbol{C} = \mu\boldsymbol{C}_0$, μ an arbitrary scalar (Van Huffel and Vandewalle, 1991).

Hung, *et al.* (2004) showed that for the 2-parameter Beta model (9) $\boldsymbol{C}_0$ can be expressed as

$$\boldsymbol{C}_0(\phi) = \begin{bmatrix} 2\phi & -\phi & 0 \\ -\phi & \phi+1 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \phi > 0 \tag{10}$$

where ϕ is the ratio of the thermocouple noise variances which is typically unity.

Given $\boldsymbol{C}_0$, the GTLS parameter estimates are obtained by computing the GSVD of the matrix pair $[\boldsymbol{X}\ \boldsymbol{Y}]$ and $\boldsymbol{W}$, where $\boldsymbol{W}$ is the Cholesky decomposition of $\boldsymbol{C}_0$:

$$gsvd([\boldsymbol{X}\ \boldsymbol{Y}], \boldsymbol{W}) \rightarrow (\boldsymbol{\Sigma}_G, \boldsymbol{G}) \tag{11}$$

and evaluating

$$\left[\hat{\boldsymbol{\theta}}^T;\quad -1\right]^T = -\frac{1}{g_{3,3}} \cdot \boldsymbol{g}_3 \,. \tag{12}$$

Here $\boldsymbol{\Sigma}_G$ is the (3×1) vector of generalised singular values, $\boldsymbol{G}$ is the (3×3) matrix of corresponding singular vectors, $\boldsymbol{g}_3$ is the third column of $\boldsymbol{G}$ and $g_{3,3}$ is the third element of $\boldsymbol{g}_3$.

3. SLIDING WINDOW CHARACTERISATION

The sensor characterisation methods described in the previous section are intended for situations where thermocouple time constants are relatively constant over the interval of interest. They can, however, be easily extended to variable time-constant scenarios by introducing a sliding data window. Then, provided the time constants are changing sufficiently slowly to be almost constant over the length of the data window, accurate time constant estimates can be obtained.

The choice of data window length (N) is a critical parameter in the performance of these methods. Since the variance of least-squares parameter estimates and the bandwidth of the estimator (with respect to time-constant variation frequency) are both inversely proportional to N it follows that choosing N becomes a trade-off between robustness to measurement noise and tracking performance. In practice, the sensitivity of sensor characterisation methods to noise severely limits the bandwidth that can be achieved.

In the next section a sliding window algorithm is presented that extends the bandwidth that can be achieved by relaxing the requirement that time constants are invariant over the data window.

3.1 Beta model with polynomial parameter fitting

Recalling that the β parameter in (7) is defined as $\beta \triangleq b_2/b_1$ and using the relationships in (4) it can be shown that

$$\beta = \frac{1-\exp(\tau_s/\tau_2)}{1-\exp(\tau_s/\tau_1)} \approx \frac{\tau_1}{\tau_2} = \alpha < 1 \tag{13}$$

Provided $\tau_s << \tau_1$. The significance of this relationship is that since α is known to be invariant with respect to gas flow velocity (Kee, *et al.*, 1999), it follows that β is also approximately invariant and can therefore be assumed to be constant over large data windows even if the time constants are not.

In light of this property of the β model (7), the following generalisation is proposed as a means of relaxing the constraint on time constant invariance over the sliding data window:

$$\Delta T_{m2}^{k} = \beta \Delta T_{m1}^{k} + b_2(k)\Delta T_{m12}^{k-1} \tag{14}$$

with

$$b_2(k) = b_{20} + kb_{21} + k^2 b_{22} + k^3 b_{23}, \tag{15}$$

where b_{2j} is the polynomial coefficient of the jth power term.

Here the constant b_2 is replaced by a 3rd order polynomial, $b_2(k)$ to capture the parameter variation within the data window. A low order polynomial is chosen in preference to other function approximators such as neural networks as it provides a reasonable compromise between algorithm complexity and approximation accuracy.

Given β and $b_2(k)$, the most reliable sliding window time-constant estimates can be obtained by evaluating $b_2(k)$ at the centre of the sliding data window, i.e. at $k = N/2$. Alternatively if the resulting $N/2$ sample delay cannot be tolerated the estimates can be computed for $k = N$.

Modifying the matrix-vector representation (9) to incorporate $b_2(k)$ gives

$$\boldsymbol{Y}_p = \boldsymbol{X}_p \boldsymbol{\theta}_p, \tag{16}$$

where

$$\begin{aligned} \boldsymbol{X}_p &= [\Delta \boldsymbol{T}_{m1}^{k}\ \Delta \boldsymbol{T}_{m12}^{k-1}\ \boldsymbol{k}\Delta \boldsymbol{T}_{m12}^{k-1}\ \boldsymbol{k}^2\Delta \boldsymbol{T}_{m12}^{k-1}\ \boldsymbol{k}^3\Delta \boldsymbol{T}_{m12}^{k-1}] \\ \boldsymbol{Y}_p &= \Delta \boldsymbol{T}_{m2}^{k}, \quad \boldsymbol{\theta}_p = [\beta\ b_{20}\ b_{21}\ b_{22}\ b_{23}]^T \end{aligned} \tag{17}$$

with the products $\boldsymbol{k}^j \Delta \boldsymbol{T}_{m12}^{k-1}$ defined as

$$\boldsymbol{k}^j \Delta \boldsymbol{T}_{m12}^{k-1} = [\Delta T_{m12}^{2}\ 2^j \Delta T_{m12}^{3} \cdots (N-1)^j \Delta T_{m12}^{N}]^T \tag{18}$$

In order to apply generalised total least squares (GTLS) to obtain unbiased parameter estimates with this extended model the corresponding noise covariance matrix must be computed as follows.

3.2 Computing the noise covariance matrix

Due to the introduction of the $b_2(k)$ polynomial the noise covariance matrix of the augmented data matrix $[\boldsymbol{X}_p\ \boldsymbol{Y}_p]$ contains terms of the form

$$E[k^j \eta_a^k \eta_b^k] \tag{19}$$

where $\eta_a^k, \eta_b^k \in \{\eta_{m1}^k, \eta_{m2}^k, \eta_{m12}^{k-1}\}$ are the random noise components of the composite signals defined in (8). If the noise components on the thermocouple measurements, $T_{m1}(k)$ and $T_{m2}(k)$, are assumed to be zero mean, white noise sequences with variances v_1 and v_2 respectively, then $\eta_a^k \eta_b^k$ will be independent of the deterministic signal k^j, hence

$$E[k^j \eta_a^k \eta_b^k] = E[k^j].E[\eta_a^k \eta_b^k]. \tag{20}$$

Taking into account the correlations between the composite noise signals it can be shown that

$$E[\eta_a^k \eta_b^k]/v_2 \in \{2\phi, \phi+1, 2\}, \tag{21}$$

where $\phi = v_1/v_2$ and is typically unity.

$E[k^j]$ can be computed over the N-sample sliding window as

$$E[k^j]=\frac{1}{N}\sum_{k=1}^{N}k^j=\frac{1}{N}\sum_{k=1}^{N}k!\binom{N+1}{k+1}S(j,k) \qquad (22)$$

where S is the Stirling number of the second kind

$$S(j,k)=\frac{1}{k!}\sum_{i=0}^{k-1}(-1)^i\binom{k}{i}(k-i)^j \cdot \qquad (23)$$

Using these expressions the noise covariance matrix can be computed as

$$\boldsymbol{C}_{p0}(\phi,N)=\begin{bmatrix} 2\phi & \phi & D_1\phi & D_2\phi & D_3\phi & 0 \\ \phi & \overline{\phi} & D_1\overline{\phi} & D_2\overline{\phi} & D_3\overline{\phi} & 1 \\ D_1\phi & D_1\overline{\phi} & D_2\overline{\phi} & D_3\overline{\phi} & D_4\overline{\phi} & D_1 \\ D_2\phi & D_2\overline{\phi} & D_3\overline{\phi} & D_4\overline{\phi} & D_5\overline{\phi} & D_2 \\ D_3\phi & D_3\overline{\phi} & D_4\overline{\phi} & D_5\overline{\phi} & D_6\overline{\phi} & D_3 \\ 0 & 1 & D_1 & D_2 & D_3 & 2 \end{bmatrix} \qquad (24)$$

where

$$\begin{aligned} &D_1=\tfrac{1}{2}\overline{N},\ D_2=\tfrac{1}{6}\overline{N}(2N+1),\ D_3=\tfrac{1}{4}(N)\overline{N}^2,\\ &D_4=\tfrac{1}{30}\overline{N}(2N+1)(3N^2+3N-1),\\ &D_5=\tfrac{1}{12}(N)\overline{N}^2(2N^2+2N-1),\\ &D_6=\tfrac{1}{42}\overline{N}(2N+1)(3N^4+6N^3-3N+1),\\ &\overline{\phi}=\phi+1 \quad \text{and} \quad \overline{N}=N+1. \end{aligned} \qquad (25)$$

4. SIMULATION RESULTS

A MATLAB® simulation of a two-thermocouple probe system (Fig. 2) was used to evaluate the performance of the proposed sliding window sensor characterisation algorithm. The thermocouples were modelled according to (1) and (2) with κ and m set to 4.9x10^5 and 0.415, respectively. Wire diameters d_1 and d_2 were chosen to be 12.5 μm and 25 μm so as to yield time constants (Fig. 3) of the order of 2.5 and 7 milliseconds, respectively. The simulated gas temperature and velocity were varied sinusoidally according to the equations:

$$\begin{aligned} T_g(t)&=75+45\sin(63t)\\ v(t)&=30+25\sin(12.5t) \end{aligned} \qquad (26)$$

and the resulting temperature measurements sampled every 2 milliseconds.

Fig. 3 shows the performance of the proposed β model polynomial parameter fitting sliding window algorithm (Beta PF) for noise free data and a

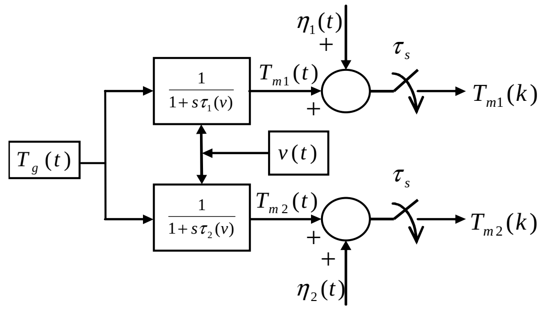

Fig. 2. Block diagram of the simulated two-thermocouple measurement system.

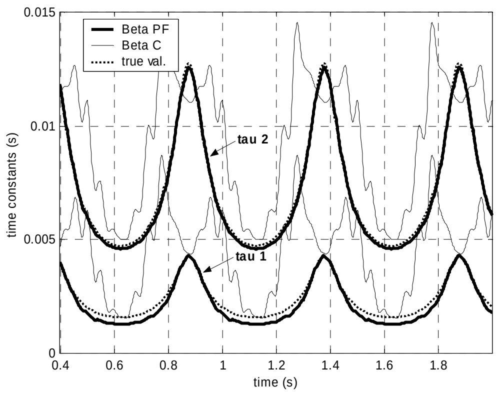

Fig. 3. Beta PF and Beta C sliding window time constant estimates for noise-free data with sinusoidal flow velocity fluctuations.

window size of 100 samples (0.2 seconds). Also included, for comparison purposes, are results for a constant parameter β model algorithm (Beta C). Note that while the velocity profile was chosen to be sinusoidal, the time constant profiles are not because of the non-linear relationship between gas flow velocity and time constant value (1).

The results clearly show the superiority of the Beta PF approach, which gives consistently good time constant estimates over the complete profile while the Beta C algorithm performs very poorly throughout.

The advantage of using polynomial rather than constant parameter fitting is further highlighted in Fig. 4. This shows the variation of b_2 over a typical data window together with the $b_2(k)$ polynomial and constant parameter estimates of this variation.

A series of 100 run Monte Carlo simulations were also performed to evaluate the performance of Beta PF in the presence of additive white measurement noise. The percentage noise level, K, defined as

$$K=100\sqrt{\frac{\mathrm{var}(\eta_j)}{\mathrm{var}(T_{mj})}}\% \qquad (27)$$

is used to quantify the amount of noise introduced. Here η_j is the measurement noise added to the *j*th thermocouple signal.

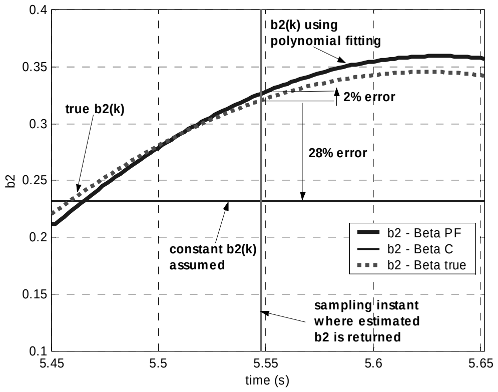

Fig. 4. Performance comparison of the Beta PF and Beta C models for a typical sliding window.

The GTLS based Beta PF results are benchmarked against a conventional least squares (LS) Beta PF implementation to illustrate the effect of bias. In addition results are presented for the constant parameter Beta algorithm (Beta C) and a benchmark time domain reconstruction (TDR) technique that relies on *a priori* knowledge of the time constant ratio, α . For details of the latter, refer to Kee, *et al.* (1999). In each case a sliding window of 100 samples is used.

Algorithm performance is measured in terms of the mean and standard deviation of $e_{\tau j}$, the percentage time constant estimation error defined as

$$e_{\tau j}(k) = \frac{100[\tau_j(k) - \hat{\tau}_j(k)]}{\tau_j(k)}\% , \qquad (28)$$

averaged over 1.4 seconds of data (700 samples) and both time constants. Here τ_j and $\hat{\tau}_j$ are the true and estimated values respectively.

The results (Table 1) clearly show that the GTLS Beta PF sliding window algorithm outperforms constant parameter alternatives at all noise levels. While TDR performs reasonably well, it does so by virtue of *a priori* knowledge of the time constant ratio, α, which allows parameter estimation to be reduced to a 1-dimensional search. This is reflected in the lower standard deviations observed with TDR. Allowing for the error that exists under noise free conditions (due to the limited approximation capabilities of cubic polynomials), a comparison of LS and GTLS Beta PF also shows that the later generates unbiased parameter estimates.

Table 1 Mean and (standard deviation) of the percentage estimation errors obtained with various sliding window sensor characterisation algorithms

Methods	*K* (%)				
	0	1	2	4	8
TDR (α)	2.57 *(0.00)*	2.80 *(0.53)*	3.54 *(1.11)*	6.40 *(2.42)*	18.62 *(6.30)*
Beta C (GTLS)	19.52 *(0.00)*	20.12 *(1.34)*	21.70 *(2.86)*	27.72 *(7.44)*	46.31 *(27.18)*
Beta PF (GTLS)	1.73 *(0.00)*	1.23 *(1.74)*	0.08 *(3.57)*	3.45 *(7.47)*	9.30 *(13.38)*
Beta PF (LS)	1.75 *(0.00)*	2.54 *(1.79)*	4.66 *(3.80)*	10.05 *(8.00)*	16.90 *(13.17)*

*Values in brackets are the mean percentage standard deviations

5. CONCLUSIONS

A novel sliding window difference equation based two-thermocouple sensor characterisation algorithm has been presented for variable velocity flow applications. By adopting a formulation that can exploit the time-constant ratio invariance property of such systems an efficient, unbiased polynomial parameter fitting method is developed to track time-constant variation within the sliding window. Simulation results confirm the superiority of the new algorithm over alternatives that assume fixed time constants over the sliding window.

6. ACKNOWLEDGEMENTS

All authors wish to acknowledge the financial support of the Virtual Engineering Centre, Queen’s University Belfast, *http://www.vec.qub.ac.uk.*

REFERENCES

Forney, L.J., G.C. Fralick (1994). Two wire thermocouple: Frequency response in constant flow. *Rev. Sci. Instrum.*, **65**, pp 3252-3257.

Hung, P. C. F., S. McLoone, G. Irwin and R. Kee (2003). A Total Least Squares Approach to Sensor Characterisations. *Proc. 13th IFAC Symposium on Sys. Id.*, Rotterdam, the Netherlands, pp 337-342.

Hung, P.C., S. McLoone, G. Irwin and R. Kee (2005). A difference equation approach to two-thermocouple sensor characterisation in constant velocity flow environments. *Rev. Sci. Instrum.*, **76**, pn 024902.

Kee R.J, P.G. O'Reilly, R. Fleck and P.T. McEntee (1999). Measurement of Exhaust Gas Temperature in a High Performance Two-Stroke Engine. *SAE Trans. J. Engines*, **107**, pn 983072.

Tagawa, M. and Y. Ohta (1997). Two-Thermocouple Probe for Fluctuating Temperature Measurement in Combustion – Rational Estimation of Mean and Fluctuating Time Constants. *Combustion and Flame*, **109**, pp 549-560.

Van Huffel S. and J. Vandewalle (1991). *The Total Least Squares Problem: Computational Aspects and Analysis,* SIAM, Philadelphia.