

# IMPACT AND FORCE CONTROL WITH SWITCHING BETWEEN MECHANICAL IMPEDANCE PARAMETERS

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Abstract: This work consists of two parts. First, a brief analysis active impedance control is made, deducing the value of each parameter in order to achieve good performance in free motion, impact and force control. Second, a new methodology of impact control is proposed. It consists in switching among different impedances, imposing for example a low mass while penetrating in the environment and a high mass while rebounding. The former dumps the impact limiting the peak of force while the latter avoids losses of contact. A similar consideration may be made for the stiffness. The dissipativity of the system is demonstrated. Copyright © 2005 IFAC

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## 1. INTRODUCTION<sup>1</sup>

In robot force control there is an abrupt change from free to constrained motion. Typically, the environment is very stiff; therefore the dynamics of the system is much faster during the constrained motion. In the transient phase (impact) there may be dangerous peaks of force and/ or bouncing.

During the first phase the velocity should be controlled, and during the second one the force. It seems logical to use different controllers for each phase. Switching among the controllers is another potential source of bouncing.

In order to make the transition from free to constrained motion as smooth as possible a third controller may be introduced. It is called impact control and it was studied exhaustively in (Brogliato 1999).

An alternative to switching among controllers is impedance control (Hogan, 1985). Its objective is to impose a desired dynamics to the robot rather than controlling directly the force. Its main advantage is that the same controller can be used for both free and restricted motion. It has been stated in many sources, for example Tarn et al. (1996) and De Schutter et al. (1997), that the main disadvantage of impedance control is that it is necessary to have an exact model of the environment in order to reach the force reference, what is impossible for real applications.

In this article is, at first, deduced the way to select the impedance parameters in order to overcome this limitation. Second, it is proposed to switch the parameters of the impedance form depending on robot position, velocity and acceleration, in order to achieve better impact control what will be better explained in section 3.

The methodology described guarantees an improvement of the damping of the system without using the force derivative. In the ideal case, it even does not need a force sensor for accurate force control.

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## 2. SOME CONSIDERATIONS REGARDING IMPEDANCE CONTROL

The mechanical impedance is the relation between the force and the velocity of the system:

$$Z(s) = \frac{F_{ext}(s)}{v(s)} = \frac{Ms^2X + BsX + KX}{sX} = Ms + B + \frac{K}{s} \quad (1)$$

Where  $Z$  represents the impedance,  $F_{ext}$  the external force,  $v$  the velocity,  $X$  the position,  $M$  the mass,  $B$  the damping and  $K$  the stiffness of the system.

The impedance control consists in imposing to the system the desired mass, damping and stiffness ( $M_d$ ,  $B_d$  and  $K_d$  respectively) instead of the real ones.

This may be realised in several ways: by lineal position feedback, inverse dynamics (both methods described in (Volpe, 1993) and (Sciavicco and Siciliano, 1996) or sliding mode control (Lu and Goldenberg, 1995). The description of these methods is beyond the scope of this article.

Several formulations are used for the mechanical impedance. The commonly used have been enumerated by (Volpe, 1990):

$$\begin{aligned} F_{ext} &= M_d(\ddot{x} - \ddot{x}_{ref}) + B_d(\dot{x} - \dot{x}_{ref}) + K_d(x - x_{ref}) \\ F_{ext} &= M_d\ddot{x} + B_d\dot{x} + K_d(x - x_{ref}) \\ F_{ext} &= M_d\ddot{x} + B_d(\dot{x} - \dot{x}_{ref}) + K_d(x - x_{ref}) \\ F_{ext} &= B_d\dot{x} + K_d(x - x_{ref}) \\ F_{ext} &= B_d(\dot{x} - \dot{x}_{ref}) + K_d(x - x_{ref}) \end{aligned} \quad (2)$$

The first formulation is the most general one and all the others may be deduced from it. It may be written in the form:

$$\begin{aligned} F_{ext} &= M_d\ddot{x} + B_d\dot{x} + K_d x - M_d\ddot{x}_{ref} + B_d\dot{x}_{ref} + K_d x_{ref} \\ F_{ext} &= M_d\ddot{x} + B_d\dot{x} + K_d x - ref \end{aligned} \quad (3)$$

Following will be made an analysis of the influence of each parameter in constrained and free motion in order to select the optimal values.

### 2.1 The constrained motion

It will be assumed that the environment is immobile, and that the deformation is elastic. Both things are true for many applications.

In this case, the reaction force of the environment will be:

$$F_{ext} = -K_e(x - x_e) - B_e\dot{x} \quad (4)$$

The dynamics of the position in constrained motion will be:

$$X(s) = \frac{ref + K_e x_e}{M_d s^2 + (B_d + B_e)s + K_d + K_e} \quad (5)$$

And the force:

$$F_{ext}(s) = \frac{(ref + K_e x_e)(K_e + B_e s)}{M_d s^2 + (B_d + B_e)s + K_d + K_e} \quad (6)$$

The roots of the characteristic polynomial are:

$$s_{1,2} = \frac{-(B_d + B_e) \pm \sqrt{(B_d + B_e)^2 - 4(K_d + K_e)M_d}}{2M_d} \quad (7)$$

and the discriminant:

$$(B_d + B_e)^2 - 4(K_d + K_e)M_d \quad (8)$$

In order to make the system as damped as possible, it is convenient to choose a high value for  $B_d$  and low values for  $M_d$  and  $K_d$ .

The final value of the force will be:

$$\begin{aligned} F_{\infty} &= K_e \frac{ref + K_d x_e}{K_d + K_e} & K_d \neq 0 \\ F_{\infty} &= ref & K_d = 0 \end{aligned} \quad (9)$$

This means that choosing  $K_d = 0$  not only damps the system, but also allows us to reach the reference value of force.

### 2.2 The free motion

In this case  $F_{ext} = 0$  and the behaviour of the system is:

$$X(s) = \frac{ref}{M_d s^2 + B_d s + K_d} \quad (10)$$

The final values will be in this case:

$$\begin{aligned} x_{\infty} &= \frac{ref}{K_d} & K_d \neq 0 \\ \dot{x}_{\infty} &= \frac{ref}{B_d} & K_d = 0 \wedge B_d \neq 0 \end{aligned} \quad (11)$$

Therefore, choosing a stiffness different of zero will take the robot to a given distance of the origin where it will remain. Choosing the stiffness equal to zero will make the system go to a constant velocity.

The latter is more practical for impact control since it does not require previous knowledge about the position of the environment.

On the other hand, the damping should be chosen the way to assure the desired final velocity  $v_{ref}$ :

$$B_d = \frac{ref}{v_{ref}} \quad (12)$$

### 2.3 Conclusions about the impedance parameters

The previous conclusions can be resumed the following way:

The value of  $ref$  should be selected equal to the reference force in order to reach it (9). This is possible only if the stiffness is zero. It should be emphasized that an asymptotic stability of the force is guaranteed regardlessly of the characteristics of the environment.

The stiffness  $K_d$  should be fixed to zero for the previously stated reason also to damp the system in constrained motion. In free motion, a stiffness different than zero would stop the system after some distance instead of guarantee constant speed.

The damping  $B_d$  should be as high as possible in constrained motion in order to damp the impact. On the other hand, in free motion its value is determined by the desired velocity and reference force value (eq.12). A very high value would make the speed of the system too slow.

Regarding the mass  $M_d$ , it is the only parameter that practically has no importance during free motion. The value assigned to the mass should be low in order to damp the system during the impact.

Briefly the values to be assigned are the following:

- $ref=F_{ref}$
- $K_d=0$ .
- $B_d$  according to equation 12 in order to attain the reference speed.
- $M_d$  as small as possible.

It is worthy emphasizing that usually the active damping is adjusted for impact control. In this case this is achieved by means of the mass. The damping is used for velocity control.

### 3. SWITCHING AMONG IMPEDANCE VALUES

Even with the parameters selected as previously explained, force overshoots and bouncing may occur.

Assuming the characteristics of the environment are unknown it is not possible to design the controller the way to guarantee the system will be overdamped.

In this section will be proposed a method for additional smoothing of the transient phase. It consists of the switching among impedance parameters.

Of the three parameters, the damping is the one whose influence is the most obvious: the impact is smoother as its value is higher. Nevertheless, to high

values make the system slower. Also, sometimes it may be hard to implement due to the very low values of speed during constrained motion.

The effect of the mass is ambiguous: in the instant of the impact it is convenient its value to be low in order to have less kinetic energy to dissipate from free motion. On the other hand, to avoid bouncing, it is convenient to have a big mass to be opposed to the reaction of the environment.

The stiffness also has an ambiguous influence: a high value will provoke high peaks of force. A low value may lead to bouncing.

In a damping- less system the total energy would be:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad (13)$$

That represents an ellipse in the phase plane.

Assuming there is no loss of energy, the intersecting points with the abscissa and the ordinate will be, respectively:

$$\begin{aligned} x(\dot{x} = 0) &= \pm \sqrt{\frac{2E}{K}} \\ \dot{x}(x = 0) &= \pm \sqrt{\frac{2E}{m}} \end{aligned} \quad (14)$$

It means that if the mass is smaller then the stiffness, the intersection points with the abscissa will be closer to the origin than the intersection points with the ordinate. Therefore, the ellipse that represents the system in the phase plane will be higher than its width, like represented in figure 1.

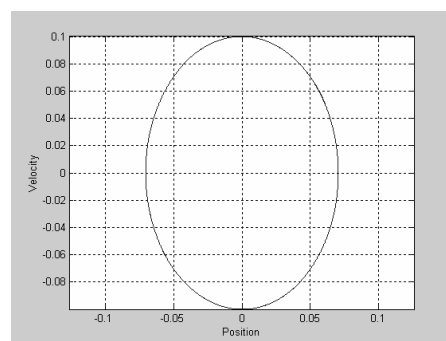


Fig. 1. Phase plane diagram of a system with high stiffness and low mass.

If the mass higher than the stiffness, the width of the ellipse will be greater than its height (see figure 2).

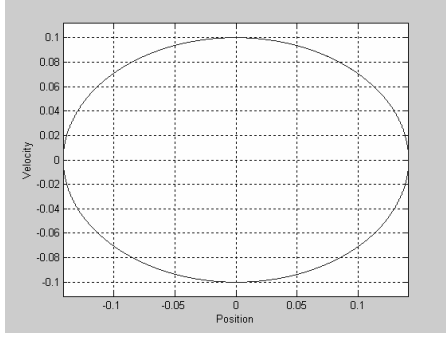


Fig. 2. System with high mass and low stiffness.

Taking into account that impedance control allows the imposition the dynamic parameters of the system, it is possible to use one system in the second and fourth quadrant (mass  $m_1$  and stiffness  $k_1$ ) and another system in the first and third quadrant. ( $m_2$  y  $k_2$ ) with:

$$m_1 > m_2, \quad k_1 < k_2 \quad (15)$$

The total energy will be:

$$E_{13} = \frac{1}{2} m_2 v^2 + \frac{1}{2} k_2 x^2 \quad (16)$$

In the first and third quadrant, and:

$$E_{24} = \frac{1}{2} m_1 v^2 + \frac{1}{2} k_1 x^2 \quad (17)$$

In the second and fourth.

Assuming the robot penetrates the environment in the second quadrant with initial energy  $E_0$ , the velocity in the intersection with the ordinate axis will be:

$$\dot{x}_0 = \sqrt{\frac{2E_0}{m_1}} \quad (18)$$

Switching to the other system in this instant will be:

$$E_1 = \frac{1}{2} m_2 \dot{x}_0^2 = \frac{m_2}{m_1} E_0 < E_0 \quad (19)$$

The next switching should take place at intersecting the abscissa. The position of the robot will be then:

$$x_1 = \sqrt{\frac{2E_1}{k_2}} \quad (20)$$

The energy after switching will be:

$$E_2 = \frac{1}{2} k_1 x_1^2 = \frac{k_1}{k_2} E_1 < E_1 \quad (21)$$

It can be deduced:

$$E_{2k} = \frac{k_1}{k_2} E_{2k-1} = \frac{k_1}{k_2} \frac{m_2}{m_1} E_{2k-2} = \dots = \left( \frac{k_1}{k_2} \frac{m_2}{m_1} \right)^k E_0$$

$$E_{2k-1} = \frac{m_2}{m_1} \left( \frac{k_1}{k_2} \frac{m_2}{m_1} \right)^{k-1} E_0 \quad (22)$$

The coordinate of the intersection points with the abscissa axe:

$$x_{2k-1} = \sqrt{\frac{2E_{2k-1}}{k_2}} = \sqrt{\frac{2 \left( \frac{k_1}{k_2} \frac{m_2}{m_1} \right)^{k-1} E_0}{k_2}} =$$

$$= \sqrt{\left( \frac{k_1}{k_2} \frac{m_2}{m_1} \right)^{k-1}} x_{2k-3} \quad (23)$$

What demonstrates each intersection of the x axe is closer to the origin than the previous, what reduces the possibility of bouncing.

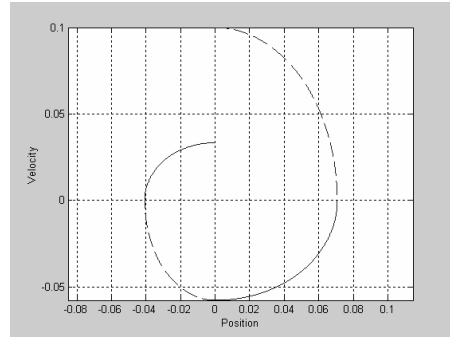


Fig. 3. Phase plane diagram of a system with switching among impedances, starting from the point (0.1, 0). The full and dashed lines represent the trajectory of the system with high mass and stiffness respectively.

The following three advantages of switching among impedances can be deduced:

1. The force peak is smaller.
2. The possibility of contact loss is also smaller.
3. The convergence of the system is faster.

As it is seen previously (Equation 9.) adding active stiffness displaces the equilibrium point of the system, which makes the previous reasoning incomplete. Also, it makes the system more underdamped. A solution could be not to use active stiffness.

In this case the switching would be done only among the masses.

The energy of the system would be then:

$$E_{2k} = E_{2k-1} = \frac{m_2}{m_1} E_{2k-2} = \dots = \left( \frac{m_2}{m_1} \right)^k E_0 \quad (24)$$

$$E_{2k-1} = \frac{m_2}{m_1} E_{2k-2} = \left( \frac{m_2}{m_1} \right)^k E_0$$

And the intersection with the  $x$  axe:

$$x_{2k-1} = \sqrt{\left(\frac{m_2}{m_1}\right)} x_{2k-3} \quad (25)$$

Comparing with (23) it may be deduced that the bouncing is more probable than if the stiffness is also switched. Nevertheless it is less probable than in the case without any switching.

All the previous analysis was made for a system without damping.

In the general case, the energy of the system is:

$$V = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \quad (26)$$

Therefore:

$$\dot{V} = m \ddot{x} \dot{x} + k x \dot{x} \quad (27)$$

In order to increase the energy dissipation the following switching laws should be applied:

$$\begin{aligned} \ddot{x} < 0 & \quad m = m_1 \\ \ddot{x} > 0 & \quad m = m_2 & \quad m_1 > m_2 \\ \dot{x} < 0 & \quad k = k_2 & \quad k_1 < k_2 \\ \dot{x} > 0 & \quad k = k_1 \end{aligned} \quad (28)$$

#### 4. EXPERIMENTAL RESULTS

An experimental platform was developed in order to verify the previous conclusions.

It consists of a Quanser linear axe on which was mounted a JR3 force/ torque sensor. In order to measure the speed, a Maxon tachometer was coupled to the axe.

The system is controlled by a PC under the real-time operating system MARTE (Aldea M. and González M., 2001). The control routines were programmed in ADA while the drivers for the data acquisition boards were made in C.

For all the experiments, the reference value ( $ref$  in the equation (3)) was 0.5 V, and the control action was saturated to  $\pm 1.0V$  for safety reasons.

The measurements were realized impacting the robot against a book, in order to reduce the risk of force overshoots that could damage the system.

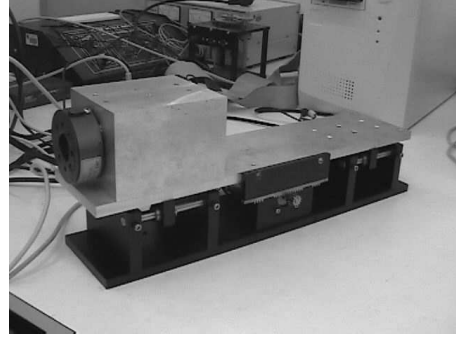


Fig. 4. Linear axe with force sensor.

Four experiments were made. The first one consisted of an impact, without any change in the system characteristics. In next two experiments, a different mass of the real one was imposed in order to verify the effect of the mass in the behaviour of the system. Concretely, in the second experiment the imposed mass was 0.9 of the real one, and in the third one it was 1.2. In the last experiment switching between the values of the mass was performed according to (28).

These values were intentionally selected in order to have a considerable overshoot rather than to achieve good performance. This is done for a better and easier comparison of the different control techniques described in this article.

The results are represented in the following figures:

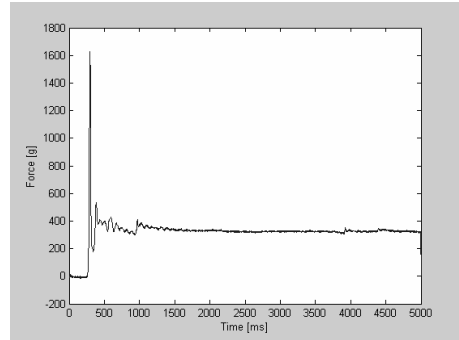


Fig. 5. Response of the system without any change of the parameters of the system.

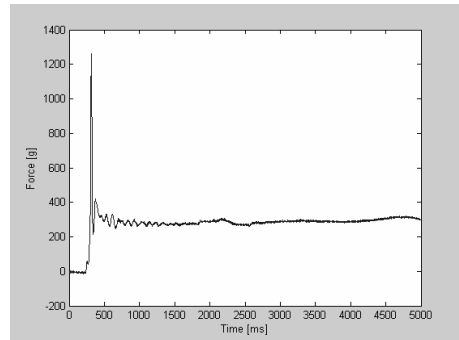


Fig. 6. Response of the system imposing a mass of 90% of the real.

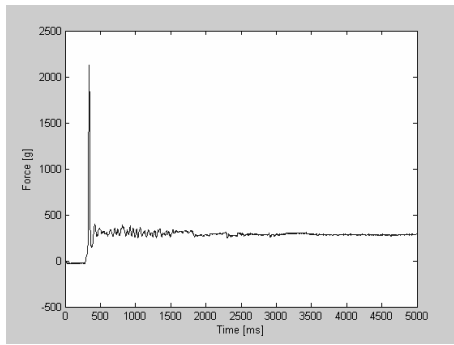


Fig. 7. Response of the system imposing a mass of 120% of the real.

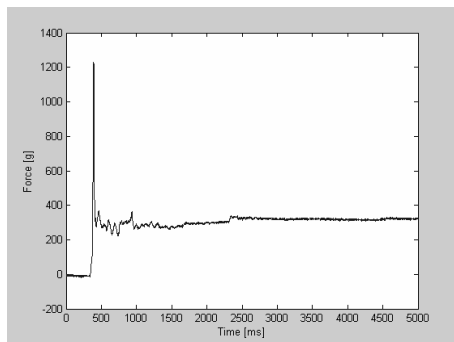


Fig. 8. Response of the system imposing switching between masses of 90% and 120% of the real one.

It can be verified in Fig. 5 and 6 that the overshoot is directly proportional with the mass.: for a the real mass the force peak is 1600, reducing the mass 0.9 times it reaches around 1200, and increasing it 1.2 times it is higher than 2000 grams.

Comparing Fig. 5 and Fig. 7 it can be seen that the first peak approximately equal in both but that the subsequent peaks are smaller in Fig. 7. The former occurs because the same mass is used in both cases until the velocity changes of direction. The latter happens due to the switching of impedances.

## 5. CONCLUSIONS

At first, an analysis of the impedance control was made. It has been demonstrated that all three phases can be controlled with the same impedance controller, even without previous knowledge of the environment characteristics.

Contrary to the usual trend in impedance control, the damping is used for velocity control in the free motion. The behaviour of the system during the impact is controlled by the mass imposed to the system.

Assigning a zero value to the stiffness allows reaching the reference value of the force.

Adjusting the three parameters this way makes possible a control of velocity in free motion, impact damping and in the ideal case the force control.

An improvement of the previous results can be made by means of switching between parameters of the mechanical impedance.

This is done basically imposing a high mass when kinetic energy is decreasing and a low one when it is increasing.

With the switching between impedances, the damping of the system can only been improved.

The conclusions were experimentally verified with a platform specially made for this purpose. It consisted basically in a linear axe with a force sensor.

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