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New Delay-dependent Stability Criteria for T-S Fuzzy Systems with a Time-varying Delay ⋆

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Abstract: This paper deals with the asymptotic stability problem of uncertain T-S fuzzy systems with time-varying delay by employing a further improved free-weighting matrix approach. The relationship among the time-varying delay, its upper bound and their difference is taken into account. As a result, some less conservative LMI-based delay-dependent stability criteria are obtained without ignoring any useful terms in the derivative of Lyapunov-Krasovskii functional. Finally, two numerical examples are given to demonstrate the effectiveness and the merits of the proposed methods.

1. INTRODUCTION

During the past two decades, the stability for Takagi-Sugeno(T-S) fuzzy systems(Takagi [1985]) has been attracted an increasing attention since it can combine the flexibility of fuzzy logic theory and rigorous mathematical theory of linear or nonlinear system into a unified framework. Lots of asymptotic stable criteria of T-S fuzzy systems have been expressed in linear matrix inequalities(LMIs) via various stability analysis methods(see Chen [2000], Tanaka [1994], Teixeira [1999], Wang [1996], and the reference therein). However, all the aforementioned criteria aim at time-delay free T-S fuzzy systems. In practice, time-delays often occur in many dynamic systems such as chemical processes, metallurgical processes, biological systems, and mechanics. Furthermore, the existence of time-delays is usually a source of instability and deteriorated performance. As a result, to study the stability analysis for T-S fuzzy systems has not only important theoretical interesting but also practical value, which has received more interesting in recent years(Akar [2000], Cao [2001], Chang [2004], Guan [2004], Jiang [2005], Li [2004]), Tian [2006], Yoneyama [2003].

Among these literatures, stability criteria for T-S fuzzy systems can be classified into two types: delay-dependent criteria and delay-independent criteria. Since delay-depend -ent criteria make use of information on the length of delays, they are less conservative than delay-independent ones, especially when time-delay is small. The delay-dependent stabilization was first discussed in Guan [2004] for nominal T-S fuzzy time-delay systems with time-invariant delay based on Lyapunov-Krasovkii functional approach and Moon et al's inequality. Although the stability problem of uncertain T-S fuzzy systems with time-varying delay has been studied in Li [2004], there also retain further room to research. For example, the derivative of $\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z\dot{x}(s)dsd\theta$ with $0 \le d(t) \le h$ is usually estimated as $h\dot{x}^T(t)Z\dot{x}(t) - \int_{t-d(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds$ and the term $\int_{t-h}^{t-d(t)}\dot{x}^T(s)Z\dot{x}(s)ds$ is ignored, which may lead to considerable conservativeness. Besides, the delay term $d(t)$ with $0 \le d(t) \le h$ is often enlarged as h, and another term $h - d(t)$ is also regarded as h, i.e. $h = d(t) + h - d(t)$ is enlarged as $2h$, which may also lead to conservativeness.

Recently, a free-weighting matrix approach (He et al. [2004], He et al. [2007]) has been employed to study the stability problem for delay systems, and some less conservative stability criteria have been derived. Enlightened by it, in this paper, we discuss the asymptotic stability for T-S fuzzy system with a time-varying delay by employing free-weighting matrix approach. Under considering the relationship among the time-varying delay, its upper bound and their difference, some improved LMI-based asymptotic stability criteria for uncertain T-S fuzzy system with a time-varying delay are obtained without ignoring any useful terms in the derivative of a Lyapunov-Krasovkii function. Finally, two numerical examples are given to

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demonstrate the effectiveness and merits of the proposed method.

Notion: Through this paper, N^T and N^{-1} stands for the transpose and the inverse of the matrix N, respectively; $\mathcal{R}^n$ denotes the $n-$dimensional euclidean space; $P > 0$ means that the matrix P is positive definite; diag$\{\cdots\}$ denotes a block-diagonal matrix; $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ stands for $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$; $\|x\|$ is the Euclidean norm of x.

2. SYSTEM DESCRIPTION

Consider fuzzy system with a time-varying delay, which is represented by a T-S fuzzy model composed of a set of fuzzy implications, and each implication is expressed by a linear system model. The ith rule of this T-S fuzzy model is of the following form:

Rule i:

If $\Theta_1(t)$ is μ_{i1} and $\cdots$ and $\Theta_p(t)$ is μ_{ip} then

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - d(t)) \\ x(t) = \phi(t) \quad t \in [-h, 0] \quad i = 1, 2, \ldots, r \end{cases} \tag{1}$$

where μ_{ij} is the fuzzy set; $x(t) \in \mathcal{R}^n$ is the state vector; A_i and A_{di} are constant real matrices with appropriate dimensions; scalar r is the number of IF-Then rules; $\Theta_1(t), \Theta_2(t), \cdots, \Theta_p(t)$ are the premise variables; $d(t)$, is the time-varying delay satisfying

$$0 \le d(t) \le h \tag{2}$$

$$\dot{d}(t) \le \mu \tag{3}$$

where μ and h are constants. In addition, the matrices $\Delta A_i(t)$ and $\Delta A_{di}(t)$ denote the uncertainties in the system and take the form of

$$[\Delta A_i(t) \ \Delta A_{di}(t)] = DF(t)[E_i \ E_{di}] \tag{4}$$

where D,E_i and E_{di} are known constant matrices and $F(t)$ is an unknown matrix function with Lesbesgue measurable elements bounded by:

$$F^T(t)F(t) \le I, \ \forall t, \tag{5}$$

where I is an appropriately dimensioned identity matrix. By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{cases} \dot{x}(t) = \dfrac{\sum_{i=1}^{r} w_i(\theta(t))[(A_i+\Delta A_i(t))x(t)+(A_{di}+\Delta A_{di}(t))x(t-d(t))]}{\sum_{i=1}^{r} w_i(\theta(t))} \\ \quad = \sum_{i=1}^{r} \rho_i(\theta(t))[(A_i + \Delta A_i(t))x(t) \\ \qquad + (A_{di} + \Delta A_{di}(t))x(t - d(t))] \\ \quad = \bar{A}_i x(t) + \bar{A}_d x(t - d(t)) \\ x(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{6}$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_p]; w_i : \mathcal{R}^p \to [0 \ 1], i = 1, \ldots, r$, is the membership function of the system with respect to the plant rule i; $\rho_i(\theta(t)) = w_i(\theta(t))/\sum_{i=1}^{r} w_i(\theta(t))$; and $\bar{A}_i = \sum_{i=1}^{r} \rho_i(\theta(t))(A_i + \Delta A_i(t)), \bar{A}_d = \sum_{i=1}^{r} \rho_i(\theta(t))(A_{di} + \Delta A_{di}(t))$. It is obvious that the fuzzy weighting functions $\rho_i(\theta(t))$ satisfy $\rho_i(\theta(t)) \ge 0, \sum_{i=1}^{r} \rho_i(\theta(t)) = 1$.

In order to obtain the main results, the following lemmas will be employed in the proofs of our results.

Lemma 1(Schur complement): Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T$, then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0$$

or

$$\begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Lemma 2: Let $Q = Q^T, H, E, \ R = R^T > 0$ and $F(t)$ satisfying $F^T(t)F(t) \le I$ are appropriately dimensional matrices,, then the following inequality

$$Q + HF(t)E + E^T F^T(t) H^T < 0$$

is true, if and only if the following inequality holds for any $\varepsilon > 0$,

$$Q + \varepsilon^{-1} H H^T + \varepsilon E^T R E < 0.$$

3. MAIN RESULTS

In this section, we shall obtain the stability criteria for T-S fuzzy system with a time-varying delay based on the improved free-weighting matrix approach. Our first result in this paper deals with the stability of (1) with $\Delta A_i(t) = 0$, and $\Delta A_{di}(t) = 0$, i.e.,

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d(t)) \\ x(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{7}$$

where $A = \sum_{i=1}^{r} \rho_i(\theta(t))A_i, A_d = \sum_{i=1}^{r} \rho_i(\theta(t))A_{di}$.

Based on the Lyapunov-Krasovkii stability theorem, the following main result is obtained.

Theorem 1. For given a scalar $h \ge 0$ and μ, the system (7) with a time-delay $d(t)$ satisfying (2) and (3) is stable if there exist $P = P^T > 0$, $Q = Q^T > 0$, $W = W^T > 0$, $Z = Z^T > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \ge 0$, and any appropriately dimensioned matrices, $N = [N_1^T \ N_2^T]^T$, and $M = [M_1^T \ M_2^T]^T$, such that the following LMIs are feasible for $i = 1, 2, \ldots, r$:

$$\Phi_i = \begin{bmatrix} \Phi_{11} & \Phi_{12} & -M_1 & hA_i^T Z \\ * & \Phi_{22} & -M_2 & hA_{di}^T Z \\ * & * & -W & 0 \\ * & * & * & -hZ \end{bmatrix} < 0 \tag{8}$$

$$\Psi_1 = \begin{bmatrix} X & N \\ * & Z \end{bmatrix} \ge 0 \tag{9}$$

$$\Psi_2 = \begin{bmatrix} X & M \\ * & Z \end{bmatrix} \ge 0 \tag{10}$$

where

$$\Phi_{11} = PA_i + A_i^T P + Q + W + N_1 + N_1^T + hX_{11}$$

$$\Phi_{12} = PA_{di} - N_1 + N_2^T + M_1 + hX_{12}$$

$$\Phi_{22} = -(1-\mu)Q - N_2 - N_2^T + M_2 + M_2^T + hX_{22}.$$

Proof : Choose the fuzzy weighting-dependent Lyapunov-Krasovskii functional candidate as:

$$\begin{aligned} V(x_t) &= x^T(t)Px(t) + \int_{t-d(t)}^{t} x^T(s)Qx(s)ds \\ &+ \int_{t-h}^{t} x^T(s)Wx(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\theta, \end{aligned} \tag{11}$$

where $P = P^T > 0$, $Q = Q^T > 0$, $W = W^T > 0$, and $Z = Z^T > 0$ are to be determined.

Using the Newton-Leibniz formula, for any appropriately dimensioned matrices N, M, the following equation is true:

$$0 = 2\zeta_1^T(t)N \cdot \left[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)ds \right] \tag{12}$$

$$0 = 2\zeta_1^T(t)M \cdot \left[x(t-d(t)) - x(t-h) - \int_{t-h}^{t-d(t)} \dot{x}(s)ds \right] \tag{13}$$

where

$$\zeta_1(t) = [x^T(t) \quad x^T(t-d(t))]^T.$$

.

On the other hand, for any semi-positive definite matrix

$$X = X^T = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$$

the following equation holds:

$$\begin{aligned} 0 &= \int_{t-h}^{t} \zeta_1^T(t)X\zeta_1(t)ds - \int_{t-h}^{t} \zeta_1^T(t)X\zeta_1(t)ds \\ &= h\zeta_1^T(t)X\zeta_1(t) - \int_{t-d(t)}^{t} \zeta_1^T(t)X\zeta_1(t)ds \\ &- \int_{t-h}^{t-d(t)} \zeta_1^T(t)X\zeta_1(t)ds. \end{aligned} \tag{14}$$

In addition, it is clear that the following equation is also true:

$$\begin{aligned} \int_{t-h}^{t} \dot{z}^T(s)Z\dot{z}(s)ds &= \int_{t-d(t)}^{t} \dot{z}^T(s)Z\dot{z}(s)ds \\ &+ \int_{t-h}^{t-d(t)} \dot{z}^T(s)Z\dot{z}(s)ds \end{aligned} \tag{15}$$

Using (15) and calculating the derivatives of $V(x_t)$ defined in (11) along the trajectories of system (7) yields

$$\begin{aligned} \dot{V}(x_t) =& x^T(t)[PA + A^TP]x(t) \\ &+2x^T(t)PA_dx(t-d(t)) + x^T(t)Qx(t) \\ &-(1-\dot{d}(t))x^T(t-d(t))Qx(t-d(t)) \\ &+x^T(t)Wx(t) - x^T(t-h)Wx(t-h) \\ &+h\dot{x}^T(t)Z\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \\ =& x^T(t)[PA + A^TP]x(t) \\ &+2x^T(t)PA_dx(t-d(t)) + x^T(t)Qx(t) \\ &-(1-\dot{d}(t))x^T(t-d(t))Qx(t-d(t)) \\ &+x^T(t)Wx(t) - x^T(t-h)Wx(t-h) \\ &+h\dot{x}^T(t)Z\dot{x}(t) - \int_{t-d(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \\ &- \int_{t-h}^{t-d(t)} \dot{x}^T(s)Z\dot{x}(s)ds \end{aligned} \tag{16}$$

Then, adding the terms on the right of equations (12), (13), and (14) to $\dot{V}(z(t))$ yields:

$$\begin{aligned} \dot{V}(x_t) \leq& x^T(t)[PA + A^TP]x(t) \\ &+2x^T(t)PA_dx(t-d(t)) + x^T(t)Qx(t) \\ &-(1-\mu)x^T(t-d(t))Qx(t-d(t)) \\ &+x^T(t)Wx(t) - x^T(t-h)Wx(t-h) \\ &+h\dot{x}^T(t)Z\dot{x}(t) - \int_{t-d(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \\ &- \int_{t-h}^{t-d(t)} \dot{x}^T(s)Z\dot{x}(s)ds \\ &+2\zeta_1^T(t)N \cdot \left[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)ds \right] \\ &+2\zeta_1^T(t)M \cdot \left[x(t-d(t)) - x(t-h) - \int_{t-h}^{t-d(t)} \dot{x}(s)ds \right] \\ &+h\zeta_1^T(t)X\zeta_1(t) - \int_{t-d(t)}^{t} \zeta_1^T(t)X\zeta_1(t)ds \\ &- \int_{t-h}^{t-d(t)} \zeta_1^T(t)X\zeta_1(t)ds \end{aligned} \tag{17}$$

$$\begin{aligned} =& \zeta^T(t)\Xi\zeta(t) - \int_{t-d(t)}^{t} \eta^T(t,s)\Psi_1\eta(t,s)ds \\ &- \int_{t-h}^{t-d(t)} \eta^T(t,s)\Psi_2\eta(t,s)ds \end{aligned} \tag{18}$$

where

$$\begin{aligned} \zeta(t) &= [x^T(t) \quad x^T(t-d(t)) \quad x^T(t-h)] \\ \eta(t,s) &= [\zeta^T(t) \quad \dot{x}^T(s)]^T \end{aligned}$$

$$\Xi = \begin{bmatrix} \widetilde{\Phi}_{11} + hA^T ZA & \widetilde{\Phi}_{12} + hA^T ZA_d & -M_1 \\ * & \widetilde{\Phi}_{22} + hA_d^T ZA_d & -M_2 \\ * & * & -W \end{bmatrix}$$
$$\widetilde{\Phi}_{11} = PA + A^T P + Q + W + N_1 + N_1^T + hX_{11}$$
$$\widetilde{\Phi}_{12} = PA_d - N_1 + N_2^T + M_1 + hX_{12}$$
$$\widetilde{\Phi}_{22} = -(1-\mu)Q - N_2 - N_2^T + M_2 + M_2^T + hX_{22}$$

and $\Psi_i (i = 1, 2)$ are defined in (9) and (10). If $\Xi < 0$ and $\Psi_i \geq 0, (i = 1, 2)$, then $\dot{V}(z(t)) < -\varepsilon \|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$. By Schur complement, $\Xi < 0$ is equivalent to the following inequality is true.

$$\widetilde{\Phi} = \begin{bmatrix} \widetilde{\Phi}_{11} & \widetilde{\Phi}_{12} & -M_1 & hA^T Z \\ * & \widetilde{\Phi}_{22} & -M_2 & hA_d^T Z \\ * & * & -W & 0 \\ * & * & * & -hZ \end{bmatrix} < 0. \tag{19}$$

That is to say, if $\widetilde{\Phi} < 0$ and $\Psi_i(i = 1, 2) \geq 0$, then $\dot{V}(z(t)) < -\varepsilon \|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$. Furthermore, (8) implies $\sum_{i=1}^{r} \rho_i(\theta(t))\Phi_i < 0$, which is equivalent to (19). Therefore, if LMIs(8), (9) and (10) are feasible, the system (7) is asymptotically stable. This completes the proof of Theorem 1.

Based on Theorem 1, we have the following result for uncertain T-S fuzzy system (1).

Theorem 2. For given a scalar $h \geq 0$ and μ, the system (1) with uncertainty described by (4) is stable if there exist $P = P^T > 0$, $Q = Q^T > 0$, $W = W^T > 0$, $Z = Z^T > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \geq 0$, and any appropriately dimensioned matrices, $N = [N_1^T \; N_2^T]^T$, and $M = [M_1^T \; M_2^T]^T$ and a scalar $\lambda > 0$, such that the following LMIs are feasible for $i = 1, 2, \ldots, r$:

$$\begin{bmatrix} \Phi_{11} + \lambda E_i^T E_i & \Phi_{12} + \lambda E_i^T E_{di} & -M_1 & hA_i^T Z & PD \\ * & \Phi_{22} + \lambda E_{di}^T E_{di} & -M_2 & hA_{di}^T Z & 0 \\ * & * & -W & 0 & 0 \\ * & * & * & -hZ & hZD \\ * & * & * & & -\lambda D \end{bmatrix} < 0 \tag{20}$$

$$\Psi_1 = \begin{bmatrix} X & N \\ * & Z \end{bmatrix} \geq 0 \tag{21}$$

$$\Psi_2 = \begin{bmatrix} X & M \\ * & Z \end{bmatrix} \geq 0 \tag{22}$$

where $\Phi_{11}, \Phi_{12}, \Phi_{13}$, and $\Psi_i(i = 1, 2)$ are defined in (8), (9) and (10).

Proof: Replacing A and A_{di} with $A_i + DF(t)E_i$ and $A_{di} + DF(t)E_{di}$ in (8), respectively, the corresponding formula of (8) of the system (1) can be written as follows:

$$\Phi + \begin{bmatrix} PD \\ 0 \\ 0 \\ hZD \end{bmatrix} F(t) \,[\, E_i \;\; E_{di} \;\; 0 \;\; 0 \,] + \begin{bmatrix} E_i^T \\ E_{di}^T \\ 0 \\ 0 \end{bmatrix} F^T(t) \left[\, D^T P \;\; 0 \;\; 0 \;\; hD^T Z \,\right] < 0. \tag{23}$$

According to *Lemma 2*, it is easy to know that (23) is true if there exist a positive number $\lambda > 0$ to make the following inequality hold:

$$\Phi + \lambda^{-1} \begin{bmatrix} PD \\ 0 \\ 0 \\ hZD \end{bmatrix} \left[\, D^T P \;\; 0 \;\; 0 \;\; hD^T Z \,\right] + \lambda \begin{bmatrix} E_i^T \\ E_{di}^T \\ 0 \\ 0 \end{bmatrix} [\, E_i \;\; E_{di} \;\; 0 \;\; 0 \,] < 0. \tag{24}$$

By Schur complement, (24) is equivalent to (20). This completes the proof of Theorem 2.

Remark 3. In Li [2004], in the derivative of Lyapunov-Krasovkii function for T-S fuzzy systems with a time-varying delay, were employed fixed weighting matrices to express the relationships between the terms in the Newton-Leibniz formula, which may lead conservativeness. In contrast, the proofs of Theorem 1 and Theorem 2 employ two free-weighting matrices N, M to express the relationships between the terms in the Newton-Leibniz formula, which may reduce the conservativeness of system.

Remark 4. In the derivative of Lyapunov-Krasovkii function for T-S fuzzy systems with a time-varying delay in Li [2004], the negative term $\int_{t-h}^{t-d(t)} \dot{x}^T(s) Z \dot{x}(s) ds$ in $\dot{V}(x(t))$ is ignored, which may lead to conservatism. In contrast, the proof of Theorem 1 and Theorem 2 show that this negative term is retained and a new free-weighting matrix M is introduced, which considers the relationship among h, $d(t)$ and their difference.

4. NUMERICAL EXAMPLES

This section provides two numerical examples that demonstrate the effectiveness of the criteria presented in this paper.

Example 1: Consider a time-delayed fuzzy system without uncertainty. The T-S fuzzy model of this fuzzy system is of the following form:

$$\dot{x}(t) = \sum_{i=1}^{2} \rho_i (A_i x(t) + A_{di} x(t - d(t)))$$

where

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \; A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.5 & 1 \\ 0 & -0.75 \end{bmatrix}, \; A_{d2} = \begin{bmatrix} -1 & 0 \\ 1 & -0.85 \end{bmatrix},$$
$$h_1 = sin^2(\theta(t)), h_2 = cos^2(\theta(t)).$$

Employing the LMIs in Li [2004] and those in Theorem 1 yield upper bounds on h that guarantee the stability of system (7) for various μ, which are listed in Table 1.

Table 1. allowable upper bound of h for μ

μ	0	0.01	0.1	0.5	unknown μ
Li [2004]	1.196	–	–	–	–
Theorem 1	1.348	1.340	1.280	1.135	1.099

It is clear that when the delay is time-invariant, i.e. $d = 0$, the obtained result in Theorem 1 is better than in Li [2004]. Furthermore, when the delay is time-varying, the corollary of Li [2004] fails to verify that the system is stable, while the Theorem 1 in this paper can obtain the upper bounds which guarantee the stability of the above fuzzy system.

Example 2: Consider a time-delayed fuzzy system with uncertainty. The T-S fuzzy model of this fuzzy system is of the following form:

$$\dot{x}(t) = \sum_{i=1}^{2} \rho_i(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - d(t)))$$

where

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix},\ E_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix},\ E_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Employing the Theorem 1 of Li [2004] and those in Theorem 2 of this paper yield upper bounds on h that guarantee the stability of system (1) with uncertainty for various μ, which are listed in Table 2.

Table 2. allowable upper bound of h for μ

μ	0	0.01	0.1	0.5	unknown μ
Li [2004]	0.950	0.944	0.892	0.637	–
Theorem 2	1.353	1.348	1.303	1.147	1.081

It is shown that when μ is known, our obtained results are better than those in Li [2004]; when μ is unknown, the Theorem 1 of Li [2004] fails to verify that the system is stable, while the Theorem 2 of this paper can also obtain the upper bound of stability is 1.081.

The reason is that our results not only retain any terms in the derivative of Lyapunov-Krasovskii function, but also consider the relationship among h, $d(t)$ and $h - d(t)$.

5. CONCLUSION

In this paper, some less conservative LMI-based asymptotic stability criteria are obtained without ignoring any terms in the derivative of Lyapunov-Krasovskii function for T-S fuzzy system with a time-varying delay. two numerical examples demonstrate that the proposed method is an improvement over the existing ones.

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