Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

H_∞ filtering of network-based systems with random delays

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Abstract: The H_∞ filtering problem is studied for a class of network-based systems with random network-induced delays in discrete-time domain. The considered random delay between the sensor and the filter may be longer than one sampling period and is modeled as a Markov chain. The filtering error system is modeled as a Markovian switched time-delay system and by using a properly constructed Lyapunov function, sufficient conditions for the existence of the H_∞ filters are presented in terms of linear matrix inequalities. Convex optimization problem is also formulated to design the desired H_∞ filter which guarantees the stochastic stability and an optimal H_∞ disturbance attenuation level for the filtering error system. An illustrative example is finally given to prove the effectiveness of the proposed method.

Keywords: H_∞ filtering; Random delays; Markov chain; Network-based systems; Linear matrix inequality (LMI)

1. INTRODUCTION

In many modern complex and distributed control systems, remotely located sensors, controllers, filters and controlled plants are often connected through a sharing communication network. Systems with such architectures are called the network-based systems, which bring a lot of advantages such as low cost, simple maintenance, high reliability and so on (Zhang et.al., 2001). In spite of these advantages, the sharing networks make the analysis and synthesis of such network-based systems challenging. Recently, the network-based control system, which is known as the NCS, has attracted much research interests (Zhang et al., 2001; Zhang et al., 2005; Nilsson et al., 1998). On the other hand, signal estimation over networks is important in many applications such as remote sensing, space exploration, and sensor networks. Therefore, the network-based signal estimation is also a potential researching field which needs to be fully investigated (Hespanha et al., 2007; Yue et al., 2006).

It is well known that when signals are transmitted through the networks, they may encounter unavoidable time delays or even packet dropouts between the senders and the receivers. Moreover, the delays and packet dropouts in the networks are often random so that traditional signal estimation methods, such as the standard Kalman filter and so on, can not be applied directly. Thus the filtering problem for the network-based systems with delays and packet dropouts has been a challenging yet interesting research topic, and has attracted increasing attention. Some results on the network-based system with packet dropouts can be found in (Smith et al., 2003; Sinopoli et al., 2004; Wang et al., 2003; Suh et al., 2007; Huang et al., 2007) and the references therein. As for the time-delay issue, much few results are available. In Wang et al., 2007, by modeling the network-induced delay as a Bernoulli process, the robust H_∞ filtering problem was studied for a class of discrete-time network-based filtering system. However, the delay considered is assumed to be shorter than one sampling period. The filtering problem of network-based system with long time-varying delay was considered in Yue et al., 2006, however, the method presented is not suitable to investigate the random delays. To the best of the authors' knowledge, the problem of network-based H_∞ filtering with random long delays has not yet been investigated, which motivates the present research.

In this paper, the H_∞ filtering problem is studied for a class of network-based systems with random network-induced delays which may be longer than one sampling period of the filtering system. The delay is modelled as a Markov chain, and the overall filtering error system is finally described as a Markovian switched time-delay system which explicitly describes the dynamic of the filtering error system. By using the Lyapunov method and the LMI technique, sufficient conditions are derived to guarantee the stochastic stability and a prescribed H_∞ performance for the filtering error system. A convex optimization problem is also formulated to design the optimal H_∞ filter. An illustrative example is finally provided to demonstrate the effectiveness of the proposed results.

2. MODELLING OF THE FILTERING ERROR SYSTEM

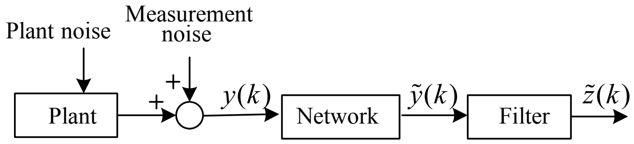

Fig.1. structure of filtering system over networks

The structure of the network-based filtering system under consideration is illustrated in Fig.1, where the plant is a discrete-time linear time-invariant system described as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bw(k) \\ y(k) &= Cx(k) + Dw(k) \\ z(k) &= Lx(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the system state, $w(k) \in R^m$ is the disturbance input belongs to $L_2[0,\infty)$, $y(k) \in R^p$ is the measured output, and $z(k) \in R^r$ is the signal to be estimated, A, B, C, D, and L are matrices of appropriate dimensions. We consider the following full order linear dynamic filter:

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f \tilde{y}(k) \\ z_f(k) &= C_f x_f(k) + D_f \tilde{y}(k) \end{aligned} \tag{2}$$

where $x_f(k) \in R^n$ is the filter state, $\tilde{y}(k) \in R^p$ is the filter input, and $z_f(k) \in R^r$ is the estimated signal, A_f, B_f, C_f, and D_f are filter parameters to be determined.

We use $\rho(k)$ to denote the random delay in the network between the sensor and the filter, and it is assumed to be bounded, that is, $0 \le \rho(k) \le N$, where N is a known integer. We model $\rho(k)$ as a Markov chain that take values in $\mathrm{N}_\rho = \{0,1,\cdots,N\}$ with known transition probability matrix $\Lambda = [\rho_{ij}]$. The transition probability ρ_{ij} is defined as follows:

$$\rho_{ij} = \Pr\{\rho(k+1) = j \,|\, \rho(k) = i\} \tag{3}$$

where $\rho_{ij} \ge 0$ and $\sum_{j=0}^{N} \rho_{ij} = 1$ for all $i, j \in \mathrm{N}_\rho$. Since the network-induced delay may be longer than one sampling period, various measured outputs may arrive at the filter side over one sampling period. We assume that the filter always uses the most recent measured output available at the filter side to update its input, and that if no measured output arrives at the filter over one sampling period, the filter input will hold at its previous value. By the above analyses and assumptions, it can be seen that the filter input $\tilde{y}(k)$ may take different values in $\{y(k), y(k-1), \cdots, y(k-N)\}$ at different sampling instants, which will result in $N+1$ different system dynamics of the filtering error system. For example, we have $\tilde{y}(k) = y(k)$ when there is no network-induced delay, then by augmenting the state variable as $\tilde{x}(k) = [x^T(k) \quad x_f^T(k)]^T$ and the disturbance input as $\tilde{w}(k) = [w^T(k) \quad w^T(k-1) \quad \cdots \quad w^T(k-N)]^T$, defining $e(k) = z(k) - \tilde{z}(k)$, we obtain the following filtering error system, which is one subsystem of the overall filtering error system.

$$\begin{aligned} \tilde{x}(k+1) &= \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} B & 0 & \cdots & 0 \\ B_f D & 0 & \cdots & 0 \end{bmatrix} \tilde{w}(k) \\ e(k) &= \begin{bmatrix} L - D_f C & -C_f \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} -D_f D & 0 & \cdots & 0 \end{bmatrix} \tilde{w}(k) \end{aligned} \tag{4}$$

The above procedures can be applied to obtain the other N subsystems of the filtering error system when $\tilde{y}(k) = y(k-i)$, $i = 1, 2, \cdots, N$. We call the subsystem (4) the 0-th subsystem, and the corresponding subsystem the i th subsystem when $\tilde{y}(k) = y(k-i)$.

It can be seen from the above analysis that over one sampling period many different situations may appear, and the filtering error system may reside in different subsystems during different sampling intervals. Moreover, during the sampling interval $[kT, (k+1)T)$, $k = 0, 1, \cdots$, the system dynamic of the filtering error system is actually determined by the states of the Markov chain $\rho(k-i)$, $i = 0, 1, \cdots, N$. Hence, a map $\chi : \rho(k), \rho(k-1), \cdots, \rho(k-N) \to \theta(k)$ is introduced to describe the filtering error system in a clear way, where $\theta(k) \in N_\rho$ represents the number of the activated subsystem. If $\rho(k-i)$, $i = 0, 1, \cdots, N$ satisfy (5) shown as follows, then the filtering error system is in the $\theta(k)$th subsystem.

$$\begin{aligned} & m < \rho(k-m), 0 \le m < \theta(k) \\ & 0 \le \rho(k-m) \le m, m = \theta(k) \\ & 0 \le \rho(k-m) \le N, \theta(k) < m \le N \end{aligned} \tag{5}$$

The indications of (5) will be explained in detail as follows. When $\rho(k) = 0$, it means that $y(k)$ arrives at the filter immediately at the sampling instant k, in this case, no matter what values $\rho(k-1), \cdots, \rho(k-N)$ take, by the assumption that the filter always uses the most recent measured output, it can be seen that the filter input is actually set to be $\tilde{y}(k) = y(k)$. From the previous analysis it can be seen that the filtering error system is running at the 0-th mode. If $\rho(k) > 0$, it means that $y(k)$ can not arrive at the filter at the sampling instant k, so the filter have to use the previous value of the measured output. Then if $\rho(k-1) \le 1$, it suggests that $y(k-1)$ is the newest measured output at the filter side so that $y(k-1)$ will be used and then the filtering error system is running in the first mode. Otherwise, if $\rho(k) > 0$ and $\rho(k-1) > 1$, both $y(k)$ and $y(k-1)$ will not be available at the filter at the sampling instant k, then if $\rho(k-2) \le 2$, we can see that the filter will use $y(k-2)$ and that the second subsystem is activating. The other situations can be carried out by following the above similar analysis procedures, and hence is omitted for conciseness. An example is given as follows to further help understand the meanings expressed by (5). We take $N = 3$, $\rho(k) = 1$, $\rho(k-1) = 2$, $\rho(k-2) = 1$ and $\rho(k-3) = 1$ as an example. By applying (5) we can obtain that $\theta(k) = 2$, therefore, the second subsystem is activating at the time step k. Next, at the sampling time $k+1$, if $\rho(k+1) = 2$, then $\theta(k+1) = 1$ and the filtering error system is running at the first mode.

By the above analyses, and taking all the possible situations into account, the overall filtering error system can be modeled as the following Markovian switched time-delay system with $N+1$ modes:

$$\begin{aligned} \tilde{x}(k+1) &= A_{\theta(k)0}\tilde{x}(k) + A_{\theta(k)1}\tilde{x}(k-\theta(k)) + B_{\theta(k)}\tilde{w}(k) \\ e(k) &= C_{\theta(k)0}\tilde{x}(k) + C_{\theta(k)1}\tilde{x}(k-\theta(k)) + D_{\theta(k)}\tilde{w}(k) \end{aligned} \tag{6}$$

where $A_{\theta(k)0} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}$, $A_{\theta(k)1} = \begin{bmatrix} 0 & 0 \\ B_f C & 0 \end{bmatrix}$,

$C_{\theta(k)0} = \begin{bmatrix} L & -C_f \end{bmatrix}$, $C_{\theta(k)1} = \begin{bmatrix} -D_f C & 0 \end{bmatrix}$, $\forall \theta(k) \in N_\rho$,

$B_0 = \begin{bmatrix} B & 0 & \cdots & 0 \\ B_f D & 0 & \cdots & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} B & 0 & \cdots & 0 \\ 0 & B_f D & \cdots & 0 \end{bmatrix}$, $\cdots$,

$B_N = \begin{bmatrix} B & 0 & \cdots & 0 \\ 0 & 0 & \cdots & B_f D \end{bmatrix}$, $D_0 = \begin{bmatrix} -D_f D & 0 & \cdots & 0 \end{bmatrix}$,

$D_1 = \begin{bmatrix} 0 & -D_f D & \cdots & 0 \end{bmatrix}$, $\cdots$, $D_N = \begin{bmatrix} 0 & 0 & \cdots & -D_f D \end{bmatrix}$

and $\theta(k)$ can be obtained by (5).

Remark1. A similar modelling method has been presented in Zhang et al., 2005 to study the control problem of networked-based system with random delays. However, the proposed model have not explicitly describes the dynamic of the closed-loop system.

Before presenting the main objective of this paper, we first introduce the following useful definitions for the filtering error system (6).

*Definition1***.** The filtering error system (6) is said to be stochastically stable if with $w(k) = 0$ for all $k \ge -N$, and for every initial state $\tilde{x}(i)$, $\rho(i)$, $i = 0, -1, \cdots, -N$, the following inequality holds

$$E\left\{ \sum_{k=0}^{\infty} \|\tilde{x}(k)\|^2 \,|\tilde{x}(i), \rho(i), \forall\, i = 0, -1, \cdots, -N \right\} < \infty \tag{7}$$

*Definition2***:** For all non-zero $w \in L_2[0,\infty)$ and a given constant $\gamma > 0$, the filtering error system (6) is said to be stochastically stable with an H_∞ disturbance attenuation level bound γ if it is stochastically stable and under zero initial condition, that is $\tilde{x}(k) = w(k) = 0$, $k = -1, -2, \cdots, -N$, and $\tilde{x}(0) = 0$, the following inequality holds

$$E\left\{ \sum_{k=0}^{\infty} e^T(k) e(k) \right\} < \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k) \tag{8}$$

The objective of the paper is to design a filter of the form (2) such that the filtering error system (6) is stochastically stable with an H_∞ disturbance attenuation level γ.

3. H_∞ PERFORMANCE ANALYSIS AND FILTER DESIGN

The following theorem gives sufficient conditions and H_∞ performance results for system (6).

*Theorem 1***.** For a given constant $\gamma > 0$, if there exist matrices $P(a_0, a_1, \cdots, a_N) > 0$, $Q_r(a_s) > 0$, and scalars $b_i > 0$, for all a_0, a_1, $\cdots$, $a_N \in \mathrm{N}_\rho$, $r \in \{1, 2, \cdots, N\}$, $s \in \mathrm{N}_\rho$, and $i \in \{0, 1, \cdots, N-1\}$, such that the following matrix inequalities hold

$$\Xi(a_0, a_1, \cdots, a_N) = \begin{bmatrix} \Omega & 0 & \Phi^T & \Theta^T \\ * & -\gamma^2 E & \Lambda^T & \Psi^T \\ * & * & \Pi & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{9}$$

Then, the filtering error system (6) is stochastically stable and has an H_∞ performance level γ, where

$$\Omega = \begin{bmatrix} Q_1(a_0) - P(a_0, a_1, \cdots, a_N) & 0 & \cdots & 0 & 0 \\ * & Q_2(a_1) - Q_1(a_1) & \cdots & 0 & 0 \\ * & * & \ddots & \vdots & \vdots \\ * & * & * & Q_N(a_{N-1}) - Q_{N-1}(a_{N-1}) & 0 \\ * & * & * & * & -Q_N(a_N) \end{bmatrix}$$

$\Phi = \begin{bmatrix} \Phi^T(0) & \Phi^T(1) & \cdots & \Phi^T(N) \end{bmatrix}^T$,

$\Theta = \begin{bmatrix} C_{\theta(k)0} + \varepsilon_0 C_{\theta(k)1} & \varepsilon_1 C_{\theta(k)1} & \cdots & \varepsilon_N C_{\theta(k)1} \end{bmatrix}$,

$\Phi(j) = \begin{bmatrix} E_1(j) + \varepsilon_0 F_1(j) & \varepsilon_1 F_1(j) & \cdots & \varepsilon_N F_1(j) \end{bmatrix}$,

$E_1(j) = \rho_{a_0 j}^{1/2} P(j, a_0, \cdots, a_{N-1}) A_{\theta(k)0}$,

$F_1(j) = \rho_{a_0 j}^{1/2} P(j, a_0, \cdots, a_{N-1}) A_{\theta(k)1}$, $j = 0, 1, \cdots, N$,

if $\theta(k) = i$, then $\varepsilon_i = 1$ and $\varepsilon_j = 0$, $\forall\, j \in N_\rho / i$,

$\Lambda = \begin{bmatrix} G^T(0) & G^T(1) & \cdots & G^T(N) \end{bmatrix}^T$, $\Psi = \begin{bmatrix} D_{\theta(k)} \end{bmatrix}$,

$G(j) = \rho_{a_0 j}^{1/2} P(j, a_0, \cdots, a_{N-1}) B_{\theta(k)}$, $j = 0, 1, \cdots, N$,

$\Pi = diag\{-P(0, a_0 \cdots, a_{N-1}), -P(1, a_0 \cdots, a_{N-1}), \cdots, -P(N, a_0 \cdots, a_{N-1})\}$

$$E = \begin{bmatrix} b_0 I & 0 & \cdots & 0 \\ 0 & b_1 I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (1 - \sum_{i=0}^{N-1} b_i) I \end{bmatrix}$$

Proof: Choose the following Lyapunov function for system (6)

$$V(k)=\tilde{x}^T(k)P(\rho(k),\rho(k-1),\cdots\rho(k-N))\tilde{x}(k)+\sum_{i=1}^{N}\tilde{x}^T(k-i)Q_i(\rho(k-i))\tilde{x}(k-i)$$

For notational convenience, we use a_i to denote $\rho(k-i)$ for all $i=0,1,\cdots,N$, and denote $\varphi(k)=\{\tilde{x}(k),\cdots,\tilde{x}(k-N),\rho(k),\cdots,\rho(k-N)\}$, then

$$\begin{aligned}
&E\{\Delta V(k)|\varphi(k)\}\\
&=E\{V(k+1)|\varphi(k)\}-V(k)\\
&=E\{\tilde{x}^T(k+1)P(\rho(k+1),a_0,a_1,\cdots,a_{N-1}))\tilde{x}(k+1)\}\\
&\quad+\tilde{x}^T(k)(Q_1(a_0)-P(a_0,a_1,\cdots,a_N))\tilde{x}(k)\\
&\quad+\tilde{x}^T(k-1)(Q_2(a_1)-Q_1(a_1))\tilde{x}(k-1)+\cdots\\
&\quad+\tilde{x}^T(k-N+1)(Q_N(a_{N-1})-Q_{N-1}(a_{N-1}))\tilde{x}(k-N+1)\\
&\quad+\tilde{x}^T(k-N)(-Q_N(a_N))\tilde{x}(k-N)\\
&=\sum_{j=0}^{N}\rho_{a_0j}(A_{\theta(k)0}\tilde{x}(k)+A_{\theta(k)1}\tilde{x}(k-\theta(k))+B_{\theta(k)}\tilde{w}(k))^T\\
&\quad\cdot P(j,a_0,\cdots,a_{N-1})(A_{\theta(k)0}\tilde{x}(k)+A_{\theta(k)1}\tilde{x}(k-\theta(k))+B_{\theta(k)}\tilde{w}(k))\\
&\quad+\sum_{s=1}^{N-1}\tilde{x}^T(k-s)(Q_{s+1}(a_s)-Q_s(a_s))\tilde{x}(k-s)\\
&\quad+\tilde{x}^T(k)(Q_1(a_0)-P(a_0,a_1,\cdots,a_N))\tilde{x}(k)\\
&\quad+\tilde{x}^T(k-N)(-Q_N(a_N))\tilde{x}(k-N)
\end{aligned}$$

Define

$$\eta(k)=\begin{bmatrix}\tilde{x}^T(k) & \tilde{x}^T(k-1) & \cdots & \tilde{x}^T(k-N)\end{bmatrix}^T,$$

$$\xi(k)=\begin{bmatrix}\tilde{x}^T(k) & \cdots & \tilde{x}^T(k-N) & \tilde{w}^T(k)\end{bmatrix}^T,$$

$$Z(j)=\rho_{a_0j}^{1/2}\begin{bmatrix}A_{\theta(k)0}+\varepsilon_0A_{\theta(k)1} & \varepsilon_1A_{\theta(k)1} & \cdots & \varepsilon_NA_{\theta(k)1}\end{bmatrix},$$

if $\theta(k)=i$, then $\varepsilon_i=1$, and $\varepsilon_j=0$ for all $j\in N_\rho/i$. then it follows that

$$\begin{aligned}
&E\{\Delta V(k)|\varphi(k)\}\\
&=\eta^T(k)\Omega\eta(k)+\sum_{j=0}^{N}\xi^T(k)\begin{bmatrix}Z^T(j)\\ \rho_{a_0j}^{1/2}B_{\theta(k)}^T\end{bmatrix}\\
&\quad\cdot P(j,a_0,\cdots,a_{N-1})\begin{bmatrix}Z(j) & \rho_{a_0j}^{1/2}B_{\theta(k)}\end{bmatrix}\xi(k)
\end{aligned}$$

When $w(k)=0$ for all $k\geq -N$, we have

$$\begin{aligned}
&E\{\Delta V(k)|\varphi(k)\}\\
&=\eta^T(k)\left(\Omega+\sum_{j=0}^{N}Z^T(j)P(j,a_0,\cdots a_{N-1})Z(j)\right)\eta(k)
\end{aligned}$$

Denote $L(a_0,a_1,\cdots a_N)=\Omega+\sum_{j=0}^{N}Z^T(j)P(j,a_0,\cdots a_{N-1})Z(j)$ and by Schur complement, (9) implies that $L(a_0,a_1,\cdots a_N)<0$, which further indicates that

$$\begin{aligned}
E\{\Delta V(k)\}&\leq-\lambda_{\min}(-L(a_0,a_1,\cdots a_N))\eta^T(k)\eta(k)\\
&\leq-\beta\eta^T(k)\eta(k)=-\beta\|\eta(k)\|^2\leq-\beta\|\tilde{x}(k)\|^2
\end{aligned}$$

where

$$\beta=\min\{\lambda_{\min}(-L(a_0,a_1,\cdots,a_N))|\forall a_i\in N_\rho,\forall i\in N_\rho\}>0$$

By the above inequality, it can seen that for any integer $T\geq 1$

$$E\{V(T+1)\}-E\{V(0)\}\leq-\beta E\left\{\sum_{k=0}^{T}\|\tilde{x}(k)\|^2\right\}$$

Then it follows from the above inequality that

$$E\left\{\sum_{k=0}^{T}\|\tilde{x}(k)\|^2\right\}\leq\frac{1}{\beta}\left(E\{V(0)\}-E\{V(T+1)\}\right)$$

Since $V(T+1)\geq 0$ and $E\{V(0)\}=\eta^T(0)\Upsilon(a_0,a_1,\cdots,a_N)\cdot\eta(0)$, we obtain

$$\begin{aligned}
&E\left\{\sum_{k=0}^{T}\|\tilde{x}(k)\|^2\right\}\\
&\leq\frac{1}{\beta}\eta^T(0)\Upsilon(\rho(0),\rho(-1),\cdots,\rho(-N))\eta(0)<\infty
\end{aligned}$$

where $\Upsilon(\rho(0),\rho(-1),\cdots,\rho(-N))=\mathrm{diag}\{P(\rho(0),\rho(-1),\cdots,\rho(-N)),Q_1(a_1),Q_2(a_2),\cdots,Q_N(a_N)\}$. Setting T to infinite we conclude by Definition1 that system (6) is stochastically stable.

On the other hand, consider the following performance function:

$$\sum_{k=0}^{M}\left\{E\{e^T(k)e(k)\}-\gamma^2w^T(k)w(k))\right\}$$

Since $w(k)=0$, $k=-1,-2,\cdots,-N$, we obtain that

$$\begin{aligned}
&\sum_{k=0}^{M}\left\{E\{e^T(k)e(k)\}-\gamma^2w^T(k)w(k))\right\}\\
&=\sum_{k=0}^{M}\left\{E\{e^T(k)e(k)\}-\gamma^2w^T(k)w(k)+E\{\Delta V(k)\}\right\}\\
&\quad-\sum_{k=0}^{M}E\{\Delta V(k)\}\\
&\leq\sum_{k=0}^{M}\xi^T(k)\left\{\begin{bmatrix}\Theta^T\\ \Psi^T\end{bmatrix}\begin{bmatrix}\Theta & \Psi\end{bmatrix}+\begin{bmatrix}\Omega & 0\\ 0 & 0\end{bmatrix}\right.\\
&\quad+\sum_{j=0}^{N}\begin{bmatrix}Z^T(j)\\ \rho_{a_0j}^{1/2}B^T{}_{\theta(k)}\end{bmatrix}P(j,a_0,\cdots,a_{N-1})\\
&\qquad\left.\cdot\begin{bmatrix}Z(j) & \rho_{a_0j}^{1/2}B_{\theta(k)}\end{bmatrix}\right\}\xi(k)-\sum_{k=0}^{M}E\{\Delta V(k)\}\\
&\quad-\left\{\sum_{k=0}^{M}b_0\gamma^2w^T(k)w(k)+\sum_{k=0}^{M}b_1\gamma^2w^T(k-1)w(k-1)\right.\\
&\qquad\left.+\cdots\sum_{k=0}^{M}(1-\sum_{i=0}^{N-1}b_i)\gamma^2w^T(k-N)w(k-N)\right\}\\
&=\sum_{k=0}^{M}\xi^T(k)\left\{\begin{bmatrix}\Theta^T\\ \Psi^T\end{bmatrix}\begin{bmatrix}\Theta & \Psi\end{bmatrix}+\begin{bmatrix}\Omega & 0\\ 0 & -\gamma^2E\end{bmatrix}\right.\\
&\quad+\sum_{j=0}^{N}\begin{bmatrix}Z^T(j)\\ \rho_{a_0j}^{1/2}B^T{}_{\theta(k)}\end{bmatrix}P(j,a_0,\cdots,a_{N-1})\\
&\qquad\left.\cdot\begin{bmatrix}Z(j) & \rho_{a_0j}^{1/2}B_{\theta(k)}\end{bmatrix}\right\}\xi(k)-\sum_{k=0}^{M}E\{\Delta V(k)\}
\end{aligned}\tag{10}$$

By Schur complement, (9) implies that

$$W(a_0,a_1,\cdots,a_N)=\begin{bmatrix}\Theta^T\\ \Psi^T\end{bmatrix}\begin{bmatrix}\Theta & \Psi\end{bmatrix}+\begin{bmatrix}\Omega & 0\\ 0 & -\gamma^2E\end{bmatrix}$$

$$+\sum_{j=0}^{N}\begin{bmatrix} Z^T(j) \\ \rho_{a_0 j} B^T{}_{\theta(k)} \end{bmatrix} P(j,a_0,\cdots,a_{N-1})\begin{bmatrix} Z(j) & \rho_{a_0 j} B_{\theta(k)} \end{bmatrix} < 0$$

Since $\sum_{k=0}^{M} E\{\Delta V(k)\} = E\{V(M+1)\} - E\{V(0)\}$, then under zero initial condition, we have that $\sum_{k=0}^{M} E\{\Delta V(k)\} \geq 0$. These together with (10) gives $\sum_{k=0}^{M}\left\{E\{e^T(k)e(k)\} -\gamma^2 w^T(k)w(k)\right\} \leq \sum_{k=0}^{M}\xi^T(k)W\xi(k)$. Thus (9) guarantees that $\sum_{k=0}^{M}\left\{E\{e^T(k)e(k)\} - \gamma^2 w^T(k)w(k))\right\} < 0$. Setting M to infinite we conclude by Definition 2 that system (6) is stochastically stable with an H_∞ disturbance attenuation level bound γ. This proof is completed.

Remark 2. In theorem 1, sufficient conditions for the stochastic stability and H_∞ performance results of the filtering error system (6) are presented in terms of nonlinear matrix inequalities, and in order to obtain the desired filter we have to transform them into LMIs.

Theorem 2: If there exists matrices $P_{a_0 a_1 \cdots a_N} > 0$, $Q_r(a_s) > 0$, R, U, V, C_f, D_f and scalars $c_i > 0$, $\delta > 0$ for all $a_0, a_1, \cdots, a_N, \in \mathrm{N}_\rho$, $r \in \{1, \cdots N\}$, $s \in \mathrm{N}_\rho$, and $i \in \{0,1,\cdots,N-1\}$, such that the following LMIs hold

$$\begin{bmatrix} \bar{\Omega} & 0 & \bar{\Phi}^T & \bar{\Theta}^T \\ * & -\bar{E} & \bar{\Lambda}^T & \bar{\Psi}^T \\ * & * & \bar{\Pi} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} P_{a_0 a_1 \cdots a_N} & R \\ * & R \end{bmatrix} > 0 \tag{12}$$

Then, system (6) is stochastically stable and has an H_∞ performance $\gamma = \sqrt{\delta}$. Moreover, the parameters of the filter are given by $A_f = R^{-1}U$, $B_f = R^{-1}V$, C_f, D_f. The detailed expressions of $\bar{\Omega}$, $\bar{\Phi}$, $\bar{\Theta}$, $\bar{E}$, $\bar{\Lambda}$, $\bar{\Psi}$, and $\bar{\Pi}$ are given in the Appendix.

Proof: By introducing new variables $P_{a_0 a_1 \cdots a_N}$, $Q_{ra_s 0}$, $Q_{ra_s 1}$, c_i, for all $a_0, a_1, \cdots, a_N, \in \mathrm{N}_\rho$, $s \in N_\rho$, $r \in \{1,2,\cdots,N\}$, $i \in \{0,1,\cdots,N-1\}$, and R, U, V, then letting $P(a_0,a_1,\cdots,a_N) = \begin{bmatrix} P_{a_0 a_1 \cdots a_N} & R \\ R & R \end{bmatrix}$, $c_i = \delta b_i$, $Q_r(a_s) = \begin{bmatrix} Q_{ra_s 0} & Q_{ra_s 1} \\ Q_{ra_s 1} & Q_{ra_s 1} \end{bmatrix}$, and $RA_f = U$, $RB_f = V$ in (9), we obtain (11), which is a set of LMIs. The proof is completed.

Remark 3. Note that (11) and (12) are LMIs over the scalar δ. This implies that δ can be included as an optimization variable to obtain the reduction of the disturbance attenuation level. So the minimum of the H_∞ disturbance attenuation level for system (1) can be obtained by minimizing δ subject to (11) and (12).

4. ILLUSTRATIVE EXAMPLES

Considering the linear time-invariant system (1) with

$$A = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, D = 1, L = \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T \tag{13}$$

Assume that $N = 2$, choose the probability transfer matrix as follow

$$\Lambda = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix} \tag{14}$$

By applying Theorem 2 and the corresponding convex optimization problem we obtain the minimum H_∞ disturbance attenuation level bound $\gamma^* = 3.3224$, associated with this minimum bound are the following filter matrices:

$$A_f = \begin{bmatrix} \text{-}0.0894 & 0.0587 \\ \text{-}0.2843 & 0.1793 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.0330 \\ 0.0600 \end{bmatrix}$$
$$C_f = \begin{bmatrix} 0.0652 & \text{-}0.0918 \end{bmatrix}, \quad D_f = \begin{bmatrix} 0.0042 \end{bmatrix}$$

5. CONCLUSIONS

The H_∞ filtering problem was studied in this paper for a class of discrete-time network-based system with long random delays. A new modeling method was presented to describe the overall filtering error system as a Markovian switched system. Sufficient conditions were derived to guarantee the stochastic stability and an H_∞ performance level for the filtering error system, and design procedures were also presented to design the optimal H_∞ filter. The effectiveness of the proposed method was finally illustrated by a numerical example.

ACKNOWLEDGEMENTS

The work was supported by the National Natural Science Funds for Distinguished Young Scholar under Grant 60525304.

REFERENCES

Zhang, W., Branicky, M. S., and Philips, S. M. (2001). Stability of networked control systems. *IEEE Control System Magazine*, 21(2), 84-99.

Zhang, L. Q., Shi, Y., et.al. (2005). A new method for stabilization of networked control system with random delays. *IEEE Transactions on Automatic Control*, 50(8): 1177–1181.

Nilsson, J., Bernhardsson, B., and Wittenmark. (1998). Stochastic analysis and control of real-time systems with random time delays, *Automatica*, 34(1), 57-64.

Wang, W., and Yang, F. W. (2007). Robust filtering for networked uncertain systems with random time delays. *Acta Automatica Sinica*, 33(5), pp 557-560.

Yue, D. and Han, Q. L. (2006). Network-based robust H_∞ filtering for uncertain linear system. *IEEE Transaction on Automatic Control*, 54(11), 4293-4301.

Hespanha and Xu, Y. G. (2007). A survey of recent results in networked control systems. *In proc of IEEE special issue on technology of Networked control system*, 95, 138-162

Smith, S.C., Seiler, P. (2003). Estimation with lossy measurements: jump estimators for jump systems. *IEEE Transaction on Automatic Control*, 48(12), 2163-2171.

Sinopoli, B., Schenato, L., et.al. (2004). Kalman Filtering With Intermittent Observations. *IEEE Transaction on Automatic Control*, 49(9), 1453-1464.

Wang, Z. D., Daniel, W. C., and Liu, X. H. (2003). Variance-Constrained Filtering for Uncertain Stochastic Systems With Missing Measurements, *IEEE Transaction on Automatic Control*, 48(7), 1254-1258.

Suh, Y. S., Nguyen, V. H., and Ro, Y. S. (2007). Modified Kalman filter for networked monitoring systems employing a send-on-delta method, *Automatica*, 43(2), 332-338.

Huang, M. Y., Subhrakanti, D. (2007) .Stability of Kalman filtering with Markovian packet losses, *Automatica*, 43(4) , 598~607.

Appendix

$$\bar{\Omega}=\begin{bmatrix} Q_{1a_0 0}-P_{a_0 a_1\cdots a_N} & Q_{1a_0 1}-R & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ * & Q_{1a_0 1}-R & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ * & * & Q_{2a_1 0}-Q_{1a_1 0} & Q_{2a_1 1}-Q_{1a_1 1} & \cdots & 0 & 0 & 0 & 0 \\ * & * & * & Q_{2a_1 1}-Q_{1a_1 1} & \cdots & 0 & 0 & 0 & 0 \\ * & * & * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & Q_{Na_{N-1}0}-Q_{N-1a_{N-1}0} & Q_{Na_{N-1}1}-Q_{N-1a_{N-1}1} & 0 & 0 \\ * & * & * & * & * & * & Q_{Na_{N-1}1}-Q_{N-1a_{N-1}1} & 0 & 0 \\ * & * & * & * & * & * & * & -Q_{Na_N 0} & -Q_{Na_N 1} \\ * & * & * & * & * & * & * & * & -Q_{Na_N 1} \end{bmatrix}$$

$\bar{\Phi}=\begin{bmatrix}\bar{\Phi}^T(0) & \bar{\Phi}^T(1) & \cdots & \bar{\Phi}^T(N)\end{bmatrix}^T$, $\bar{\Theta}=\Theta$,

$\bar{\Lambda}=\begin{bmatrix}\bar{\Lambda}^T(0) & \bar{\Lambda}^T(1) & \cdots & \bar{\Lambda}^T(N)\end{bmatrix}^T$, $\bar{\Psi}=\Psi$,

$\bar{\Phi}(j)=\begin{bmatrix}E_2(0)+\varepsilon_0 F_2(0) & \varepsilon_1 F_2(1) & \cdots & \varepsilon_N F_2(N)\end{bmatrix}$

$\bar{\Lambda}(j)=\begin{bmatrix}E_3(0)+\varepsilon_0 F_3(0) & \varepsilon_1 F_3(1) & \cdots & \varepsilon_N F_3(N)\end{bmatrix}$

$$E_2(j)=\begin{bmatrix}\rho_{a_0 j}^{1/2}P_{ja_1\cdots a_{N-1}}A & \rho_{a_0 j}^{1/2}V \\ \rho_{a_0 j}^{1/2}RA & \rho_{a_0 j}^{1/2}V\end{bmatrix},\quad F_2(j)=\begin{bmatrix}\rho_{a_0 j}^{1/2}UC & 0 \\ \rho_{a_0 j}^{1/2}UC & 0\end{bmatrix}$$

$$E_3(j)=\begin{bmatrix}\rho_{a_0 j}^{1/2}P_{ja_1\cdots a_{N-1}}B \\ \rho_{a_0 j}^{1/2}RB\end{bmatrix},\quad F_3(j)=\begin{bmatrix}\rho_{a_0 j}^{1/2}UD \\ \rho_{a_0 j}^{1/2}UD\end{bmatrix}$$

if $\theta(k)=i$, then $\varepsilon_i=1$ and $\varepsilon_j=0$, $\forall j\in N_\rho / i$

$$\bar{\Pi}=\begin{bmatrix} -P_{0a_0\cdots a_{N-1}} & -R & 0 & 0 & \cdots & 0 & 0 \\ * & -R & 0 & 0 & \cdots & 0 & 0 \\ * & * & -P_{1a_0\cdots a_{N-1}} & -R & \cdots & 0 & 0 \\ * & * & * & -R & \cdots & 0 & 0 \\ * & * & * & * & \ddots & \vdots & \vdots \\ * & * & * & * & * & -P_{Na_0\cdots a_{N-1}} & -R \\ * & * & * & * & * & * & -R \end{bmatrix}$$

$$\bar{E}=\begin{bmatrix} c_0 & 0 & 0 & 0 \\ * & \ddots & 0 & 0 \\ * & * & c_{N-1} & 0 \\ * & * & * & \delta-\sum_{i=0}^{N-1}c_i \end{bmatrix}$$