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# On robust position control of DC motors by $\epsilon$-PID controller and its application to humanoid robot arms ⋆

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**Abstract:** In this paper, we propose a new $\epsilon$-PID controller for DC motors. We provide a systematic design steps of selecting the gains of the proposed $\epsilon$-PID controller. We also analytically show that the proposed controller provides robustness against system parameter uncertainty and reduces the effect of unknown load torque to the order of $\epsilon$. The benefit of our control approach is that the PID gains are tuned conveniently by adjusting a single $\epsilon$ gain-factor. An experiment of DC motor control with its application to humanoid robot arms demonstrates the practical aspect of the proposed control method.

## 1. INTRODUCTION

In this paper, we consider a robust position control problem of DC motors (MAXON DC motor series) in the presence of parameter uncertainty and unknown load torque. These DC motors are used as actuators at the joints of humanoid robots such as arm, leg joints, etc. Among various linear/nonlinear robust control approaches, the PID control approach (or PI controller for a simplified version [12]) is one that has been continuously acknowledged and used in many practical systems. There are linear or nonlinear form of PID controllers [2],[5] and the practicability and robustness of PID controllers are well-documented in the control and industrial literature. Also, since there is usually no need to estimate the load torque with the PID control scheme, the controller form is much simpler when compared with control schemes which use the additional load torque estimator [8]-[10], adaptive method [3], or nonlinear observer [11].

Here, we propose a new PID controller with an aim of achieving good robust tracking performance with much simplified gain design process and rigorous robust stability analysis. First, we utilize a feedback linearizing control approach which erases *known* input matching terms. Then, the $\epsilon$ gain-factor control method of [1] with an extended integral term is utilized as an internal controller. The resulting combined controller is a new $\epsilon$-PID controller for the considered DC motors. The benefits of our proposed control scheme is verified through a single DC motor experiment and position control experiment of the humanoid robot arms. Throughout the letter, the Euclidean norm is used. Otherwise, it will be specifically denoted by a subscript.

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## 2. MODELING OF DC MOTORS

The DC motors (MAXON DC motors, e.g., RE35(11879), etc) dynamics is given by

$$0 = \frac{K_m}{rR}V - \frac{J_m}{r^2}\ddot{q} - \frac{B_m + K_b K_m/R}{r^2}\dot{q} - Q(t) \quad (1)$$

where $q$ is the position of link, $\dot{q}$ is the velocity of link, $\ddot{q}$ is the acceleration of link, $K_m$ is the torque constant, $J_m$ is the motor inertia, $B_m$ is the damping coefficient, $K_b$ is the back emf constant, $R$ is the armature resistance, $r$ is the gear ratio, $Q(t)$ is the load torque, and $V$ is the control input in voltage.

The control goal is to send $q$ to $q_d$. Let $x_1 = q$, $x_2 = \dot{q}$, $u = V$. Then, we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{B_m + K_b K_m/R}{J_m}x_2 + \frac{rK_m}{J_m R}u - \frac{r^2}{J_m}Q(t) \end{aligned} \quad (2)$$

For convenience, we set $\dot{q}_d = 0$. Defining $e_1 = x_1 - q_d$, $e_2 = x_2$, we have an error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -ae_2 + bu - cQ(t) \end{aligned} \quad (3)$$

where

$$a = \frac{B_m + K_b K_m/R}{J_m}, \quad b = \frac{rK_m}{J_m R}, \quad c = \frac{r^2}{J_m} \quad (4)$$

Practically, there is uncertainty in the system parameters. Thus, the actual system parameters are expressed as sums of the nominal terms and uncertain terms, e.g., $B_m = \bar{B}_m + \delta B_m$ where $\bar{B}_m$ denotes a nominal value and $\delta B_m$ denotes any possible uncertain value. The same is applied to other terms $K_b$, $K_m$, etc. Then, the error dynamics (3) is rewritten as follows

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -\bar{a}e_2 - \delta a e_2 + (\bar{b} + \delta b)u - cQ(t) \end{aligned} \quad (5)$$

where $\bar{a}$, $\bar{b}$ denote the parts consisting of nominal system parameters and $\delta a$, $\delta b$ denote the collecting terms of uncertain system parameters.

Moreover, a direct measurement of the load torque is difficult because high cost equipment is required [10]. Thus, the load torque $Q(t)$ is usually considered to be unknown. Now, our control goal is to make $|e_1| \le \epsilon$ for $t \ge t_f$ in the presence of uncertain system parameters and unknown load torque.

## 3. ϵ-PID CONTROLLER

First, we define $\mu := \delta b/\bar{b}$ and assume that $|\mu| < 1$. This assumption is reasonable because the nominal value usually takes the large portion of the actual value. Also, we assume that the load torque $Q(t)$ is differentiable and there exists a finite constant $\rho$ such that $\dot{Q}(t) \le \rho$ for all $t \ge 0$. Take a time-derivative of (5). Then,

$$\begin{aligned} \ddot{e}_1 &= \dot{e}_2 \\ \ddot{e}_2 &= -\bar{a}\dot{e}_2 - \delta a \dot{e}_2 + (\bar{b} + \delta b)\dot{u} - c\dot{Q}(t) \end{aligned} \quad (6)$$

Let $v = \dot{u}$ and define $z = [z_1, z_2, z_3]^T = [\dot{e}_1, \dot{e}_2, e_1]^T$. Then, since $\dot{z}_3 = z_1$, we obtain the following state equation using (6)

$$\dot{z} = Az + B\{-\bar{a}z_2 - \delta a z_2 + (\bar{b} + \delta b)v - c\dot{Q}(t)\} \quad (7)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (8)$$

Now, by using the nominal system parameters, we apply the following feedback linearizing controller

$$v = \frac{\bar{a}z_2 + \omega}{\bar{b}} \quad (9)$$

to the system (7). Then, we obtain

$$\dot{z} = Az + B\{(\mu\bar{a} - \delta a)z_2 + (1 + \mu)\omega - c\dot{Q}(t)\} \quad (10)$$

Now, for the internal controller $\omega$, we set

$$\omega = \frac{k_1}{\epsilon^2}z_1 + \frac{k_2}{\epsilon}z_2 + \frac{k_3}{\epsilon^3}z_3 = \epsilon^{-2}KE_\epsilon z \quad (11)$$

where $K = [k_1, k_2, k_3]$ and $E_\epsilon = \mathrm{diag}[1, \epsilon, \epsilon^{-1}]$, $\epsilon > 0$.

From (9) and (11), the resulting controller is

$$v = \frac{k_1}{\bar{b}\epsilon^2}z_1 + \frac{1}{\bar{b}}\left(\frac{k_2}{\epsilon} + \bar{a}\right)z_2 + \frac{k_3}{\bar{b}\epsilon^3}z_3 \quad (12)$$

From $\dot{u} = v$ and letting $e = e_1$, the $\epsilon$-PID controller is summarized as follows

$$u = K_P(\epsilon)e + K_D(\epsilon)\dot{e} + K_I(\epsilon)\int_0^t e(\tau)d\tau \quad (13)$$

where

$$K_P(\epsilon) = \frac{k_1}{\bar{b}\epsilon^2},\ K_D(\epsilon) = \frac{k_2}{\bar{b}\epsilon} + \frac{\bar{a}}{\bar{b}},\ K_I(\epsilon) = \frac{k_3}{\bar{b}\epsilon^3} \quad (14)$$

## 4. CONTROLLER DESIGN RULE AND ROBUST STABILITY ANALYSIS

First, we state the following design steps for the proposed $\epsilon$-PID controller.

*Design steps:*

(1) (Basic gain selection) Select $K$ such that $A_K = A + B(1 + \mu)K$ is Hurwitz.
(2) (Lyapunov equation solution) Obtain the solution $P$ of $A_K^T P + PA_K = -I_n$ where $I_n$ is an $n \times n$ identity matrix.
(3) ($\epsilon$-PID gain tuning) Select $\epsilon$ such that $\epsilon^{-1} - 2\gamma_1 > 0$ where $\gamma_1 = 2|\mu\bar{a} - \delta a|\|P\|$.

Note that design step 1 is equivalent to selecting $K$ to make all roots of the following polynomial have negative real parts.

$$0 = s^3 - (1 + \mu)k_2 s^2 - (1 + \mu)k_1 s - (1 + \mu)k_3 \quad (15)$$

where $|\mu| < 1$. The selection of such $K$ can be obtained systematically by utilizing Kharitonov's theorem as approached in [4].

**Lemma 1.** If $A_K$ is Hurwitz, then $A_K(\epsilon) := A + B(1 + \mu)\epsilon^{-2}KE_\epsilon$ is Hurwitz for all $\epsilon > 0$.

**Proof:** First, between $A_K$ and $A_K(\epsilon)$, the following relation holds

$$A_K = \epsilon E_\epsilon A_K(\epsilon)E_\epsilon^{-1} \quad (16)$$

Here, we note that $\lambda(E_\epsilon A_K(\epsilon)E_\epsilon^{-1}) = \lambda(\epsilon^{-1}A_K)$ and $\lambda(\epsilon^{-1}A_K) = \epsilon^{-1}\lambda(A_K)$ where $\lambda(M)$ denotes the eigenvalues of a matrix $M$. This assures the Hurwtiz property of $A_K(\epsilon)$ for all $\epsilon > 0$ given that $A_K$ is Hurwtiz. □

Now, we state the main theorem.

**Theorem 1.** Suppose that $K$ and $\epsilon$ are selected by following the design steps. Then, with the $\epsilon$-PID controller (13), the origin of (10) is ultimately bounded by $O(\epsilon)$. Moreover, when $Q(t) = Q$, the origin of (10) is exponentially stable.

**Proof:** From (10), the closed-loop system is written as

$$\dot{z} = A_K(\epsilon)z + B\{(\mu\bar{a} - \delta a)z_2 - c\dot{Q}(t)\} \quad (17)$$

Since $A_K$ is Hurwitz, we have a Lyapunov equation

$$A_K^T P + PA_K = -I_n \quad (18)$$

By applying the relation (16) to (18), we obtain a new Lyapunov equation as

$$A_K^T(\epsilon)P_\epsilon + P_\epsilon A_K(\epsilon) = -\epsilon^{-1}E_\epsilon^2 \quad (19)$$

where $P_\epsilon = E_\epsilon P E_\epsilon$.

This derived Lyapunov equation (19) is valid because $A_K(\epsilon)$ is Hurwitz by Lemma 1 and $P_\epsilon$ is positive definite. Now, we set a Lyapunov function $V(z) = z^T P_\epsilon z$. Then, along the trajectory of (17), its time-derivative is

$$\dot{V}(z) = -\epsilon^{-1}\|E_\epsilon z\|^2 + 2z^T E_\epsilon P E_\epsilon B\{(\mu\bar{a} - \delta a)z_2 - c\dot{Q}(t)\} \quad (20)$$

From (20), we have $E_\epsilon B\{(\mu\bar{a} - \delta a)z_2 - c\dot{Q}(t)\} \leq |\mu\bar{a} - \delta a|\|E_\epsilon z\| + \epsilon c\rho$. Then, we have the following inequality

$$\begin{aligned}\dot{V}(z) &\leq -\epsilon^{-1}\|E_\epsilon z\|^2 + \gamma_1\|E_\epsilon z\|^2 + \epsilon\gamma_2\|E_\epsilon z\| \\ &= -(\frac{1}{2}\epsilon^{-1} - \gamma_1)\|E_\epsilon z\|^2 \\ &\quad -\epsilon^{-1}(\frac{1}{2}\|E_\epsilon z\| - \epsilon^2\gamma_2)\|E_\epsilon z\| \end{aligned} \quad (21)$$

where $\gamma_1 = 2|\mu\bar{a} - \delta a|\|P\|$ and $\gamma_2 = 2c\rho\|P\|$ which are $\epsilon$-independent constants.

When we select $\epsilon$ such that $\epsilon^{-1} - 2\gamma_1 > 0$, $\|E_\epsilon z\|$ is ultimately bounded by $O(\epsilon^2)$ [6]. From the definition of $E_\epsilon$, this implies that $\|z\|$ is ultimately bounded by $O(\epsilon)$. When the load torque is constant ($Q(t) = Q$), we have $\rho = 0$. Then, the exponential stability is achieved from the quadratic form of Lyapunov function. □

The selection of controller gain is divided into two steps. The following is the observation on the meaning of each gain selection step.

- The selection of $K$ provides a way of robust design in the presence of input uncertainty ($\delta b$ term). Moreover, from the positions of the assigned eigenvalues of $A_K$, the behavior of the controlled system can be roughly expected by examining the corresponding damping ratio, etc.
- The selection of $\epsilon$ provides a way to robustly suppress the parameter uncertainty and unknown load torque without much restriction on $\gamma_1$. Moreover, with the role of $\epsilon$, the effect of load torque $Q(t)$ is reduced to the order of $\epsilon$. The fine tuning of the proposed $\epsilon$-PID controller is done by conveniently adjusting only a single $\epsilon$ gain factor under the condition of step 3.

## 5. EXPERIMENTAL RESULT

Three MAXON DC motors(RE30(148877), RE35(11879), 2642W-048CR) are considered for the experiment. These DC motors are used as actuators at the joints of a humanoid robot such as arm, leg joints, etc. In particular, as shown in Fig. 1, these motors are placed at the elbow, shoulder, and wrist joints of humanoid robots (named as MAHRU and AHRA [7]). So, when each motor tracks certain positions, the whole arms can make a certain form such as a greeting form, etc.

(i) *Single DC motor control:* First, by using a particular DC motor (RE35(11879))) , we illustrate the gain-tuning process of our proposed $\epsilon$-PID controller. The experimental setup is shown in Fig. 2. Our developed DSP board is directly connected to the DC motor to provide a designed control input signal. For RE35(11879), the nominal system parameters are as follows: $B_m = 2.68042 \times 10^{-5} Nms$, $K_b = 0.0603Vs$, $K_m = 0.060438586Nm/A$, $R = 1.16\Omega$,

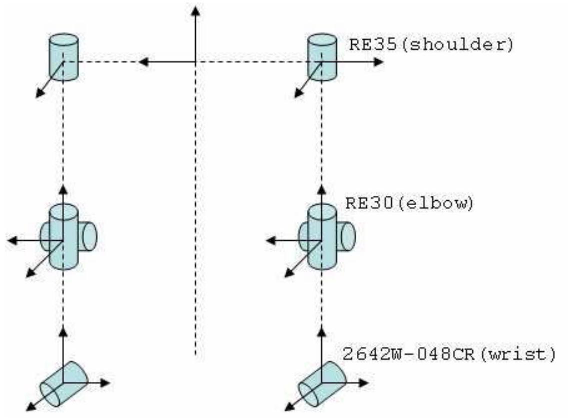



Fig. 1. DC Motors placed at the joints of humanoid robot arms

$J_m = 0.0000134Kgm^2$, and $r = 1$. As the first design step, we select $K = [-3, -3, -1]$ to place the eigenvalues at $-1$. Then, the $\epsilon$ gain-factor is selected with four choices and each corresponding position response $q$ is observed as shown in Fig. 3. The dotted lines denote the reference position $q_d$ and solid lines denote the measured output position $q$. As consistent with our theorem, the system response becomes faster and the tracking error is reduced as $\epsilon$ is decreased. In particular, when $\epsilon$ is less than 0.05, the tracking errors become small such that the tracking error can be observed only in tracking error plots shown in Fig. 3b. During the experiment, we could not make $\epsilon$ smaller than 0.01 as the DSP board suffers a voltage saturation. As a result, the PID gains are conveniently tuned by only adjusting $\epsilon$ and tracking error is reduced proportional to $\epsilon$.

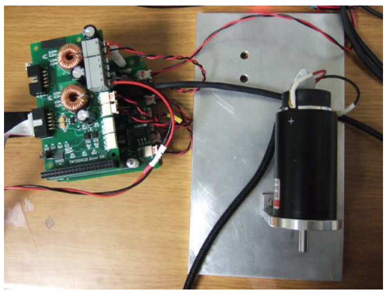
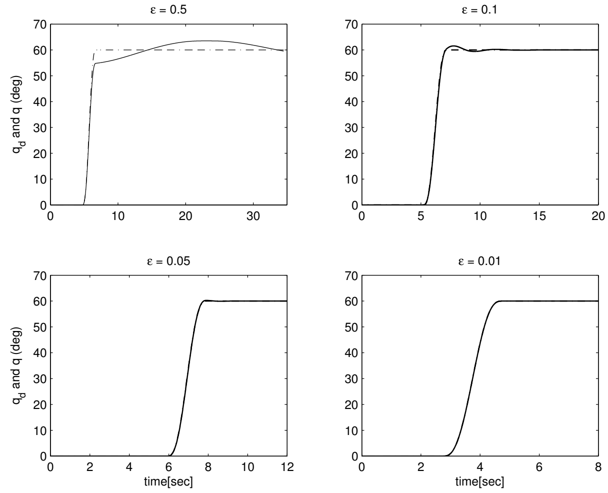
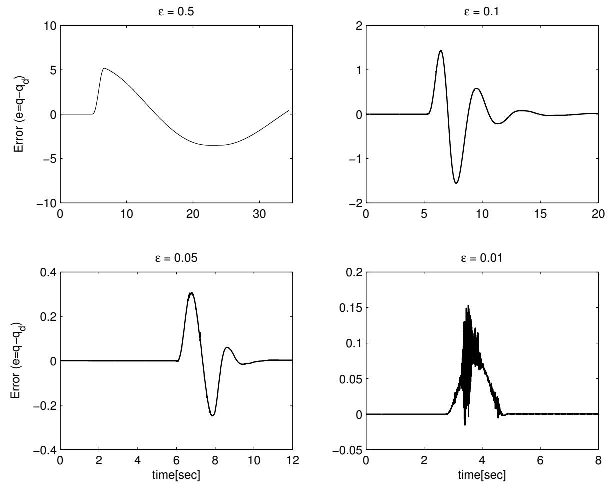


Fig. 2. Experiment setup

(ii) *Multiple DC motors control:* Once the tuning process of the $\epsilon$-PID controller is checked for a single motor (RE35(11879)), the $\epsilon$-PID controller gains for all other motors are easily tuned by putting each nominal system parameters and simply adjusting the $\epsilon$ gain-factor with the same $K$. The final tuned value of $\epsilon$ is 0.01 for three motors. The ultimate target reference angles for each motor (RE35, RE30, 2642W-048CR) are set as 25deg, 30deg, and 20deg, respectively. As shown in Fig. 4, the proposed $\epsilon$-PID controllers are applied to each of three motors and the experimental result shows good robust tracking performances for each of three motors. In particular, the tracking errors are small such that $q_d$ and $q$ are almost overlapped and the small errors can be seen in Fig. 4b. By tracking these reference angles at the same time, the humanoid robot arms form a greeting position from an initial stand-up position as shown in Fig. 5




(a)



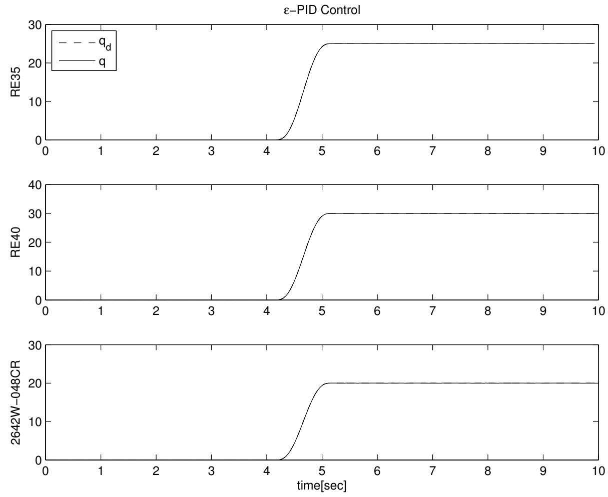

(b)

Fig. 3. Control results with various values of $\epsilon$: (a) plots of $q_d$ and $q$, (b) plots of tracking errors

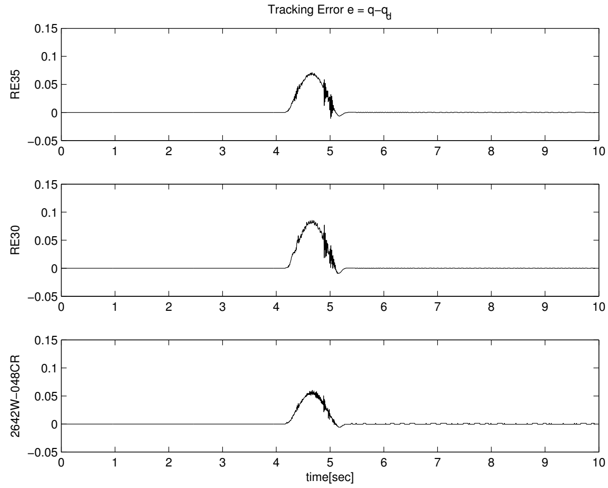
(iii) *Comparison with the inverse optimal PID control [2]:* In [2], the authors suggest an inverse optimal PID control design method for robust tracking problems. Their gain-tuning process is done by coarse/fine performance tuning laws and there are mainly two tuning variables $k$ and $\gamma$. Then, $K_P$, $K_I$, and $K_D$ are determined once $k$ and $\gamma$ are set. Thus, several tuning variables are mixed up with each other in [2] whereas our tuning process is simpler and straightforward. By following the design steps of [2], the optimally selected gains $(k, \gamma, K_P, K_I, K_D)$ of the inverse optimal PID control are set as $(120, 0.09, 40, 200, 1)$ for RE35, $(80.0, 0.1, 40, 200, 1)$ for RE30, and $(40.0, 0.2, 40, 200, 1)$ for 2642W-048CR, respectively. As shown in Fig. 6, our proposed controller shows a clearly improved performance (better convergence, reduced error) with less gain-tuning effort. Also, note that our $\epsilon$-PID controller is designed based on pole-placement approach such that the system output behavior is somewhat predictable unlike [2].

## 6. CONCLUSIONS

In this paper, we propose a new $\epsilon$-PID controller for DC motor systems. The robustness of the controller is analytically shown. Especially, the single $\epsilon$ gain-factor is effectively used in showing the robustness. Then, based on the analysis, the controller gain design steps are suggested. It turns out that the PID gains can be conveniently tuned by adjusting the single $\epsilon$ gain-factor. The experimental results agree with our analysis and demonstrate the improved performance of our control method.




(a)




(b)

Fig. 4. Control results of three DC motors, $\epsilon = 0.01$: (a) plots of $q_d$ and $q$, (b) plots of tracking errors

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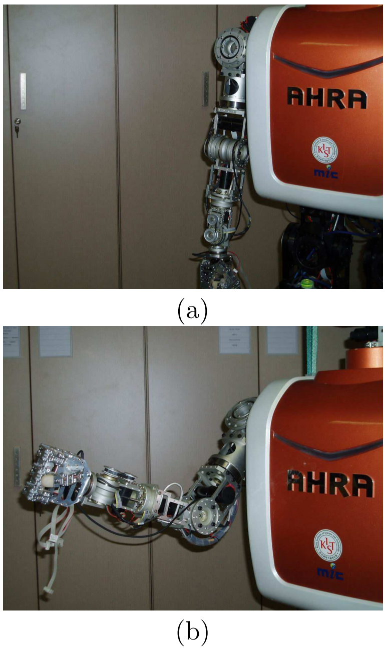


Fig. 5. Humanoid arm position control: (a) stand-up position, (b) greeting position

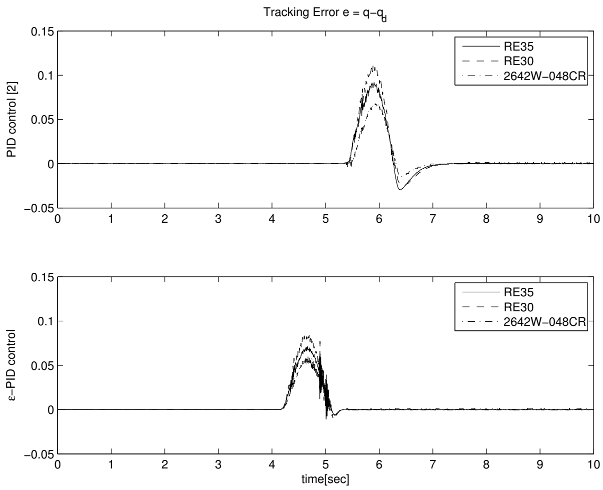


Fig. 6. Tracking error comparison between the inverse optimal PID control [2] and our $\epsilon$-PID control

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