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On Trading of Equities: A Robust Control Paradigm

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Abstract: The objective of this paper is describe a new paradigm for the trading of equities. In our formulation, the control corresponds to a feedback law which modulates the amount invested $I(t)$ in stock over time. The controller also includes a saturation limit I_{max} corresponding to a limit on the value at risk. The admissible stock price evolution $p(t)$ over time is modelled as a family $\mathcal{P}$ of uncertain inputs against which we seek robust returns. Motivated by the fact that back-testing of candidate trading strategies involves significant cost and effort associated with computational simulation over sufficiently diverse markets, our paradigm involves the notion of *synthetic prices* and some idealizations involving the volatility of prices and trading liquidity. Our point of view is that a robust performance certification in this somewhat idealized market setting serves as a filter to determine if a trading strategy is worthy of the considerable time and expense associated with full-scale back-testing. The paper also includes a description of a so-called *saturation reset controller*. This controller is used to illustrate how the model works in practice and the attainment of robustness objectives over various sub-classes of $\mathcal{P}$.

Keywords: finance, markets, trading of stock, economic problems, robust control

1. INTRODUCTION

This paper relates to a body of literature commonly known as *technical analysis*; e.g., see Archelis (2000) for an introduction. In this setting, the so-called market technician typically begins with past prices and trading volume in order to generate predictions about future prices. Having a degree of confidence in such a prediction suggests making an investment with the goal of producing "excess returns." That is, based on prediction of future prices, a *trading rule*, which times one's degree of investment in the market, is aimed at generating stock returns which exceed some standard benchmark such as buy-and-hold.

The notion of trading rules based on technical analysis appears to originate in the late eighteen hundreds in the reporting of Wall Street Journal editor Charles Dow. In more recent times, early studies such as those of Alexander (1961) and Fama and Blume (1966) concentrate on simple "filter rules" which generate buy signals when the equity or index under consideration moves a certain amount up from a recent low. Taking transaction costs into account, which were much more significant then versus now, these authors argue such rules are not capable of generating excess returns; see also Dooley and Shafer (1983) and Sweeney (1986) where similar findings were obtained in a foreign exchange rate context.

Consistent with this early work on technical analysis was the widely held point of view in the academic community that the attainment of excess returns, except in some rare exceptional cases, is an impossibility. This view was based largely on the notion of *efficient capital markets*; e.g., see Fama (1970). That is, if one accepts the argument that current prices incorporate all relevant information about the future, there is nothing to be gained via technical analysis; e.g., see Malkiel (1981). On the flip side, technical analysts argue that the notion of efficient capital markets is an abstraction which is not applicable in most real-world trading situations.

Following the trading rule literature into the eighties and nineties, we see many papers arguing that technical analysis can indeed be efficacious. That is, we see a swinging of the pendulum away from the efficient market point of view. In this regard, Brown and Jennings (1989) provide motivation by describing a scenario under which technical analysis can be powerful. Subsequently, Frankel and Froot (1990) provide evidence for the usefulness of charts (for example, stock price and volume versus time) and we see papers by Sweeney (1988) and Neftci (1991) studying the attainability of excess returns using trading rules based on moving averages. In this vain, the paper by Brock, Lakonishok and LeBaron (1992) is heavily cited. By back-testing two of the simplest and most popular trading rules involving moving averages and trading range breaks, they provide strong statistical evidence that technical analysis might lead to excess returns; see also Blume, Easley and O'Hara (1994) and Lee and Swaminathan (2000) where the role of volume information is studied.

Finally, to complete this abbreviated review, we note that the literature of the nineties also includes a number of papers raising the possibility that technical anal-

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ysis may be more powerful in foreign exchange markets than equity markets; e.g., see Levich and Thomas (1993), Neely (1997) and Gencay (1998), LeBaron (1999) and Neely and Weller (2001). In addition, the literature also includes a number of well known results further bolstering the arguments for technical analysis in an equity context. This is illustrated by the paper by Lo, Mamaysky and Wang (2000) which analyzes the effectiveness of the famous "head and shoulders" chart pattern and the paper by Kavajecz and Odders-White (2004) where support and resistance levels are studied.

1.1 Motivation for This Paper

Looking at the technical analysis literature described above, we see that various trading strategies are proposed and the "proof" of their efficacy involves making a case that "excess returns" are obtained in comparison to some standard method which does not involve the use of market timing. For example, authors often make the case that their technical trading rule out-performs the classical buy-and-hold strategy. In this regard, it is widely recognized that the pitfalls of "data snooping" are to be avoided. Roughly speaking, this means the following: If back-testing of a trading rule is not carried out on a diverse and rich enough set of market scenarios, there is a tendency for researchers to obtain results which are consistent with their natural predilections; e.g., see Sweeney (1988).

Motivation for this paper is provided by the fact that "proper" back-testing of candidate trading strategies involves significant cost associated with computational simulation over sufficiently diverse markets. To this end, our new paradigm involves the notion of *synthetic prices* and some idealizations involving the volatility of prices, trading liquidity and transaction costs. In addition, we specify a class of admissible price variations $\mathcal{P}$ and then establish robustness results with respect to various subclasses; see Section 4. In this way, before testing a trading algorithm on real data as in Section 5, it is first certified in a theoretical context. We view this as a first step in the process of identifying potentially efficacious trading strategies. Our point of view is that a robust performance certification in our idealized synthetic market setting serves as a filter to determine if a trading strategy is worthy of the considerable time and expense associated with full-scale back-testing.

1.2 Control Theory Point of View

Taking the point of view of robust control, our approach does not involve any type of stochastic modelling or the use of volatility measures; e.g., see Black and Scholes (1973). Instead, consistent with the tenets of robust control, we seek "certification" of the performance of a trading strategy with respect to a class $\mathcal{P}$ of admissible stock price variations. As a simple illustration, if a stock begins at price p_0 at time $t = 0$ and does a round-trip leading to price, $p(T) = p_0$ at some future time T, will the trading strategy perform better than buy and hold? In this first paper introducing our new paradigm, we analyze a trading rule in the context of three classes of price variations which we define: *bullish price variations, bearish price variations* and *round-trip price variations*: The expansion of this paradigm to other robustness scenarios is relegated to future research.

Motivated by our earlier discussion about the costliness and labor intensive aspects of back-testing, our new paradigm involves the notion of *synthetic price variations*. Using this theoretical construct before bringing real market data into the picture, we are able to separate the "wheat from the chaff" in the following sense: We can rapidly determine which trading strategies are worthy of more serious consideration. In this idealized setting, we limit volatility by including a smoothness assumption about price variations, a liquidity assumption about the rate at which trades can be executed and an assumption that brokerage costs are negligible. Our point of view is summarized as follows: Before one invests inordinate time and effort back-testing a candidate strategy, a necessary condition is that its efficacy should be established under these idealized market conditions.

2. THE IDEALIZED SYNTHETIC PRICE MODEL

We assume continuously differentiable price variations $p(t)$ on the time interval $[0, T]$. Notice that this assumption rules out price gaps which may occur following various events such as earnings announcements or major news. In addition, it is assumed that the trader can react immediately to observed price variations with zero transaction cost. That is, the stock position can be updated almost instantaneously as price changes occur. Note that such an assumption idealizes the situation faced by the day trader and is more analogous to programmed trading. It is a type of perfect liquidity assumption with no gap between the bid and ask prices. Our most general formulation also assumes a margin account which and a money market rate which is available for uninvested funds.

2.1 Classes of Price Variations

Henceforth, we use $\mathcal{P}$ to denote the *class of admissible price variations* described above. For the purposes of robustness analysis, we also define various sub-classes of $\mathcal{P}$ as follows: Indeed, the *class of bullish price variations* is defined by

$$\mathcal{P}_+ \doteq \{p \in \mathcal{P} : p(T) > p_0\},$$

the *the class of bearish price variations* is defined by

$$\mathcal{P}_- \doteq \{p \in \mathcal{P} : p(T) < p_0\},$$

and the the *class of round-trip price variations* by

$$\mathcal{P}_0 \doteq \{p \in \mathcal{P} : p(T) = p_0\}.$$

We also parameterize each of these classes in terms of the maximal price

$$p_{max} \doteq \max_{t \in [0,T]} p(t).$$

That is, for $\gamma \geq 1$, we define

$$\mathcal{P}_+(\gamma) \doteq \{p \in \mathcal{P}_+ : \frac{p_{max}}{p_0} = \gamma\},$$

$$\mathcal{P}_-(\gamma) \doteq \{p \in \mathcal{P}_- : \frac{p_{max}}{p_0} = \gamma\},$$

and

$$\mathcal{P}_0(\gamma) \doteq \{p \in \mathcal{P}_0 : \frac{p_{max}}{p_0} = \gamma\}.$$

2.2 Investment, Value, Trading Gains and Feedback

Henceforth, at time $t \in [0, T]$, we let $V(t)$ denote the value of the trader's account. We further decompose $V(t)$ into two components. The first component, denoted $I(t)$, and denotes the amount invested. For simplicity of exposition, we assume $I(t) \geq 0$ while noting that this assumption is being made solely for simplicity of exposition; i.e., the paradigm to follow is readily modified to accommodate $I(t) < 0$ with the appropriate interpretation that stock is being shorted. At any time, the idle cash in the account, $V(t) - I(t)$, is assumed to increase at a specified money market rate m. For the case, when $V(t) - I(t) < 0$, this cash deficit will accrue interest at the broker's margin rate. Finally, we assume initial conditions $V(0) = V_0$ and $I(0) = I_0$ and keep track of gains or losses as the trade evolves. That is, the gain or loss, $g(t)$, with its initial condition $g(0) = 0$ is given by $g(t) = V(t) - V(0)$.

Our paradigm allows I to be a feedback control which processes the available states such as stock price, account value or trading gains and losses. By continuously modulating the amount invested, our objective is to maximally increase the account value $V(t)$ while satisfying various conditions along the way. Finally, to limit the value at risk, a consideration which is critical to most traders, our model also includes a saturation limit $I(t) \leq I_{max}$ on the amount which can be invested or shorted at any time. A classical feedback setup illustrating one possibility for the scenario above is given in Figure 1.

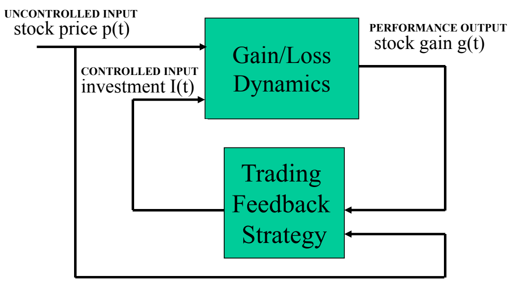

Figure 1: Control System Point of View

2.3 The Saturation Reset Controller

At the most general level, the amount invested $I(t)$ is a feedback which can be rather arbitrary. In this paper, to demonstrate the type of robustness results which are possible, we consider a simple modification of the classical linear time-invariant state feedback controller. Roughly speaking, subject to the saturation limit I_{max}, this controller exploits a pure gain K to add to the amount invested when the stock is faring well and reduces one's position as the setting becomes bearish. More precisely, when $dg < 0$, the amount invested is decreased according to the requirement $dI = Kdg$. On the other hand, if $dg > 0$, there are two possibilities: When operating outside the saturation regime, an infinitesimal trading gain dg dictates again dictates an incremental change in investment $dI = Kdg$. This time, however, this change corresponds to increasing the amount invested. When operating inside the saturation regime with $dg > 0$, then the invested amount is reduced to maintain the saturation limit $I(t) = I_{max}$. In other words, this gain dg is banked at the money market rate. To summarize these ideas in terms of time evolution, prior to saturation we have

$$\frac{dI}{dt} = K\frac{dg}{dt},$$

with its solution $I(t) = I_0 + Kg$ and during saturation, the we require $I(t) = I_{max}$; i.e., the invested funds are held fixed.

Finally, associated with this controller, we define the *initial trading allocation constant*

$$\Delta \doteq \frac{I_{max}}{I_0}.$$

Note that this constant can be viewed as a measure of conservatism of the controller. The most aggressive trader might select $\Delta = 1$; i.e., invest all available funds at $t = 0$. On the opposite extreme, a very conservative investor might begin with most funds in the fixed income money market and select a small percentage of V_0 to be subjected to risk; i.e., use a large Δ.

3. CLOSED LOOP TRADING DYNAMICS

We consider an infinitesimal time increment dt over which we update both the trading gain g and the account value V. We first consider the case when $I \leq V$ and with m being the money-market rate and dp being the corresponding stock price increment, we first obtain incremental contributions to trading gains given by

$$dg = \frac{dp}{p}I.$$

Now, noting that the incremental contribution to the account value comes from both the stock gains and idle or borrowed cash, we obtain

$$dV = dg + m(V - I)dt$$

Note that when $I > V$, we can use the same equation above provided that m is interpreted to be the broker's margin rate.

3.1 Non-Saturation Regime

When $I < I_{max}$, recalling that the incremental investment is given by $dI = Kdg$, we have $I = I_0 + Kg$ and the incremental gain above is rewritten in closed loop form as

$$dg = \frac{dp}{p}(I_0 + Kg).$$

Now, by re-arranging the equation above as

$$\frac{dg}{I_0 + Kg} = \frac{dp}{p},$$

a straightforward integration leads to the trading gain formula

$$g(t) = \frac{I_0}{K}\left[\left(\frac{p(t)}{p_0}\right)^K - 1\right]$$

and resulting investment, as a function of price, given by

$$I(t) = I_0 \left(\frac{p(t)}{p_0} \right)^K .$$

Notice that the non-negativity of $I(t)$ assures that this type of controller never leads to a short position. It now remains to obtain a solution for the associated account value $V(t)$. Indeed, rewriting the infinitesimal dV equation in time derivative form and substituting the solution for $I(t)$ above, after some algebra, we obtain the *closed loop account value equation*

$$\frac{dV}{dt} = mV + I_0 \left(\frac{p}{p_0} \right)^K \left[\frac{1}{p}\frac{dp}{dt} - m \right]$$

whose solution is given by

$$V(t) = e^{mt}V_0 + I_0 \int_0^t \left(\frac{p(\tau)}{p_0} \right)^K \left[\frac{1}{p(\tau)}\frac{dp(\tau)}{d\tau} - m \right] e^{m(t-\tau)} d\tau.$$

3.2 Saturation Regime

Letting $[t_*, t]$ denote a time interval over which we have $I = I_{max}$, the incremental trading gains above become $dg = \frac{dp}{p} I_{max}$ which integrates to

$$g(t) = g(t_*) + I_{max} \log \left(\frac{p(t)}{p(t_*)} \right)$$

where t_* is the time at which this saturation begins. From the analysis for the non-saturation regime above, it is easy to see that the first such time is given when the price reaches level

$$p_* = \Delta^{\frac{1}{K}} p_0$$

and in a manner analogous to the non-saturating case, the corresponding value function $V(t)$ can be obtained.

4. PROPERTIES AND ROBUSTNESS RESULTS

The saturation reset controller described in the preceding section has a number of robustness properties. That is, via the next two lemmas and the theorem which follows, we establish properties that are enjoyed by $V(t), I(t)$ and $g(t)$ which are guaranteed to hold no matter what price realization $p(t)$ occurs within the designated subclass of $\mathcal{P}$. Due to space limitations, only sketches of the proofs are given. The first such property is that one never requires the use of margin. That is, along any admissible price trajectory $p(t)$, the condition $V(t) \geq I(t)$ is always satisfied.

4.1 No-Margin Lemma

For any $p \in \mathcal{P}$, it follows that the saturation reset controller results in a closed loop system satisfying $V(t) \geq I(t)$ for all $t \in [0, T]$.

Sketch of Proof: It suffices to prove the result with zero money market rate $m = 0$. That is, the $V(t)$ resulting from $m = 0$ will be a lower bound and $I(t)$ is invariant to m. Now, for notational simplicity, we work with the relative price $\rho(t) \doteq \frac{p(t)}{p_0}$. It suffices to show that the price $\rho_* = \Delta^{\frac{1}{K}}$ at which saturation occurs is no greater than the price, call it ρ_M, at which use of margin is triggered. Indeed, by setting $V(t) = I(t)$ to trigger margin, a straightforward computation leads to

$$\rho_M = \left[\left(\frac{K}{K-1} \right) \left(\Delta - \frac{1}{K} \right) \right]^{\frac{1}{K}} .$$

To complete the proof, one calculates the quotient

$$\frac{\rho_M}{\rho_*} = \left(\frac{K}{K-1} \left[1 - \frac{1}{K\Delta} \right] \right)^{\frac{1}{K}}$$

from which it can be deduced, using $\Delta \geq 1$, that $\rho_M \geq \rho_*$.

The second robustness property which we now establish is that the saturation reset controller results in trading gains which depend only on the initial price p_0, the maximal price p_{max} and the final price $p(T)$. That is, the trading gains $g(T)$ are invariant to "wiggles" in $p(t)$ for $0 \leq t \leq T$.

4.2 Price Wiggle Invariance Lemma

The saturation reset controller results in terminal gain $g(T)$ which depends only on p_0, p_{max} and $p(T)$.

Sketch of Proof: For the case when $p(t)$ never enters the saturation regime, the result follows from the formulae given in Section 3. In this case p_{max} does not enter $g(T)$. On the other hand, for the case when saturation occurs, we consider the subcase when $p(t)$ increases monotonically to p_{max} and then decreases monotonically to $p(T)$. Using the formulae in Section 3, a lengthy computation, leads to

$$g(T) = \frac{1}{K} \left(\frac{p(T)}{p_{max}} \right)^K + \left(\frac{p_{max}}{p_0} \right) \Delta^{-\frac{1}{K}} - \frac{1}{K\Delta} - 1.$$

In other words, the gains are invariant to the time variations in the price. Finally, to complete the sketch of the proof, we note that the case of non-monotonic price variations can be reduced to the monotonic case.

4.3 Arbitrage-Like Possibilities

The saturation reset controller has an important arbitrage-like property which we now illustrate in the context of round-trip price variations. In the full version of this paper, analogous arbitrage results will also be given for the classes of bullish and bearish price variations. More specifically, the main result in this section is that the saturation reset controller will outperform a buy-and-hold strategy whenever a round trip in prices occurs which includes saturation. Whereas the buy-and-hold leads to $g(T) = 0$, we show below that a the saturation reset controller will result in a strictly positive gain $g(T) > 0$. While this property is a plus over this class of round-trip variations, it should be noted that there is no "free lunch." In this regard, we note that there are cases for which a buy-and-hold strategy will result in superior performance over saturation reset control; e.g., a suitably large monotonic bullish price variation which outweighs the gains attributable to money market. On the flip side, for a bearish price variation, as illustrated by examples in the examples to follow, the saturation reset control results in superior performance over buy-and-hold. This is important when capital preservation is emphasized.

4.4 Arbitrage Theorem

For $\gamma > \Delta^{\frac{1}{K}}$ and $p \in \mathcal{P}_0(\gamma)$, the trading gain is given by

$$g(T) = \gamma\Delta^{-\frac{1}{K}} + \frac{1}{K\gamma^K} - \frac{1}{K\Delta} - 1.$$

Moreover, for such a round-trip price variation, it follows that $g(T) > 0$.

Proof: To focus on the dependence on γ, we use the shorthand notation $G(\gamma)$ to denote the value of $g(T)$ with all parameters but γ fixed. With $\gamma_* \doteq \Delta^{\frac{1}{K}}$ denoting the saturation value for γ, observing that that $G(\gamma_*) = 0$, to complete the proof, it suffices to show that $dG/d\gamma > 0$ for all $\gamma > \Delta^{\frac{1}{K}}$. Indeed, a straightforward differentiation yields

$$\frac{dG}{d\gamma} = \Delta^{-\frac{1}{K}} - \frac{1}{\gamma^K \log(\gamma)}.$$

Since $p_{max} > p_0$ forces $\gamma > 1$, using lower bound $\gamma > \Delta^{\frac{1}{K}}$, the positivity of $G(\gamma)$ and hence $g(T)$ is immediate.

4.5 Example

To illustrate the use of synthetic prices in the context of saturation reset control and the Arbitrage Theorem, we consider the synthetic price variation

$$p(t) = 10 + 3\sin 2\pi t + 0.5\sin 9\pi t.$$

with terminal time $T = 1$ representing one year and money market rate of 5% per annum. We implement the saturation reset controller with feedback gain $K = 4$, maximum investment $I_{max} = \$10,000$ and initial investment $I_0 = \$5,000$. Note that the price variation above is in $\mathcal{P}_0(\gamma)$ with $\gamma \approx 1.346$. Hence the Arbitrage Theorem predicts that we should improve upon the break-even performance of buy-and-hold. This is verified in Figures 2 and 3 where trading gains and investment are plotted. From these two figures, one can also observe the controller performance with respect to the classes of bullish and bearish price variations. For example, over the period between that the maximum price and its first zero crossing, we observe the investment is rapidly attenuated and leads to a loss which is far less than the one resulting from buy-and-hold.

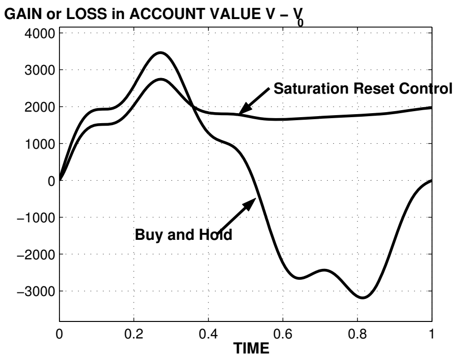

Figure 2: Saturation Reset Versus Buy and Hold

5. PRACTICAL IMPLEMENTATION

As noted in the introduction, the use of synthetic prices $p(t)$ with their required smoothness is an idealization. In this section, we consider the saturation reset controller in a more real world environment. To this end, we now introduce sampling time between trading opportunities and note that the scale of the inter-sample times has no effect on the model equations below provided the appropriate rate constant m is used; e.g., the constant m could equally well be an interest rate per day or an interest rate per annum. Indeed, we let $p(k), V(k), I(k)$ and $g(k)$ denote the discrete-time counterparts of $p(t), V(t), I(t)$ and $g(t)$ respectively. Now, beginning from the initial state $p(0) = p_0, V(0) = V_0, I(0) = I_0$ and $g(0) = 0$ and defining the one period rate of return

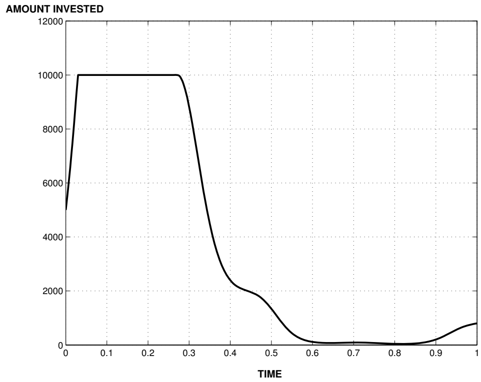

Figure 3: Investment Using Saturation Reset

$$\rho(k) \doteq \frac{p(k+1) - p(k)}{p(k)},$$

it follows from the continuous-time analysis that in the non-saturation regime, the dynamic update equations are

$$\begin{aligned} g(k+1) &= g(k) + \rho(k)I(k); \\ V(k+1) &= (1+m)V(k) + (\rho(k) - m)I(k); \\ I(k+1) &= (1 + K\rho(k))I(k). \end{aligned}$$

Note that when the controller saturates, we simply use $I(k+1) = I_{max}$ above.

5.1 Example (NASDAQ Round Trip)

In this example, we illustrate the use of the saturation reset controller in the context of trading an exchange traded fund, the QQQQ, representing the NASDAQ. We consider turbulent fourteen month period beginning on October 18, 1999. During the first half of this period, the raging dot com bull market was in full swing and the index approximately doubled. During the second half of this round-trip, we see the beginning of the dot com bust. Hence, the buy and hold investor breaks even. To simulate the performance of the saturation reset controller, we used maximum investment $I_{max} = \$10000$, initial investment $I_0 = 7500$, controller gain $K = 4$ and a money market rate of 5% per annum. Closing prices for the QQQQ were used in the simulation. Whereas the buy and hold cannot generate a profit on a round trip price variation, the saturation reset controller results in a a gain of \$3582; note that this result is consistent with the Arbitrage Theorem; see also Figure 5 where the amount invested is shown.

6. CONCLUSION

In this paper, a new paradigm for trading of equities was introduced. Instead of assuming a stochastic model for

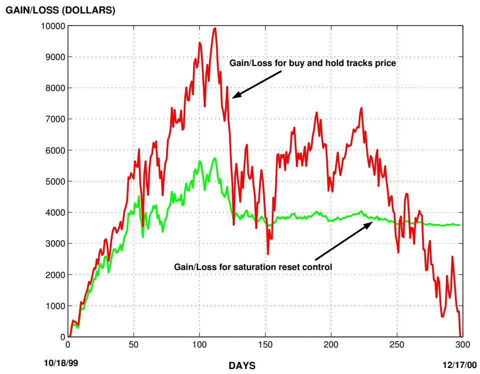

Figure 4: Nasdaq Roundtrip

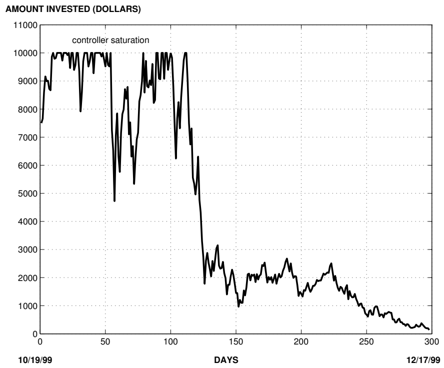

Figure 5: Amount Invested

stock price variations, a robustness approach was pursued. The saturation reset controller was introduced in order to demonstrate the "potential" for use of control concepts in an equity trading context. It may well be the case that future researchers can derive other simple control laws which result in robust performance which is better than that provided here. Finally, it should also be mentioned that this initial work suggests further research on many new and important problems. For example, the choice of the controller gain K, while de-emphasized in this initial work, would be important to consider in the future.

7. ACKNOWLEDGMENT

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