

Dynamic Analysis and Robust Control Design for Stewart Platform with Moving Payloads

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Abstract: The novelty of the paper lies in an extended nonlinear model for the Stewart platform with moving payload. The models found in the existing literature are most of the time only valid for static loads with symmetric configurations, as reflected by the presumed assumption on Moments of Inertia (MOI) being static and symmetric. Such assumption precludes a wide range of systems having asymmetric or moving payloads e.g. stabilized platforms used to stabilize satellite antenna trackers or surgical tables etc. In this paper the actual MOI of the Stabilizing Stewart platform with moving payload are computed and used to parameterize the extended nonlinear model. This model is subsequently used for designing sliding mode controller. The control performance of the proposed algorithm is verified with computer simulations. These simulations demonstrate better stability properties and performance of the proposed dynamic model with much lower uncertainty bounds, as compared to that of the controller designed on the prevalent nonlinear model.

1. INTRODUCTION

The parallel link manipulators attracted many researchers in the resent years due to its precision, rigidity and high load-toweight ratio. A Stewart platform (Stewart, 1965) is a parallel manipulator that provides six-degree-of-freedom (6DOF) i.e. roll, pitch, yaw, surge, sway and heave. Its practical usage is for disturbance isolation, precise machining and flight simulators.

This platform has six variable-length electro-mechanical actuators connecting a top plate to a base plate with spherical joints. Angular and translational motion of the top plate with respect to the base plate is produced by reducing or extending the actuator lengths. The proper coordination of the actuators length enables the top plate to follow the desired trajectory with high accuracy. Thus the six inputs to the Stewart platform in terms of torque are calculated by the controller. The outputs of the Stewart platform are the upper plate's angular and translational positions (in surge, sway, heave, roll, pitch and yaw) sensed by highly precise sensors or estimated by the motor's encoders.

Robust control for parallel manipulators was successfully demonstrated by various authors (Lee and Kim, 1998), (Kim *et al*, 2000), (Huang and Fu, 2004) and (Iqbal and Bhatti, 2006, 2007*a*) etc. The dynamical model used for these controllers is based on the assumption of symmetric and static payloads, which results in constant and diagonal MOI matrix. This supposition looks quite fine for serial robots because payload is static most of the time. But parallel robots are used primarily as base for different payloads e.g. satellite antenna, surgery tables etc. These payloads may be static or dynamics and may also be symmetric or asymmetric. These properties of payload produce a significant effect on the

dynamical equation of parallel robots. Due to these reasons, MOI tensor is no more diagonal and its derivatives may also have significant values.

In papers (Nguyen and Pooran, 1989) and (Lebret et al, 1993), Euler-Lagrange method was used for the dynamical equation of Stewart platform; they considered the Stewart platform dynamics only and took MOI as diagonal matrix. They did not consider the payload dynamics and ignored offdiagonal terms. In work (Dasgupta and Mruthyunjaya, 1998), the dynamics were solved by Newton-Euler method. They also included leg dynamics so inertia matrix was no more diagonal. In reality the leg inertia was very small as compared to payload inertia and it would not cater for the uncertainties in MOI due to payload variation. In research (Liu et al, 2000), the dynamic model of Gough-Stewart platform was analyzed using Kane's equations; they also used MOI as a diagonal matrix. The dynamical model of the Stewart platform CNC machine using Euler-Lagrange method (Ting et al 2004), in the analysis MOI was again a diagonal matrix. In article (Guo and Li, 2006), researcher used Newton-Euler method with Lagrange formulation including the dynamics of legs; they also used MOI as diagonal matrix for the derivation of dynamical equation. The authors (Iqbal et al, 2007b) used Euler-Lagrange method to formulate dynamical equations of Stewart platform for symmetric and asymmetric cases, but their analysis was for stationary payloads. They did not consider the dynamical equation with moving payloads. Generally sufficient accuracy cannot be attained except when the inertia of the payload is also considered in dynamical model of Stewart platform. In particular, for the case where the platform was used as base of the moving payload, the effect of varying MOI is no longer negligible and thus accurate modelling is required to include the moving payload dynamics for better

control performance. If the off-diagonal and time varying terms were ignored then this gives a major source of uncertainty and forcing higher gains for the controller. The controller should be more robust so that it could cater for the uncertainties due to asymmetric nature of the payload, but at the cost of losing performance.

Regarding the control aspect, in recent years many researchers worked on robust controllers for the Stewart platform. In paper (Kang et al, 1996), a Lyapunov based approach for designing robust PD controller was proposed in presence of uncertainties. In article (Lee and Kim, 1998), the model based sliding mode control with perturbation was considered. In work (Kim et al, 2000), robust tracking control design was discussed in the presence of time varying uncertainties. In study (Huang and Fu, 2004), tracking errors drive to zero asymptotically with help of sliding mode controller design. In work (Iqbal and Bhatti, 2006, 2007a), a simple way to calculate control law using sliding-mode technique was suggested. So far, the inclusion of movable pavload dynamics in the controller design is not found by the authors. The cited works considered dynamics of Stewart platform without payload or with static payloads and proceed for controller design. However in the present work the authors have attempted to abridge this gap by quantifying the uncertainties arising from the payload variations and movement.

The payload used for the Stewart platform is satellite antenna with moving parabolic dish. This antenna can perform 2DOF motion, in elevation and azimuth, thus payload can become symmetric as well as asymmetric with movement of the dish. Table 1 shows the characteristics of satellite antenna.

| Elevation Angle (degree) | -5 to 90 | |
|--------------------------|----------|--|
| Azimuth Angle (degree) | 0 to 360 | |
| Dish diameter (mm) | 2400 | |
| Hex Column height (mm) | 1245 | |
| Hex Column radius (mm) | 210 | |
| Total weight (Kg) | 300 | |

Table 1. Characteristic of Satellite Antenna

The schematic diagram of Stewart platform with movable satellite antenna is shown in Fig. 1. Considering the angle with the horizontal axis, the dish can move from -5° to 90° in elevation.

The payload is symmetric when the dish is at 90° elevation. However for other angles the antenna structure becomes asymmetric. Similar study can also be performed for the antenna movement in horizontal plane (azimuth movement). The variation in MOI matrix with moving payload can be seen in Fig. 2. The motion of the payload structure causes variation in the MOI tensor. The asymmetric shape of the antenna around 45° elevation gives rise to significant off-diagonal terms in the MOI matrix. This analysis motivated the authors for extension of the nonlinear model which includes moving payload dynamics.



Fig. 1. The Stewart Platform with moving Load



Fig. 2. MOI for the Moving Payload (in Kg-m²)

The novelty in this paper is an extended nonlinear model for Stewart platform with moving payload using Euler-Lagrange method and calculation of uncertainty bounds for sliding mode control. The rest of this paper is structured as follows; dynamics of Stewart platform with moving payload are explained in Section 2. Section 3 deals with direct slidingmode controller design, and uses a thin layer for chatteringfree control action. Section 4 addresses the computation of MOI and their consequent use in the determination of uncertainty bounds. The design is validated through full envelope nonlinear simulations in Section 5. Conclusions are drawn in Section 6.

2. DYNAMICS WITH MOVING PAYLOADS

The dynamical equation of Stewart platform with symmetric and static payload can be written (Lebret *et al*, 1993) as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau \tag{1}$$

where $q \in R^{6xl}$ is the state vector that represents surge, sway, heave, roll, pitch and yaw motions and can be represented as: $q = [x \ y \ z \ \alpha \ \beta \ \gamma]^T$. The inertial matrix $M \in R^{6x6}$ represents mass and MOI effects. The Coriolis matrix $C \in R^{6x6}$ introduces Centripetal and Coriolis torques. The $\tau \in R^{6xl}$ is the input torque vector.

When the payload is static and symmetric, the MOI is constant and diagonal matrix; off-diagonal terms are almost zero and can be neglected, furthermore MOI derivatives are also zero. Detailed elements of inertial and Coriolis matrices with symmetric and static payload can be seen in (Lebret *et al*, 1993).

For asymmetric and moving payload, off-diagonal terms in MOI matrix have significant values and cannot be neglected as usually ignored by researchers working in this area; likewise MOI derivatives also have significant values, e.g. the considerable MOI derivatives arise from the motion of satellite tracking antenna mounted on the Stewart platform. Inertia matrix for this situation can be seen in appendix A (Iqbal *et al*, 2007*b*). It can be observed from the new inertia matrix that it has off-diagonal MOI terms too and it is sum of both terms as

$$M = M_{\text{nominal}} + M_{\text{off-diag}} \tag{2}$$

e.g. for instance 4x4th element of inertia matrix M becomes:

$$M_{44} = I_{xx} \cos^2 \gamma + I_{yy} \sin^2 \gamma + 2I_{xy} \cos \gamma \sin \gamma$$

It can easily be broken up into two parts as follows

$$(M_{\text{nominal}})_{44} = I_{xx} \cos^2 \gamma + I_{yy} \sin^2 \gamma$$
$$(M_{\text{off-diag}})_{44} = 2I_{xy} \cos \gamma \sin \gamma$$

where M_{nominal} and $M_{\text{off-diag}}$ matrices are due to diagonal MOI and off-diagonal MOI respectively.

The changed Coriolis matrix for moving payload can be seen in appendix A (Iqbal *et al*, 2007*b*). It has some extra MOI derivative terms as well as off-diagonal MOI terms, which are simply adding into nominal Coriolis matrix, so new Coriolis matrix can be identified as

$$C = C_{\text{nominal}} + C_{\text{off-diag}} + C_{\text{varying}}$$
(3)

e.g. consider 4x4th element of Coriolis matrix C

$$C_{44} = \frac{1}{2}\dot{\gamma} \{ (I_{yy} - I_{xx}) \sin 2\gamma + I_{xy} \cos 2\gamma \}$$

+ $\dot{I}_{xx} \cos^2 \gamma + 2\dot{I}_{xy} \cos \gamma \sin \gamma + \dot{I}_{yy} \sin^2 \gamma$
It can easily be broken an interthere part as follows

It can easily be broken up into three parts as follows

$$(C_{\text{nominal}})_{44} = \frac{1}{2}\dot{\gamma} \{ (I_{yy} - I_{xx}) \sin 2\gamma \}$$
$$(C_{\text{off-diag}})_{44} = \dot{\gamma} (I_{xy} \cos 2\gamma)$$

$$(C_{\text{varying}})_{44} = \dot{I}_{xx} \cos^2 \gamma + 2\dot{I}_{xy} \cos \gamma \sin \gamma + \dot{I}_{yy} \sin^2 \gamma$$

where C_{nominal} , $C_{\text{off-diag}}$ and C_{varying} matrices include terms due to diagonal, off-diagonal and time varying MOI respectively. Finally the dynamical equation of Stewart platform with moving payload can be written as

$$\left(M_{\text{nominal}} + M_{\text{off-diag}}\right)\ddot{q} + \left(C_{\text{nominal}} + C_{\text{off-diag}} + C_{\text{varying}}\right)\dot{q} = \tau$$
(4)

3. ROBUST SLIDING-MODE DESIGN

In sliding-mode controller design a hyper-plane is defined as a sliding-surface. This design approach comprises of two stages; first is the reaching phase and second is the sliding phase. In the reaching phase, states are driven to a stable manifold by the help of appropriate control law and in the sliding phase states slide to an equilibrium point. One advantage of this design approach is that the effect of nonlinear terms which may be construed as a disturbance or uncertainty in the nominal plant has been completely rejected. Another benefit accruing from this situation is that the system is forced to behave as a reduced order system; this guarantees absence of overshoot while attempting to regulate the system from an arbitrary initial condition to the designed equilibrium point. A hyper-plane in R^6 as the sliding surface for the Stewart platform is defined as

$$s = \Lambda q + \dot{q} \tag{5}$$

where $q \in R^6$ is state vector. $\Lambda \in R^{6x6}$ is diagonal and positive definite matrix. Equation (5) can also written as

$$\dot{q} = -\Lambda q + s$$

The above system equation is stable if s=0. The rate of convergence of system depends upon the eigen values of matrix Λ . The Lyapunov candidate function (Slotine, 1991) is

$$V = \frac{1}{2} s^T M s \tag{6}$$

It is a positive definite function. Its time derivative is as follows

$$\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s$$
$$\dot{V} = s^T (M \Lambda \dot{q} + M \ddot{q}) + \frac{1}{2} s^T \dot{M} s$$

From (1) and by using skew symmetric property of $(\dot{M} - 2C)$ matrix, the above equation becomes:

$$\dot{V} = s^{T} \left(M \Lambda \dot{q} + C \Lambda q + \tau \right) \tag{7}$$

The M and C matrices are defined in (2) and (3). From (7) control law can be deduced as

$$\tau = -M\Lambda\dot{q} - C\Lambda q - Ksat(s) \tag{8}$$

where vector $K \in \mathbb{R}^{6xl}$, and sat(s) is a saturation function, that provides a very smooth and chatter-free control action and can be defined as follow (Slotine, 1991)

$$sat(s) = \begin{cases} \frac{s(t)}{\|s(t)\|} & \text{if } s > \delta \\ \frac{s(t)}{\|s(t) + \delta\|} & \text{if } s < \delta \end{cases}$$

where δ is a small positive number.

Let perturbation and bounded uncertainties be applied on M and C matrices in (2) and (3), we obtain uncertain matrices with ΔM and ΔC as

$$\tilde{M} = M + \Delta M; \quad \tilde{C} = C + \Delta C$$

The \tilde{M} matrix has some perturbation in nominal terms and also in the off-diagonal terms. Same as \tilde{C} matrix has some perturbation in nominal, off-diagonal and varying terms.

So \tilde{M} and \tilde{C} matrices can also be written as:

$$M = M_{\rm nominal} + \Delta M_{\rm nominal} + M_{\rm off-diag} + \Delta M_{\rm off-diag}$$

 $\tilde{C} = C_{\text{nominal}} + \Delta C_{\text{nominal}} + C_{\text{off-diag}} + \Delta C_{\text{off-diag}} + C_{\text{varying}} + \Delta C_{\text{varying}}$ Put the control law form (8) in (7), we get

$$\dot{V} = s^{T} (\Delta M \Lambda \dot{q} + \Delta C \Lambda q - Ksat(s))$$

Now if $k_{i} > \left\| [\Delta M \Lambda \dot{q} + \Delta C \Lambda q]_{i} \right\|$ (9)
 $\dot{V} = -s^{T} s$

Here \dot{V} always be negative definite for non-zero s. The above condition assures that the sliding surface variable reaches zero in finite time and once the trajectories are on the sliding surface they remain on the surface, and approaches to the equilibrium points exponentially.

4. UNCERTAIN BOUNDS COMPUTATION

Uncertainties in the system can be introduced through two sources which may be model uncertainties or parameter variation due to motion of payloads. Both uncertainties are dealt with in this Section.

As we know, position (q) and velocity (\dot{q}) vectors in the dynamical model always remain bounded due to the mechanical structure limitations therefore system uncertainty always remains bounded. Owing to the factors described above, mass and MOI undergo variations which are assumed as:

$$\begin{split} m &= m + \Delta m ; \\ \tilde{I}_{xx} &= I_{xx} + \Delta I_{xx} ; \\ \tilde{I}_{yy} &= I_{yy} + \Delta I_{yy} ; \\ \tilde{I}_{zz} &= I_{zz} + \Delta I_{zz} ; \\ \tilde{I}_{yz} &= I_{xy} + \Delta I_{yz} ; \\ \tilde{I}_{yz} &= I_{yz} + \Delta I_{yz} ; \\ \end{split}$$

where $\Delta m > \zeta_1$ and $\Delta I_i > \zeta_2$ such that ζ_1 and ζ_2 are positive real numbers. The variation in these parameters causes change in Inertial and Coriolis matrices:

$$\tilde{M} = M + \Delta M;$$
 $\tilde{C} = C + \Delta C$

The ΔM and ΔC matrices can be seen in Appendix B (Iqbal *et al*, 2007*b*). For moving payload, MOI have perturbation therein. These matrices therefore kept generic as given in equations *B-1* and *B-2* (Iqbal *et al*, 2007*b*). According to equation (9), we get general uncertainty matrix inequality

$$\begin{bmatrix} k_{1} \\ k_{2} \\ k_{3} \\ k_{4} \\ k_{5} \\ k_{6} \end{bmatrix} > \begin{bmatrix} \lambda_{1} \Delta m \dot{x} \\ \lambda_{2} \Delta m \dot{y} \\ \lambda_{3} \Delta m \dot{z} \\ \lambda_{4} (\Delta M_{44} - \Delta C_{44}) \dot{\alpha} + \lambda_{5} (\Delta M_{45} - \Delta C_{45}) \dot{\beta} + \lambda_{6} (\Delta M_{46} - \Delta C_{46}) \dot{\gamma} \\ \lambda_{4} (\Delta M_{54} - \Delta C_{54}) \dot{\alpha} + \lambda_{5} (\Delta M_{55} - \Delta C_{55}) \dot{\beta} + \lambda_{6} (\Delta M_{56} - \Delta C_{56}) \dot{\gamma} \\ \lambda_{4} (\Delta M_{64} - \Delta C_{64}) \dot{\alpha} + \lambda_{5} (\Delta M_{65} - \Delta C_{65}) \dot{\beta} + \lambda_{6} (\Delta M_{66} - \Delta C_{66}) \dot{\gamma} \end{bmatrix}$$
(10)

Fig. 2 depicts the significant variation that is taking place in MOI for the payload under different elevation and azimuth angles. From these graphs I_{xx} and other MOI perturbations can be computed and used in the gain computations as in equation (10).

5. SIMULATION RESULTS

For simulation, gain vector *K* of control law can be calculated with the help of uncertainty through the following steps;

- I. Compute the maximum I_{ii} for a asymmetric load (Fig. 2)
- II. Calculate the variation in ΔI_{ii} for the payload with tolerance of $\pm 10\%$.
- III. Get maximum limits of $q \in R^6$ and $\dot{q} \in R^6$ vectors of mechanical structure (Table 2).
- IV. Put these values in equations *B-1* and *B-2* (Iqbal *et al*, 2007*b*) and get the values of ΔM and ΔC matrices.
- V. Compute $\Lambda \in R^{6x6}$ diagonal matrix and put fast dynamics for roll and pitch.
- VI. Put these values in (10) for the bounds of control law K.

Table 2: MOOG's (6DOF5000E) Specifications

| DOF | DISPLAC- | VELOC- | ACCELE- |
|---------------|----------|-----------|---------------------------|
| | EMENT | ITY | RATION |
| | q | ġ | ÿ |
| Surge x | ±0.25 m | ±0.50 m/s | +0.8 G |
| Sway y | ±0.49 m | ±0.76 m/s | +0.8 G |
| Heave z | ±0.52 m | ±0.76 m/s | +0.8 G |
| Roll α | ±24 deg | ±40 deg/s | $\pm 250 \text{ deg/s}^2$ |
| Pitch β | ±22 deg | ±35 deg/s | $\pm 250 \text{ deg/s}^2$ |
| Yaw <i>γ</i> | ±29 deg | ±50 deg/s | $\pm 250 \text{ deg/s}^2$ |

Simulation was performed in order to examine the effectiveness of the proposed controller design. The platform

can perform rotational and translation motion i.e. surge, sway, heave, roll, pitch and yaw.

Simulations were carried out in two phases. In the first phase the model used for the sliding mode controller design was the traditional 6DOF model with no consideration for payload asymmetry or dynamics. Consequently the uncertainty bounds would be much higher as those were supposed to be bigger than the size of uncertainty arising from asymmetry and moving payload. The results are shown in Fig. 3 and Fig. 4.



Fig. 3. Regulation of 6DOF Platform with model based on diagonal terms only



Fig. 4. Control Action for regulation of 6DOF Platform based on model having diagonal terms only

In the second phase the extended 6DOF model as proposed in this paper was used. The extended model contains the asymmetry and moving payload effects. The results are shown in Fig. 5 and Fig. 6. Comparison of the two sets of simulations shows marked improvement in the case of extended model in the form of reduced largest settling times (ca 9 sec versus ca 6 sec) and reduced largest control actions (approximately 300 N-m versus 100 N-m approx). In summary the controller which includes moving payload dynamics achieve stability earlier and perform better.



Fig. 5. Regulation of 6DOF Platform with model based on off-diagonal and dynamic terms (extended model)



Fig. 6. Control action for regulation of 6DOF Platform with model based on off-diagonal and dynamic terms (extended model)

6. CONCLUSIONS

A realistic way of calculating sliding mode controller gains through the determination of uncertainty bounds resulting from the payload motion, leads to judicious choice of sliding mode gains. This obviates the need of keeping the sliding mode controller gains arbitrarily high, thus reducing controller effort significantly. The work showed a realistic way of analyzing model uncertainties and controller robustness for the Stewart platform with moving payload. The proposed extended model can find its use in industry where these platforms are expected to cope with a variety of payload variations.

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