

Backstepping based PID Control Strategy for an Underactuated Aerial Robot

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Abstract: In this paper a nonlinear model of an underactuated quad rotor aerial robot is derived, based on Newton-Euler formalism, and backstepping based PID control strategy is implemented for the derived model. Model derivation comprises determining equations of motion of the quad rotor in three dimensions and seeking to approximate actuation forces through modeling of aerodynamic coefficients and electric motor dynamics. The derived MIMO model, constituted of translational and rotational subsystem, is dynamically unstable. A nonlinear control strategy is therefore implemented for the quad rotor aerial robot. The control strategy includes integral backstepping control for the translational subsystem and backstepping based PID control for the rotational subsystem. The stability of the control design is ensured by Lyapunov stability theorem. The performance of the nonlinear control strategy is evaluated using nonlinear simulation. The simulation results, obtained from backstepping based PID, are compared with conventional optimized PID controller. For the conventional PID controller, the optimization algorithm used is to minimize the Integral of Absolute Error (IAE). Results of comparison validate effectiveness of the backstepping based PID control strategy for the underactuated aerial robot near quasi stationary flight.

1. INTRODUCTION

In the last few years, there has been major interest in developing stabilizing algorithms for underactuated systems. Underactuated systems are systems with fewer independent control actuators than degrees of freedom to be controlled. The interest comes from the need to stabilize systems like ships, underwater vehicles, helicopters, aircraft, airships, hovercrafts, satellites, walking robots, etc., which may be underactuated by design.

Several control strategies based on passivity, Lyapunov theory, feedback linearization, etc. have been developed for the fully actuated case. However the techniques developed for fully actuated systems do not apply directly to the case of underactuated nonlinear systems.

Quad rotor aerial robot is an underactuated system since it has six degrees of freedom (position (x, y, z) , pitch, roll and yaw) and only four control inputs (pitching, rolling and yaw moments and main rotor thrust). Quad rotor aerial robots exhibit a number of important physical effects such as aerodynamic effects, inertial counter torques, gravity effect, gyroscopic effects and friction etc. Due to these effects it is difficult to design a real-time control for aerial robots.

The free body diagram and axes of quad rotor aerial robot is as shown in fig. 1.

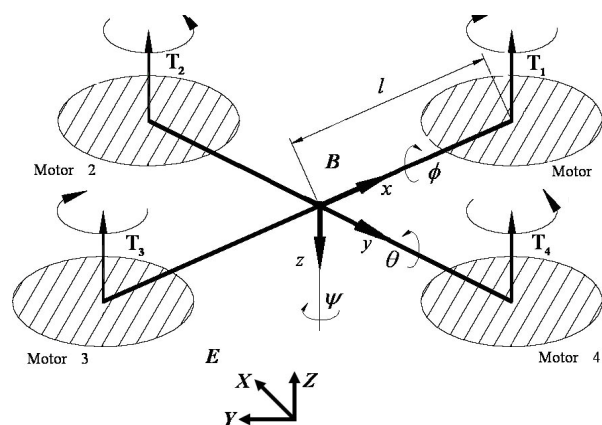


Fig. 1. Forces and moments acting on aerial robot.

In fig. 1, l represents distance of motor from pivot centre. ϕ, θ and ψ represent Euler angles about x, y, z body axis respectively. T_n represents Thrust force produced by each propeller, for $n = [1, 2, 3, 4]$. Earth fixed frame is represented by $E = \{X, Y, Z\}$ and body fixed frame is represented by $B = \{x, y, z\}$.

Increasing or decreasing speed of the four motors together generates vertical motion. When motor pair (3, 1) is allowed

to operate independently then the pitch angle θ (rotation about the y-axis) can be controlled along with the indirect control of motion along the x axis. Similarly when motor pair (2, 4) is allowed to operate independently then the roll angle ϕ (rotation about the x-axis) can be controlled along with the indirect control of motion along the y axis. Finally when motor pair (3, 1) is rotating clockwise and motor pair (2, 4) rotating counter-clockwise, the yaw angle ψ (rotation about the z-axis) can be controlled. The quad rotor aerial robot has now six degrees of freedom.

In most recent works, (McKerrow, 2004) provided a theoretical analysis of a 6 DOF quad rotor. (Pounds et al., 2002) designed a control structure based on internal linearization while (Tayebi et al., 2004) developed a quaternion based PD feedback control scheme for attitude stabilization of quad rotor. However the underactuated quad rotor is treated in such a manner that the underactuated control problem is degenerated to a full actuation one.

When the roll and yaw angles are set to zero, a hovering quad rotor can be viewed as a Planer Vertical Takeoff and Landing (PVTOL) aircraft. Therefore based on the dynamics of a PVTOL aircraft, (Castillo et al., 2004) designed controller for yaw angular displacement and pitch and roll movements of a hovering quad rotor aerial robot. (Mian et al., 2008) provided a mathematical model of an underactuated quad rotor and used affine nonlinear control for the aerial robot.

In this paper backstepping based PID flight control strategy is implemented for rotational subsystem of the quad rotor aerial robot. The main idea is to bring together the robustness against disturbances offered by backstepping (Bouabdallah et al., 2005) and robustness against model uncertainties offered by integral action. Integral action in backstepping was proposed for linear systems by (Kanellakopoulos et al., 1993) and (Krstic et al., 1995).

Main contribution of this paper is to obtain a complete dynamic model of quad rotor aerial robot, based on (Koo et al., 1998), and design integrator backstepping control for translational subsystem and backstepping based PID control for the rotational subsystem of the quad rotor aerial robot. Lyapunov theorem is used to ensure the stability of the system. The results, obtained from backstepping based PID controller, are compared with conventional optimized PID controller to validate the effectiveness of the control strategy for quad rotor aerial robot near quasi stationary flight.

2. QUAD ROTOR DYNAMICS

The main forces and moments acting on quad rotor are those produced by propellers. The aerodynamic forces and moments are derived using a combination of momentum and blade element theory, (Castillo et al., 2005) and (Prouty, 1995). Two propellers in the system are counter rotating propellers such that total torque of the system is balanced. Quad rotor has four motors with propellers. A voltage applied to each motor results in a net torque being applied to the rotor shaft, Q_i , which results in a thrust, T_i . If the rotor disk is moving, there is a difference in relative velocity between the

blade and air when moving through the forward and backward sweep, resulting in a net moment about the roll axis, R_i . Forward velocity also causes a drag force on the rotor that acts opposite to the direction of travel, D_i . These forces and moments can all be related to the square of angular velocity of the blade, Ω , through aerodynamic coefficients C_T , C_D , C_Q and C_R . Let A be blade area, ρ density of air and be r radius of the blade then,

$$T = C_T \rho A r^2 \Omega^2 \quad (1)$$

$$Q = C_Q \rho A r^2 \Omega^2 r \quad (2)$$

It can be assumed at hover, as in (McKerrow, 2004), that,

$$\begin{cases} T_i = b \Omega_i^2 \\ Q_i = d \Omega_i^2 \end{cases} \quad (3)$$

In (3), b and d are constants.

Consider quad rotor as a single rigid body. Assuming that earth is flat and neglecting ground effect, equations of motion for a rigid body subject to body force, $\mathbf{f}^b \in \mathfrak{R}^3$, and body moment, $\boldsymbol{\tau}^b \in \mathfrak{R}^3$, applied at center of mass and expressed in Newton-Euler formalism, as in (Koo et al., 1998), are,

$$\begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}^b \\ \dot{\boldsymbol{\omega}}^b \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^b \times m\mathbf{v}^b \\ \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b \end{bmatrix} = \begin{bmatrix} \mathbf{f}^b \\ \boldsymbol{\tau}^b \end{bmatrix} \quad (4)$$

In (4), $\mathbf{v}^b \in \mathfrak{R}^3$ is body velocity vector, $\boldsymbol{\omega}^b \in \mathfrak{R}^3$ is body angular velocity vector, $m \in \mathfrak{R}$ specifies total mass, $\mathbf{I} \in \mathfrak{R}^{3 \times 3}$ is an identity matrix, and $\mathbf{J} \in \mathfrak{R}^{3 \times 3}$ is an inertial matrix.

2.1 Translational Dynamics

Translational dynamics of the quad rotor are given by,

$$\mathbf{f}^b = \boldsymbol{\omega}^b \times m\mathbf{v}^b + \mathbf{f}_{\text{tot}} \quad (5)$$

In (5), \mathbf{f}_{tot} is defined as,

$$\mathbf{f}_{\text{tot}} = -C_{x,y,z} \left((\mathbf{v}^b)^2 \right) + mg\mathbf{Z} + \sum_{i=1}^4 [-T_i z - D_i(x \ y)] \quad (6)$$

The first term in (6) represents the friction force on quad rotor body during horizontal motion with $C_{x,y,z}$ representing longitudinal drag coefficients, Z defines the vertical axis in inertial coordinates and vector $(x \ y)$ defines the direction of velocity. g represents force due to gravity.

At hover D_i is zero. Let us define u_1 as vertical force input to quad rotor as,

$$u_1 = \sum_{i=1}^4 T_i = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (7)$$

Neglecting friction force and the effect of body moments on the translational dynamics, an expression of forces acting on the quad rotor, from (5), (6) and (7), expressed in inertial axis is given by,

$$m\ddot{X} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)u_1 \quad (8)$$

$$m\ddot{Y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)u_1 \quad (9)$$

$$m\ddot{Z} = mg - (\cos \phi \cos \theta)u_1 \quad (10)$$

2.2 Rotational Dynamics

Assuming that the inertia tensor is diagonal (symmetric design of quad rotor), the rotational dynamics of quad rotor are given by,

$$\boldsymbol{\tau}^b = \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b + \boldsymbol{\tau}_{\text{total}} \quad (11)$$

In (11), $\boldsymbol{\tau}_{\text{total}}$, is defined as,

$$\boldsymbol{\tau}_{\text{total}} = \left[\begin{array}{l} \left(\sum_{i=1}^4 [\mathcal{Q}_i z + R_i(x \ y) + D_i h(-y \ x)] \right) + \\ l(-T_2 + T_4)x + l(T_1 - T_3)y \end{array} \right] \quad (12)$$

In (12), h is vertical distance between propeller centre and CG of quad rotor.

At hover D_i and R_i are zero. Let us define u_2 , u_3 and u_4 as roll actuator input, pitch actuator input and yaw moment input, respectively, to the quad rotor as,

$$\left. \begin{array}{l} u_2 = b(\Omega_4^2 - \Omega_2^2) \\ u_3 = b(\Omega_3^2 - \Omega_1^2) \\ u_4 = d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{array} \right\} \quad (13)$$

Then, from (11), (12) and (13), the rotational dynamics of quad rotor in body axis are given by,

$$J_x \ddot{\phi} = \dot{\theta}\dot{\psi}(J_y - J_z) + lu_2 \quad (14)$$

$$J_y \ddot{\theta} = \dot{\phi}\dot{\psi}(J_z - J_x) + lu_3 \quad (15)$$

$$J_z \ddot{\psi} = \dot{\phi}\dot{\theta}(J_x - J_y) + u_4 \quad (16)$$

3. ENGINE MODEL

On the electrical side of DC motor, a current I flows through the armature according to drive voltage V_a , motor's inductance L , resistance R and back emf voltage V_{emf} , then,

$$V_a - V_{emf} = L \frac{dI}{dt} + RI \quad (17)$$

Motor converts electrical armature current into a mechanical torque, T_m , applied to shaft by, $T_m = K_T I$.

The applied torque produces angular velocity ω_m according to inertia J and motor load T_l , given by,

$$T_m = J \frac{d\omega_m}{dt} + T_l \quad (18)$$

Defining $V_{mf} = K_e \omega_m$, neglecting inductance of the small motor and introducing propeller and gearbox models, then from (17) and (18) we have

$$\dot{\omega}_m = -\frac{K_T K_e}{RJ} \omega_m - \frac{d}{\eta r_g^3 J} \omega_m^2 + \frac{K_T}{RJ} V_a \quad (19)$$

In (18), η is gear box efficiency, d is drag factor, K_e and K_T are constants and r_g is gear box reduction ratio.

4. CONTROL DESIGN STRATEGY

A nonlinear control strategy is implemented to stabilize the quad rotor near quasi stationary flight. The altitude of the quad rotor is stabilized by using the vertical force input u_1 . The desired roll and pitch angles are generated to the rotational controller, from position subsystem. The rotational controller is used to stabilize the quad rotor near quasi stationary flight with inputs u_2, u_3 and u_4 .

4.1 Altitude Control

Altitude subsystem of the quad rotor is given by (10). An integral backstepping control is implemented for altitude subsystem. Let altitude tracking error and its derivative, with Z_d representing desired altitude, be defined as,

$$e_{a1} = Z_d - Z \quad (20)$$

$$\dot{e}_{a1} = \dot{Z}_d - \dot{Z} \quad (21)$$

There is no control input in (21). \dot{Z} represents altitude rate of the quad rotor. Let us consider \dot{Z} be virtual control. Defining desired virtual control $(\dot{Z})_d$ as,

$$(\dot{Z})_d = c_{a1} e_{a1} + K_{a1} \Gamma_1 + \dot{Z}_d \quad (22)$$

In (22), c_{a1} and K_{a1} are positive constants for increasing the convergence speed of the altitude tracking loop and Γ_1 represents integral of altitude error, given by, $\Gamma_1 = \int e_{a1} dt$.

The virtual control $(\dot{Z})_d$ represents the altitude rate of quad rotor and has its own error given by,

$$e_{a2} = (\dot{Z})_d - \dot{Z} = (c_{a1} e_{a1} + K_{a1} \Gamma_1 + \dot{Z}_d) - \dot{Z} \quad (23)$$

From (21) and (22),

$$\dot{e}_{a1} = -c_{a1} e_{a1} - K_{a1} \Gamma_1 + e_{a2} \quad (24)$$

Taking derivative of (23),

$$\dot{e}_{a2} = \left[\begin{array}{l} [c_{a1}(-c_{a1} e_{a1} - K_{a1} \Gamma_1 + e_{a2}) + K_{a1} e_{a1} + \ddot{Z}_d] - \\ \left[g - \left(\frac{\cos \phi \cos \theta}{m} \right) u_1 \right] \end{array} \right] \quad (25)$$

The desirable dynamics of \dot{e}_{a2} are,

$$\dot{e}_{a2} = -c_{a2}e_{a2} - e_{a1} \quad (26)$$

Equation (25) will be negative if u_1 is given by,

$$u_1 = \frac{m}{\cos \phi \cos \theta} \left[\begin{array}{l} g - e_{a1} + c_{a1}^2 e_{a1} - K_{a1} e_{a1} - \\ c_{a1} e_{a2} - c_{a2} e_{a2} - \ddot{Z}_d + c_{a1} K_{a1} \Gamma_1 \end{array} \right] \quad (27)$$

Stability analysis of the proposed method is performed using Lyapunov theory. The candidate Lyapunov function chosen is,

$$V = K_1 \frac{1}{2} \Gamma_1^2 + \frac{1}{2} e_{a1}^2 + \frac{1}{2} e_{a2}^2 \quad (28)$$

Taking derivative of (28) and from (24) and (26), we obtain,

$$\dot{V} = -c_{a1} e_{a1}^2 - c_{a2} e_{a2}^2 \leq 0 \quad (29)$$

Equation (29) is negative semi-definite. For global asymptotic stability of the system, consider Lyapunov global stability theorem, as in (Li et al., 1991). With the help of Lyapunov theorem we can ensure an asymptotical stability starting from a point in a set around the equilibrium. To ensure global asymptotic stability sufficient conditions are fulfilled in our case.

4.2 Position Control

Let \dot{x}_d and \dot{y}_d be desired speeds in x and y direction respectively. Then error in desired and actual speeds is,

$$e_x = \dot{x}_d - \dot{x} \quad (30)$$

$$e_y = \dot{y}_d - \dot{y} \quad (31)$$

Desired roll and pitch angles, in term of the error between actual and desired speed, are thus given by,

$$\phi_d = \sin^{-1}(u_{ex} \sin \psi - u_{ey} \cos \psi) \quad (32)$$

$$\theta_d = \sin^{-1} \left[\frac{u_{ex}}{\cos \phi \cos \psi} - \frac{\sin \phi \sin \psi}{\cos \phi \cos \psi} \right] \quad (33)$$

In (32) and (33), u_{ex} and u_{ey} are given by,

$$u_{ex} = \frac{K_x e_x m}{u_1}, \quad u_{ey} = \frac{K_y e_y m}{u_1}$$

K_x and K_y are positive constants and u_1 is vertical force input from altitude control.

4.3 Rotational Control

Backstepping based PID control technique is implemented for rotational subsystem in which the control inputs u_2 , u_3 and u_4 control the quad rotor at hover.

Let the roll tracking error, with ϕ_d as desired roll, be,

$$e = \phi - \phi_d \quad (34)$$

The first error to be considered in the backstepping design is,

$$z_1 = K_1 e + K_2 \int e dt \quad (35)$$

In (35), K_1 and K_2 are positive tuning parameters and $\int e dt$ represents integral of roll error.

Lyapunov theorem is considered by using the Lyapunov function z_1 positive definite and its time derivative negative semi definite as,

$$V_1 = \frac{1}{2} z_1^2 \quad (36)$$

The derivative of (36) is given by,

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 (K_1 \dot{\phi} - K_1 \dot{\phi}_d + K_2 e) \quad (37)$$

There is no control input in (37). Let us consider $\dot{\phi}$ as the virtual control. Then desired virtual control $(\dot{\phi})_d$ is,

$$(\dot{\phi})_d = \dot{\phi}_d - \frac{K_2}{K_1} e - \frac{c_1 z_1}{K_1} \quad (38)$$

In (38), c_1 is positive constant for increasing the convergence speed of the roll tracking loop.

The virtual control $\dot{\phi}$ represents the roll rate of quad rotor and has its own error given by,

$$z_2 = \dot{\phi} - (\dot{\phi})_d = \frac{1}{K_1} [\dot{z}_1 + c_1 z_1] \quad (39)$$

The augmented Lyapunov function and its derivative is,

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \quad (40)$$

$$\dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2 \quad (41)$$

From (41),

$$\dot{V}_2 = \left[\begin{array}{l} z_2 \left[e \left(K_1^2 + \frac{c_1 K_2}{K_1} \right) + \int e dt (K_1 K_2) + e \left(\frac{K_2}{K_1} + c_1 \right) + \right. \\ \left. \dot{\theta} \dot{\psi} \left(\frac{J_y - J_z}{J_x} \right) + \frac{l}{J_x} u_2 - \ddot{\phi}_d \right] \\ \left. - z_1 [c_1 K_1 e + c_1 K_2 \int e dt] \right] \quad (42)$$

The desirable dynamics are,

$$\dot{V}_2 = -c_2 z_2 = -\frac{c_2}{K_1} [\dot{z}_1 + c_1 z_1] \quad (43)$$

c_2 is a positive tuning parameter in (43).

$$\dot{V}_2 = -\dot{e}(c_2) - e \left(\frac{c_2 K_2}{K_1} + c_1 c_2 \right) - \int e dt \left(\frac{c_2 c_1 K_2}{K_1} \right) \quad (44)$$

Desirable dynamics ensure negative definiteness of position tracking error, its integration and velocity tracking error.

Equation (42) is negative if,

$$u_2 = \frac{J_x}{l} \left[\begin{array}{l} -e \left(\frac{c_2}{K_1} K_2 + c_2 c_1 + K_1^2 + \frac{K_2 c_1}{K_1} \right) - \\ \int edt \left(\frac{c_2 c_1 K_2}{K_1} + K_1 K_2 \right) - \\ \dot{e} \left(c_2 + \frac{K_2}{K_1} + c_1 \right) + \ddot{\theta}_d - \dot{\theta} \dot{\psi} \left(\frac{J_y - J_z}{J_x} \right) \end{array} \right] \quad (45)$$

In regulation, (45) is a PID where the gains of each mode are given by,

$$P = \left(\frac{c_2}{K_1} K_2 + c_2 c_1 + K_1^2 + \frac{K_2 c_1}{K_1} \right),$$

$$D = \left(c_2 + \frac{K_2}{K_1} + c_1 \right) \text{ and } I = \left(\frac{c_2 c_1 K_2}{K_1} + K_1 K_2 \right).$$

Let us consider the characteristic equation of regulation dynamics. The rotational subsystem is both observable and controllable. Pole placement technique is used by placing the poles at desired location to solve for roots of the characteristic equation. Selecting larger values for c_1 and c_2 makes derivative of the Lyapunov function more negative and thus making the regulation dynamics faster.

Similarly u_3 and u_4 are computed

$$u_3 = \frac{J_y}{l} \left[\begin{array}{l} -e \left(\frac{c_4 K_4}{K_3} + c_4 c_3 + K_3^2 + \frac{K_4 c_3}{K_3} \right) - \\ \int edt \left(\frac{c_4 c_3 K_4}{K_3} + K_3 K_4 \right) - \\ \dot{e} \left(c_4 + \frac{K_4}{K_3} + c_3 \right) + \ddot{\theta}_d - \dot{\theta} \dot{\psi} \left(\frac{J_z - J_x}{J_y} \right) \end{array} \right] \quad (46)$$

$$u_4 = \frac{J_z}{l} \left[\begin{array}{l} -e \left(\frac{c_6 K_6}{K_5} + c_6 c_5 + K_5^2 + \frac{K_6 c_5}{K_5} \right) - \\ \int edt \left(\frac{c_6 c_5 K_6}{K_5} + K_5 K_6 \right) - \\ \dot{e} \left(c_6 + \frac{K_6}{K_5} + c_5 \right) + \ddot{\psi}_d \end{array} \right] \quad (47)$$

5. RESULTS AND DISCUSSION

The closed loop system is simulated with nonlinear control algorithm. The angles and their time derivatives of rotational subsystem do not depend on translational components, as evident from 6-DOF equations, however the translations depend on the angles. Rotational control keeps the 3D orientation of the quad rotor aerial robot to the desired value. The rotational controller task is to compensate the initial error, stabilize roll, pitch and yaw angles and maintain them at zero. This is accomplished with the backstepping based PID control strategy.

Table 1 summarizes different system parameters of the quad rotor aerial robot.

Table 1. Quad rotor parameters

Parameter	Value	Units
l	0.3050	m
J_x	0.0154	Kg m ²
J_y	0.0154	Kg m ²
J_z	0.0309	Kg m ²
m	0.6150	kg

The initial conditions for nonlinear simulation are $\phi = \theta = \psi = 30^\circ$, $\dot{\phi} = \dot{\theta} = \dot{\psi} = 30^\circ/\text{sec}$ and $z = 1$ meters. The reference input to the controller are, $\dot{x}_d = \dot{y}_d = 0$, $z_d = 1$ and $\psi_d = 0$. Fig. 2 shows the response of the nonlinear controller to stabilize the quad rotor at hover.

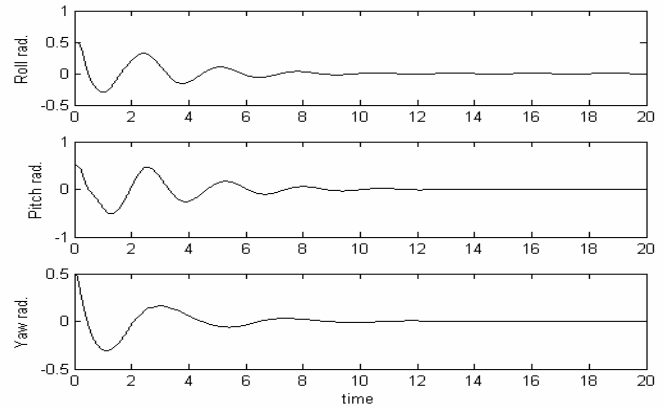


Fig. 2: Attitude control of quad rotor aerial robot

To show effectiveness of the proposed control method, the nonlinear simulation results obtained from backstepping based PID controller are compared with conventional PID controller. For conventional PID, gyroscopic effects are neglected, thus removing the cross coupling while the motor dynamics are included in the dynamic model. An optimization algorithm is used to find the best possible set of parameters. To obtain the parameters for the roll, the pitch and the yaw, the objective function used for the optimization algorithm was to minimize the Integral of the Absolute Error (IAE), as in (Smith et al., 1997). The IAE is a performance criterion that considers the difference between the set point and the output that exists when a system is excited by a step input. The optimization toolbox of matlab was used to obtain the controller's gain for the PID controllers.

Comparison of results from backstepping based PID controller and conventional optimized PID controller are as shown in fig. 3. The backstepping based PID controller is hence more robust and presents better transients than the basic PID controller.

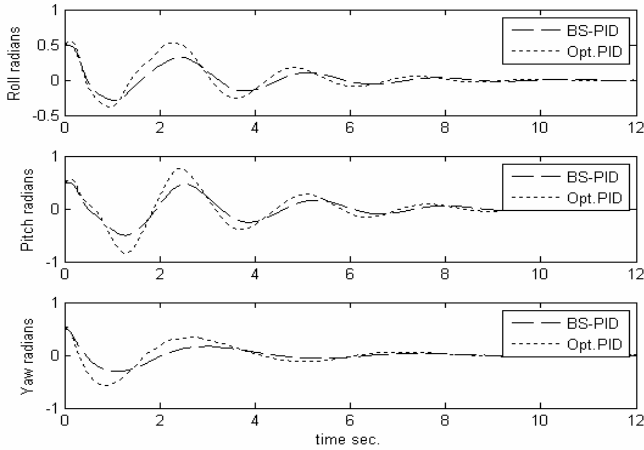


Fig. 3: Comparison of results obtained by Backstepping based PID (BS-PID) and Optimized PID (Opt. PID).

The altitude and position (x, y) response of quad rotor aerial robot is shown in fig. 4 while the control vector response of quad rotor aerial robot is as shown in fig. 5.

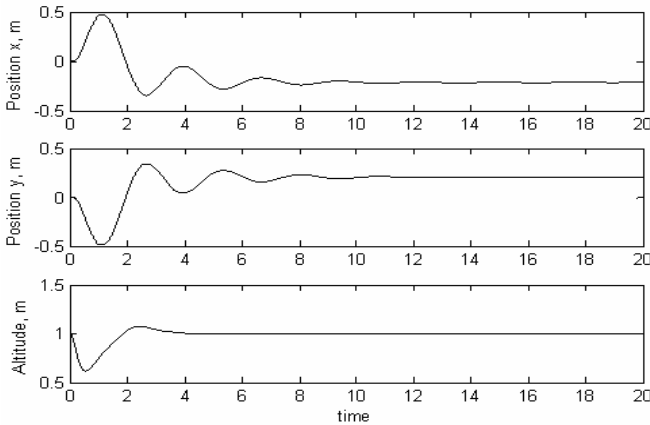


Fig. 4: Altitude and position control of quad rotor aerial robot

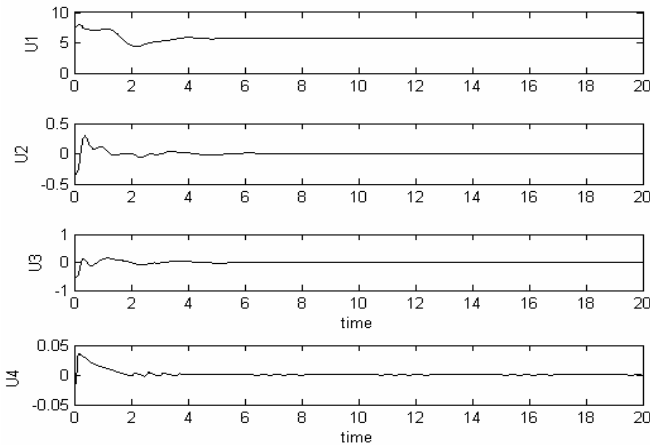


Fig. 5: Control vector response of quad rotor aerial robot

Results from fig. 4 indicate that the position controller effectively drives the attitude controller to maintain the quad rotor over a given point. The integral term in the

backstepping control helps eliminate the steady state error. Results from fig. 5 show that the roll actuator input u_2 , pitch actuator input u_3 , and yaw moment input u_4 stabilize the attitude angles while the vertical force input u_1 overcomes the weight of the quad rotor to hover at a given point.

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