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Quantized Observer-Based Coordination of Linear Multi-Agent Systems ⋆

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Abstract: For the coordination of multi-agent systems with bandwidth limited digital channels, if there are unmeasurable states in agents' dynamics, then to get the estimates of other agents' states by properly designed encoding-decoding scheme is the objective of the cooperative communication and is also the foundation of cooperative control design. In this paper, the concept of cooperatability of digital multi-agent networks is proposed with respect to quantized-observer based communication protocols and certainty equivalence principle based control protocols to characterize the cooperative communication and control in an integrative framework. The cooperatability for linear discrete-time multi-agent systems is investigated with unmeasurable states and limited data rate. Some necessary conditions and sufficient conditions are given for the existence of admissible communication and control protocols to ensure both the cooperative state observation and cooperative stabilization. It is shown that these conditions are quantitatively related to the stabilizability and detectability of agents' dynamics and the topology of the communication network.

1. INTRODUCTION

Coordination of a group of autonomous agents means to achieve some given collective behaviors by schemes of cooperative communication and control among agents. For the coordination of multi-agent systems with digital communication networks, the cooperative communication, which aims at obtaining neighbors' state information as precise as possible, is usually the foundation of the cooperative control design, so the effectiveness of the cooperative control law depends deeply on the quality of the cooperative communication. For real digital networks, communication channels only have finite capacities and the communication between different agents is a process which consists of encoding, information transmitting and decoding. For this case, the instantaneously precise communication is generally impossible and people may seek encoding-decoding schemes to achieve asymptotically precise communication.

Most of the early works on quantized coordination concentrated on quantized averaging or distributed averaging with quantized communication, which assumed that the states of each agent are fully measurable (Kashyap et al., 2007; Frasca et al., 2009; Carli et al., 2010 and Li et al. 2011). For many situations, due to the limited capacity and cost of equipments, one may encounter high-order systems with partially measurable or even unmeasurable states such as unmanned underground vehicles with unmeasurable velocities and mechanical systems with nonholonomic constrains (Qu 2009). The quantized output feedback strategies are suitable and necessary for this case.

Different from the quantized feedback control of single-agent systems, the quantized cooperative control of multi-agent systems with unmeasurable states requires each agent to observe not only state of itself but also those of its neighbors, which may need their input information. What's more, the information exchange between different agents is imprecise due to the limit capacity of digital networks. These make the coordination of multi-agent systems with quantized dynamic output feedback control much more complicated. Li & Xie (2012) proposed a quantized observer-based encoding-decoding scheme for the coordination of second-order multi-agent systems with partially measurable states and limited date rate, which integrated the state observation and communication mechanism together. They proved that if the communication topology graph is connected, then each pair of neighbors only need to transmit two bits information per communication to achieve exponentially fast cooperative stabilization.

All the literature mentioned above (Kashyap, Basar & Srikant 2007; Frasca et al. 2009; Carli, Bullo & Zampieri 2010; Li & Xie 2011 and Li & Xie 2012 etc.) focused on designing specific cooperative communication and control protocols and analyzing the closed-loop performances for

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specific systems. However, a fundamental problem of the coordination of multi-agent systems with digital communication is for what kinds of dynamic networks, there exist admissible communication and control protocols which can guarantee the closed-loop system to achieve given cooperative communication and control objectives jointly, which is proposed as cooperatability in this paper.

For the case with precise communication, the consentability of linear multi-agent systems were studied. The concept of consentability was first proposed by Zhang & Tian (2009) for discrete-time second-order multi-agent systems. Ma & Zhang (2010) studied the consentability of continuous-time linear systems with respect to linear relative state feedback and static output feedback control protocols. It was shown that the agent dynamics and the communication topology have a joint influence on the consentability, for which necessary and sufficient conditions were given. You & Xie (2011) and Gu, Marinovici & Lewis (2012) studied the consentability of single-input linear discrete-time systems. Sufficient conditions were given with respect to (w. r. t.) relative state feedback control protocols in You & Xie (2011) and w. r. t. filtered relative state feedback control protocols in Gu, Marinovici & Lewis (2012).

In this paper, we propose the concept of cooperatability to integrally characterize the cooperative state observation and cooperative stabilization. We consider the cooperatability of linear discrete-time multi-agent systems with unmeasurable states and limited data rate. We propose a class of admissible communication protocols based on quantized-observer type encoders and decoders and a class of admissible control protocols based on the relative state feedback control law and the certainty equivalence principle. We give some necessary conditions and sufficient conditions for achieving cooperative state observation and cooperative stabilization jointly w. r. t. the admissible communication and control protocols. It is shown that the cooperatability of multiagent systems is related to the simultaneous stabilizability and the detectability of the dynamics of agents and the structure of the communication graph.

The rest of this paper is organized as follows. In section 2, we first give a universal framework for the cooperatability problem of general multi-agent systems with unmeasurable states over digital networks. Then we formulate the problem to be investigated and present the admissible communication and control protocols. In section 3, we give the main results of this paper. In section 4, some concluding remarks and future research topics are given.

The following notation will be used. Denote the column vectors or matrices with all elements being 1 and 0 by $\mathbf{1}$ and $\mathbf{0}$, respectively. Denote the identity matrix with an appropriate dimension by I. Denote the sets of positive integers, real numbers, positive real numbers and conjugate numbers by $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{C}$, respectively, and $\mathbb{R}^n$ denotes the n-dimensional real space. For a given vector or matrix X, its transpose(conjugate transpose) is denoted by $X^T(X^*)$, its Euclidean norm is denoted by $\|X\|$ and its infinite norm is denoted by $\|X\|_\infty$. Denote the kth element of any given vector X by $[X]_k$. For a given matrix A, $\rho(A)$ denotes its spectral radius. Define $\mathscr{B}_r^{n\times m} = \{X \in \mathbb{R}^{n\times m} | \|X\| < r\}$ and $\mathscr{B}_r^n = \{x \in \mathbb{R}^n | \ \|x\|_\infty < r\}$, where $r \in \mathbb{R}^+ \bigcup \{+\infty\}$. We use $I(\cdot)$ to represent the identical function with a proper dimension, which means $\forall\ x \in \mathbb{R}^n$, $I(x) = x$.

2. BASIC CONCEPTS AND PROBLEM FORMULATION

In this section, we first give a universal framework to describe the cooperatability of general multi-agent systems over digital networks. Then based on this framework, we formulate the problem to be studied in this paper.

2.1 Basic Concepts

For networked multi-agent systems with N agents, generally, the dynamics of each agent is given by:

$$\begin{cases} x_i(t+1) = f_i(x_i(t), u_i(t)), \\ y_i(t) = g_i(x_i(t), u_i(t)), \end{cases} \quad i = 1, \cdots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ and $y_i(t) \in \mathbb{R}^p$ are the state, input and output of agent i, respectively. Here, the state $x_i(t)$ is not measurable and the output $y_i(t)$ can be measured by agent i. To achieve group coordination, the agents need to communicate and exchange information with each other. The overall communication structure of the system is represented by a directed graph $\mathscr{G} = \{V, \mathscr{E}, \mathscr{A}\}$, where $V = \{1, \cdots, N\}$ is the node set and each node represents an agent; $\mathscr{E}$ denotes the edge set and there is an edge $(j, i) \in \mathscr{E}$ if and only if there is a communication channel from j to i, then, agent i is called the receiver and agent j is called the sender, or i's neighbor. The concepts of neighbor set N_i of agent i, weighted adjacent matrix $\mathscr{A}$ and Laplacian matrix $\mathscr{L}$, which can be referred to (Olfat-Saber & Murray 2004), are omitted here. The eigenvalues of $\mathscr{L}$ in an ascending order of real parts are denoted by $\lambda_1(\mathscr{L}) = 0$, $\lambda_i(\mathscr{L})$, $i = 2, \cdots, N$. The agent dynamic equation (1) and the communication topology graph $\mathscr{G}$ are jointly called a dynamic network denoted by $(f_i, g_i, i = 1, \cdots, N, \mathscr{G})$ (Olfati-Saber & Murray 2004).

In real digital networks, due to the limited capacity of communication channels, only finite bits of data can be transmitted at each time step, therefore, each agent need to first quantize and encode their output into finite symbols before transmitting them. Each pair of adjacent agents uses a digital encoding-decoding scheme to exchange information. For each digital communication channel (i, j), there is an encoder/decoder pair $H_{ij} = (\Theta_{ij}, \Psi_{ij})$ associate with it. The encoder Θ_{ij} maintained by agent i is given by

$$\begin{cases} \xi_{ij}(t+1) = F_{ij}(\xi_{ij}(t), s_{ij}(t)), \\ s_{ij}(t) = Q_{ij}(V_{ij}(\xi_{ij}(t), y_i(t))), \end{cases} \tag{2}$$

where $\xi_{ij}(t) \in \mathbb{R}^l$ and $s_{ij}(t)$ are the inner state and the output of Θ_{ij}, respectively, $Q_{ij}(\cdot)$ is a quantizer, $V_{ij}(\cdot,\cdot)$ is the input function of the quantizer. The decoder Ψ_{ij} maintained by agent j is given by

$$\begin{cases} \zeta_{ij}(t+1) = \tilde{F}_{ij}(\zeta_{ij}(t), s_{ij}(t)), \\ \hat{x}_{ij}(t) = \tilde{G}(\zeta_{ij}(t)), \end{cases} \tag{3}$$

where $s_{ij}(t)$, $\zeta_{ij}(t) \in \mathbb{R}^l$ and $\hat{x}_{ij}(t) \in \mathbb{R}^n$ are the input, the inner state and the output of Ψ_{ij}, respectively. At each time step $t = 0, 1, ...$, agent i generates the symbolic data $s_{ij}(t)$ by the encoder Θ_{ij} and sends $s_{ij}(t)$ to agent j

through the communication channel (i, j). After $s_{ij}(t)$ is received, by the decoder Ψ_{ij}, agent j calculates $\hat{x}_{ij}(t)$ as an estimate of agent i's state. Denote $E_{ij}(t) = x_i(t) - \hat{x}_{ij}(t)$ as the state estimation error.

For the dynamic network $(f_i, g_i, i = 1, \cdots, N, \mathscr{G})$, $\{H_{ji}, i = 1, \cdots, N, j \in N_i | H_{ji} = (\Theta_{ji}, \Psi_{ji})\}$ is called a communication protocol, and the collection of admissible communication protocols is denoted by the admissible communication protocol set $\mathscr{H}$. Here, due to the encoding-decoding errors, the instantaneously precise communication is generally impossible and the ultimate goal of the communication between different agents can be characterized by the concept of *cooperative state observation*. We say that the dynamic network achieves asymptotic cooperative state observation if

$$\lim_{t\to\infty}(x_i(t) - \hat{x}_{ij}(t)) = 0,\ i = 1, \cdots, N, j \in N_i. \quad (4)$$

Accordingly, a cooperative control protocol of the dynamic network is denoted by $\{u_i(t),\ t = 0, 1, ...,\ i = 1, 2, \cdots, N\}$. The control protocol is called distributed if for each i, $u_i(t)$ depends only on the information of agent i and its neighbors j, $j \in N_i$, that is,

$$\begin{aligned} u_i(t) = k_i(&y_i(t), \xi_{il_1}(t), \cdots, \xi_{il_{m_i}}(t), \\ &\hat{x}_{j_1 i}(t), \cdots, \hat{x}_{j_{n_i} i}(t)),\ i = 1, \cdots, N. \end{aligned} \quad (5)$$

Here, $\xi_{il_h}(t), \hat{x}_{j_k i}(t), h = 1, \cdots, m_i, k = 1, \cdots, n_i$ are the inner states of the encoders and the outputs of the decoders maintained by agent i. The collection of admissible control protocols is denoted by the admissible control protocol set $\mathscr{U}$. For the most fundamental case, the objective of the cooperative control is to make the dynamic network achieve cooperative stabilization (also called synchronization), which means $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$, $i, j = 1, \cdots, N$. (Qu 2009). Based on the definition of cooperative state observation and cooperative stabilization, we have the definition of cooperability of a dynamic network.

We say that the dynamic network $(f_i, g_i, i = 1, \cdots, N, \mathscr{G})$ is locally cooperatable w. r. t. $\mathscr{H}$ and $\mathscr{U}$ if for any given positive constants C_1, C_2, C_3, there exist $H \in \mathscr{H}$ and $U \in \mathscr{U}$, such that for any $x_i(0) \in \mathscr{B}^n_{C_1}$, $\xi_{ji}(0) \in \mathscr{B}^l_{C_2}$, and $\zeta_{ji}(0) \in \mathscr{B}^l_{C_3}$, $i \in 1, \cdots, N$, $j \in N_i$, the closed-loop system achieves cooperative state observation and cooperative stabilization under H and U. The dynamic network is called globally cooperatable w. r. t. $\mathscr{H}$ and $\mathscr{U}$, if there exist a communication protocol $H \in \mathscr{H}$ and a control protocol $U \in \mathscr{U}$, such that for any given initial condition, the closed-loop system achieves cooperative state observation and cooperative stabilization under H and U.

2.2 Problem Formulation

In this paper, we consider the cooperatability of linear discrete-time multi-agent systems. The dynamics of each agent is given by

$$\begin{cases} x_i(t+1) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \quad t = 0, 1, \cdots, \quad (6)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$. The communication structure among agents is represented by a directed graph $\mathscr{G}$ as well. Here, the agent dynamics (6) together with the communication topology graph $\mathscr{G}$ is called a dynamic network and denoted by $(A, B, C, \mathscr{G})$.

We propose the following admissible communication protocol set:

$$\begin{aligned} &\mathscr{H}(\rho, L_G) \\ &= \Big\{ H(\gamma, \alpha, \alpha_u, L, L_u, G) = \{H_{ji} = (\Theta_j, \Psi_{ji}), \\ &\quad i = 1, \cdots, N, j \in N_i\}, \gamma \in (0, \rho), \alpha \in (0, 1], \alpha_u \in (0, 1], \\ &\quad L \in \mathbb{N}, L_u \in \mathbb{N}, G \in \mathscr{B}^{n\times p}_{L_G} \Big\}, \end{aligned} \quad (7)$$

where the constants $L_G \in \mathbb{R}^+ \bigcup \{+\infty\}$, $\rho \in (0, 1]$ are the given parameters of the admissible communication protocol set, and γ, G, α, α_u, L, L_u are the parameters of a given communication protocol to be designed. The encoder is given by

$$\Theta_j = \begin{cases} \hat{x}_j(0) = \hat{x}_{j0},\ \hat{u}_j(0) = \hat{u}_{j0}, \\ s_j(t) = Q(\dfrac{y_j(t-1) - C\hat{x}_j(t-1)}{\gamma^{t-1}}), \\ \hat{x}_j(t) = A\hat{x}_j(t-1) \\ \qquad + \gamma^{t-1}Gs_j(t) + B\hat{u}_j(t-1), \\ \hat{u}_j(t) = \hat{u}_j(t-1) + \gamma^{t-1}s_{u,j}(t), \\ s_{u,j}(t) = Q_u(\dfrac{u_j(t) - \hat{u}_j(t-1)}{\gamma^{t-1}}), \end{cases} \quad (8)$$

and the decoder is given by

$$\Psi_{ji} = \begin{cases} \hat{x}_{ji}(0) = \hat{x}_{j0},\ \hat{u}_{ji}(0) = \hat{u}_{j0}, \\ \hat{x}_{ji}(t) = A\hat{x}_{ji}(t-1) \\ \qquad + \gamma^{t-1}Gs_j(t) + B\hat{u}_{ji}(t-1), \\ \hat{u}_{ji}(t) = \hat{u}_{ji}(t-1) + \gamma^{t-1}s_{u,j}(t). \end{cases} \quad (9)$$

Here, $Q(\cdot)$ and $Q_u(\cdot)$ are finite-level uniform quantizers:

$$Q(y) = \begin{cases} 0, & -\frac{1}{2}\alpha \le y < \frac{1}{2}\alpha, \\ i\alpha, & i\alpha - \frac{1}{2}\alpha \le y < i\alpha + \frac{1}{2}\alpha, \\ L\alpha, & y \ge L\alpha - \frac{1}{2}\alpha, \\ -Q(-y), & y < -\frac{1}{2}\alpha, \end{cases}$$

$$Q_u(y) = \begin{cases} 0, & -\frac{1}{2}\alpha_u \le y < \frac{1}{2}\alpha_u, \\ i\alpha_u, & i\alpha_u - \frac{1}{2}\alpha_u \le y < i\alpha_u + \frac{1}{2}\alpha_u, \\ L_u\alpha_u, & y \ge L_u\alpha_u - \frac{1}{2}\alpha_u, \\ -Q_u(-y), & y < -\frac{1}{2}\alpha_u. \end{cases}$$

Remark 2.1. Here the encoder maintained by each agent is a broadcasting type encoder. By (8), all of the encoders associated with the communication channels originated from agent j are the same one, which is denoted by Θ_j. Denote $\xi_j(t) = (\hat{x}_j^T(t), \hat{u}_j^T(t))^T$, $\zeta_{ji}(t) = (\hat{x}_{ji}^T(t), \hat{u}_{ji}^T(t))^T$, then $\xi_j(t)$ is the inner state of Θ_j, $\zeta_{ji}(t)$ is the inner state of Ψ_{ji} and $\hat{x}_{ji}(t)$ is the output of Ψ_{ji}. So the encoder (8) and decoder (9) proposed here are special cases of (2) and (3). From (8) and (9), it's easy to verify that $\hat{x}_{ji}(t) = \hat{x}_j(t)$. So $E_{ji}(t) = x_j(t) - \hat{x}_j(t)$ and is denoted by $E_j(t)$ for short.

For the case with precise communication, Olfati-Saber & Murray (2004) proposed a class of control protocols based on relative state feedback:

$$u_i(t) = K\sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)), \quad i = 1, \cdots, N. \quad (10)$$

Based on (10) and the certainty equivalence principle, we propose the following admissible control protocol set:

$$\mathscr{U}(L_K) = \Big\{U(K) = \{u_i(t) = K\sum_{j\in N_i} a_{ij}(\hat{x}_{ji}(t) - \hat{x}_i(t)), \; t = 0,1,..., \; i = 1,\cdots,N\}, \; K \in \mathscr{B}^{m\times n}_{L_K}\Big\}, \quad (11)$$

where the constant $L_K \in \mathbb{R}^+ \bigcup\{+\infty\}$ is the given parameter of the admissible control protocol set and the gain matrix K is the parameter of a given control protocol to be designed.

Remark 2.2. Different from You & Xie (2011) and Gu, Marinovici & Lewis (2012), here we consider the cooperatability of linear multi-agent systems with unmeasurable states and limited data rate. A quantized-observer based encoding-decoding scheme is proposed to perform the estimation of neighbors' states while decoding. From (9), the decoder has a similar structure as the Luenberger observer. For the case with precise communication, the quantizers degenerate to identical functions and the decoder degenerates to the Luenberger observer. Thus, for each agent $j = 1, \cdots, N$, the output of decoder Ψ_{ji}, $\hat{x}_{ji}(t)$ is an estimate of $x_j(t)$. Note that there is the same observer embedded in encoder Θ_j, so $\hat{x}_j(t)$ is the estimate of $x_j(t)$ by itself, and $E_j(t)$ is the state estimation error.

By the definition of cooperatability in Section 2.1, we say that the dynamic network $(A, B, C, \mathscr{G})$ is cooperatable w. r. t. $\mathscr{H}(\rho, L_G)$ and $\mathscr{U}(L_K)$ for given constants ρ, L_G and L_K with proper range of values, if there exist a communication protocol $H \in \mathscr{H}(\rho, L_G)$ and a control protocol $U \in \mathscr{U}(L_K)$, such that under H and U, the closed-loop system achieves cooperative state observation and cooperative stabilization, that is,

$$(a) \quad \lim_{t\to\infty}(x_i(t) - x_j(t)) = 0, \; i, j = 1, \cdots, N,$$

$$(b) \quad \lim_{t\to\infty} E_j(t) = 0, \; j = 1, \cdots, N.$$

For the coordination of dynamic network $(A, B, C, \mathscr{G})$, it is natural and necessary to make clear whether some admissible communication protocol and control protocol exist before designing them. It is a fundamental problem to investigate quantitatively the factors which will affect the cooperatability of $(A, B, C, \mathscr{G})$. We will investigate these questions in the next section.

3. MAIN RESULTS

In this section, w. r. t. the admissible communication protocol set (7) and admissible control protocol set (11), we give some necessary conditions and sufficient conditions which ensure $(A, B, C, \mathscr{G})$ to be cooperatable. The following assumptions will be used.

A1) There exists $K \in \mathbb{R}^{m\times n}$ such that the eigenvalues of $A - \lambda_i(\mathscr{L})BK$, $i = 2, \cdots, N$ are all inside the open unit disk of the complex plane.

A2) (A, C) is detectable.

For $y \in \mathbb{R}^n$, we denote $Q(y) = (Q(y_1), \cdots, Q(y_n))^T$. Denote $\Delta_j(t-1) = \frac{y_j(t-1) - C\hat{x}_j(t-1)}{\gamma^{t-1}} - s_j(t)$ and $\Delta_{u,j}(t-1) = \frac{u_j(t) - \hat{u}_j(t-1)}{\gamma^{t-1}} - s_{u,j}(t)$ as the quantization errors of $Q(\cdot)$ and $Q_u(\cdot)$, respectively. Denote $\Delta(t) = (\Delta_1^T(t), \cdots, \Delta_N^T(t))^T$, $X(t) = (x_1^T(t), \cdots, x_N^T(t))^T$, $\hat{X}(t) = (\hat{x}_1^T(t), \cdots, \hat{x}_N^T(t))^T$, $U(t) = (u_1^T(t), \cdots, u_N^T(t))^T$, $\hat{U}(t) = (\hat{u}_1^T(t), \cdots, \hat{u}_N^T(t))^T$, $E(t) = X(t) - \hat{X}(t)$, $H(t) = U(t) - \hat{U}(t)$, $\delta(t) = X(t) - \left(\frac{1}{\mathbf{1}^T\pi}\mathbf{1}\pi^T \otimes I_n\right)X(t)$, where π^T is the nonnegative left eigenvector w. r. t. the eigenvalue 0 of $\mathscr{L}$ and it is easy to verify that π^T has at least one nonzero element. Here, $\delta(t)$ is called the cooperative stabilization error. We have the following theorems.

Theorem 3.1. For the dynamic network $(A, B, C, \mathscr{G})$ and $L_G = +\infty$, $L_K = +\infty$ and $\rho = 1$, suppose that Assumptions A1) and A2) hold. Then, for any given positive constants C_x, $C_{\hat{x}}$ and $C_{\hat{u}}$, there exist a communication protocol $H(\gamma, \alpha, \alpha_u, L, L_u, G) \in \mathscr{H}(\rho, L_G)$ and a control protocol $U(K) \in \mathscr{U}(L_K)$ such that for any $X(0) \in \mathscr{B}^{nN}_{C_x}$, $\hat{X}(0) \in \mathscr{B}^{nN}_{C_{\hat{x}}}$ and $\hat{U}(0) \in \mathscr{B}^{mN}_{C_{\hat{u}}}$, the dynamic network $(A, B, C, \mathscr{G})$ achieves cooperative state observation and cooperative stabilization under H and U, and there exist positive constants W and W_u independent of γ, α, α_u, L, L_u, G and K, such that $\sup_{t\geq 0}\sup_{1\leq i\leq N}\|\Delta_i(t)\|_\infty \leq W$ and $\sup_{t\geq 0}\sup_{1\leq i\leq N}\|\Delta_{u,i}(t)\|_\infty \leq W_u$.

The conclusion of Theorem 3.1 follows directly from Theorem 3.1 of Meng and Li (2014), in which the constructive procedure of admissible communication and control protocols are given under the conditions of Theorem 3.1.

Remark 3.1. Theorem 3.1 says that if Assumptions A1) and A2) hold, then there exist admissible communication and control protocols to ensure the cooperative state observation and cooperative stabilization. Furthermore, the quantization errors are uniformly bounded, which implies that the inputs of the quantizers are always bounded. Obviously, the boundedness of the inputs of the quantizers is good for the physical realization of the protocols.

Theorem 3.1 shows that Assumptions A1) and A2) are sufficient conditions for the cooperatability of $(A, B, C, \mathscr{G})$ w. r. t. the quantized-observer based communication protocol set and the certainty equivalence principle based control protocol set. What's more, we find that they are also necessary conditions if $\rho < 1$.

Theorem 3.2. For $(A, B, C, \mathscr{G})$ and finite $L_G > 0$, $L_K > 0$ and $\rho \in (0, 1)$, if for any given positive constants C_x, $C_{\hat{x}}$ and $C_{\hat{u}}$, there exist a communication protocol $H(\gamma, \alpha, \alpha_u, L, L_u, G) \in \mathscr{H}(\rho, L_G)$ and a control protocol $U(K) \in \mathscr{U}(L_K)$, such that for any $X(0) \in \mathscr{B}^{nN}_{C_x}$, $\hat{X}(0) \in \mathscr{B}^{nN}_{C_{\hat{x}}}$ and $\hat{U}(0) \in \mathscr{B}^{mN}_{C_{\hat{u}}}$, the closed-loop system achieves cooperative state observation and cooperative stabilization under H and U, and $\sup_{t\geq 0}\sup_{1\leq i\leq N}\|\Delta_i(t)\|_\infty \leq W$ and $\sup_{t\geq 0}\sup_{1\leq i\leq N}\|\Delta_{u,i}(t)\|_\infty \leq W_u$, where W and W_u are positive constants independent of γ, α, α_u, L, L_u, G and K, then Assumptions A1) and A2) hold.

Proof: We will use reduction to absurdity. Suppose that for any positive constants $C_x, C_{\hat{x}}, C_{\hat{u}}$, there exist a communication protocol $H(\gamma, \alpha, \alpha_u, L, L_u, G) \in \mathscr{H}(\rho, L_G)$ and a control protocol $U(K) \in \mathscr{U}(L_K)$ such that under these protocols, for any $X(0) \in \mathscr{B}^{nN}_{C_x}$, $\hat{X}(0) \in \mathscr{B}^{nN}_{C_{\hat{x}}}$ and $\hat{U}(0) \in \mathscr{B}^{mN}_{C_{\hat{u}}}$, the closed-loop system satisfies $\lim_{t\to\infty} E(t) = 0$, $\lim_{t\to\infty} \delta(t) = 0$, $\sup_{t\geq 0}\sup_{1\leq i\leq N} \|\Delta_i(t)\|_\infty \leq W$ and $\sup_{t\geq 0}\sup_{1\leq i\leq N} \|\Delta_{u,i}(t)\|_\infty \leq W_u$, however, A1) or A2) would not hold. Select a constant a satisfying

$$a > \frac{4W_u\|B\|\sqrt{mN}}{1-\rho} + \frac{4L_G W\sqrt{nN}}{1-\rho}. \tag{12}$$

Take $C_x > \sqrt{n(2N-1)}a\|\Phi^{-1}\|$, $C_{\hat{x}} > \sqrt{nN}C_x + a\sqrt{nN}$ and $C_{\hat{u}} > \sup_{K\in\mathscr{B}_{L_K}} \|\mathscr{L}\otimes K\| C_{\hat{x}}\sqrt{nN}$. Now we prove that for such C_x, $C_{\hat{x}}$ and $C_{\hat{u}}$, under any admissible communication protocol and control protocol, there exist $X(0) \in \mathscr{B}^{nN}_{C_x}$, $\hat{X}(0) \in \mathscr{B}^{nN}_{C_{\hat{x}}}$ and $\hat{U}(0) \in \mathscr{B}^{mN}_{C_{\hat{u}}}$ such that the dynamic network can't achieve cooperative state observation and cooperative stabilization jointly, which leads to the contradiction.

Denote the lower triangular Jordan canonical of $\mathscr{L}$ by $diag(0, J_2, \cdots, J_N)$ where J_i is the Jordan chain with respect to $\lambda_i(\mathscr{L})$. We know that there is $\Phi \in \mathbb{R}^{N\times N}$, consisting of the left eigenvectors of $\mathscr{L}$, such that $\Phi\mathscr{L}\Phi^{-1} = diag(0, J_2, \cdots, J_N)$. Let $\Phi = (\pi, \phi_2, \cdots, \phi_N)^T$. Denote $\bar{\Phi} = (\phi_2, \cdots, \phi_N)^T$. Denote $\tilde{\delta}(t) = (\Phi\otimes I)\delta(t)$. Let $\tilde{\delta}(t) = (\tilde{\delta}1(t), \tilde{\delta}_2(t))^T$ where $\tilde{\delta}_1(t) \in \mathbb{R}^n$. By the definition of $\delta(t)$, we can see that $\tilde{\delta}_1(t) \equiv 0$. Suppose that $H(\gamma, \alpha, \alpha_u, L, L_u, G)$ is a given admissible protocol and $U(K)$ is a given control protocol. Similarly to Theorem 3.1 of Meng and Li (2014), substituting $H(\gamma, \alpha, \alpha_u, L, L_u, G)$ and $U(K)$ to the system (6) leads to

$$\begin{aligned}\begin{pmatrix} E(t+1) \\ \tilde{\delta}_2(t+1)\end{pmatrix} &= A(K,G)\begin{pmatrix} E(t) \\ \tilde{\delta}_2(t)\end{pmatrix} \\ &+ \begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes B)H(t) \\ &+ \begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes G)\gamma^t\Delta(t),\end{aligned} \tag{13}$$

where $A(K,G) = \begin{pmatrix} J(G) & \mathbf{0} \\ (\bar{\Phi}\otimes I_n)(\mathscr{L}\otimes BK) & \bar{J}(K)\end{pmatrix}$, $\bar{J}(K) = I_{N-1}\otimes A - diag(J_2, \cdots, J_N)\otimes BK$ and $J(G) = diag(A - GC, \cdots, A - GC)_{nN\times nN}$. Since A1) and A2) don't hold simultaneously, we have $\rho(A(K,G)) \geq 1$ for any admissible communication and control protocol. Transform $A(K,G)$ to its Schur canonical, that is, select a unitary matrix P ($P^* = P^{-1}$), such that

$$P^*A(K,G)P = \begin{pmatrix} \lambda_1(A(K,G)) & & \mathbf{0} \\ \times & \ddots & \\ \times & \times & \lambda_{(2N-1)n}(A(K,G))\end{pmatrix}.$$

Here $\times$ represents the elements below the diagonal of the Schur canonical, and $\lambda_i(A(K,G))$, $i = 1, ..., (2N-1)n$, are eigenvalues of $A(K,G)$ with $|\lambda_1(A(K,G))| = \rho(A(K,G))$.

Denote $Z(t) = P^*\begin{pmatrix} E(t) \\ \tilde{\delta}_2(t)\end{pmatrix}$. From (13), we know that

$$\begin{aligned}&[Z(t+1)]_1 \\ &= \lambda_1(A(K,G))[Z(t)]_1 + \left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes B)H(t)\right]_1 \\ &\quad +\gamma^t\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes G)\Delta(t)\right]_1 \\ &= \lambda_1^{t+1}(A(K,G))[Z(0)]_1 \\ &\quad + \sum_{i=1}^{t}\lambda_1^{t-i}(A(K,G))\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes B)H(i)\right]_1 \\ &\quad + \sum_{i=0}^{t}\lambda_1^{t-i}(A(K,G))\gamma^i\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes G)\Delta(i)\right]_1 \\ &\quad + \lambda_1^{t}(A(K,G))\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes B)H(0)\right]_1.\end{aligned} \tag{14}$$

Let $P = \begin{pmatrix} P_1 \\ P_2\end{pmatrix}$ with $P_1 \in \mathbb{R}^{nN\times n(2N-1)}$ and $P_2 \in \mathbb{R}^{n(N-1)\times n(2N-1)}$. Take $X(0) = (\Phi^{-1}\otimes I_n)\begin{pmatrix}\mathbf{0} \\ P_2\mathbf{a}\end{pmatrix}$ where $\mathbf{a} = a\mathbf{1} \in \mathbb{R}^{n(2N-1)}$ and $\mathbf{0} \in \mathbb{R}^n$, then $\|X(0)\|_\infty \leq \sqrt{n(2N-1)}a\|\Phi^{-1}\|\|P_2\|$. Note that $\|P_2\| \leq \|P\| = 1$, we have $\|X(0)\|_\infty \leq \sqrt{n(2N-1)}a\|\Phi^{-1}\| < C_x$, implying $X(0) \in \mathscr{B}^{nN}_{C_x}$. Take $\hat{X}(0) = X(0) - P_1\mathbf{a}$ and $\hat{U}(0) = -(\mathscr{L}\otimes K)\hat{X}(0)$. Similarly, one can see that $\hat{X}(0) \in \mathscr{B}^{nN}_{C_{\hat{x}}}$ and $\hat{U}(0) \in \mathscr{B}^{mN}_{C_{\hat{u}}}$. By the definition of $\delta(t)$, we have

$$\begin{aligned}\delta(0) &= \left(\left(I_N - \frac{1}{\sum_{i=1}^N \pi_i}\mathbf{1}\pi^T\right)\otimes I_n\right)X(0) \\ &= \left(\left(I_N - \frac{1}{\sum_{i=1}^N \pi_i}\mathbf{1}\pi^T\right)\otimes I_n\right)(\Phi^{-1}\otimes I_n)\begin{pmatrix}\mathbf{0} \\ P_2\mathbf{a}\end{pmatrix}.\end{aligned}$$

Since π^T is the first row of Φ, we can see that $\tilde{\delta}(0) = \begin{pmatrix}\mathbf{0} \\ P_2\mathbf{a}\end{pmatrix}$ and $\tilde{\delta}_2(0) = P_2\mathbf{a}$. By the definition of $E(t)$ and $H(t)$, we know that $E(0) = X(0) - \hat{X}(0) = X(0) - (X(0) - P_1\mathbf{a}) = P_1\mathbf{a}$, and $H(0) = U(0) - \hat{U}(0) = -(\mathscr{L}\otimes K)\hat{X}(0) + (\mathscr{L}\otimes K)\hat{X}(0) = \mathbf{0}$. Since $Z(0) = P^*\begin{pmatrix} E(0) \\ \tilde{\delta}_2(0)\end{pmatrix}$, we have $Z(0) = \mathbf{a}$ and $[Z(0)]_1 = a$.

From (12), we know that

$$\begin{aligned}&\left|\sum_{i=1}^{t}\lambda_1^{t-i}(A(K,G))\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes B)H(i)\right]_1 \right.\\ &\quad\left. + \sum_{i=0}^{t}\lambda_1^{t-i}(A(K,G))g(i)\left[P^*\begin{pmatrix} I_{nN} \\ \mathbf{0}\end{pmatrix}(I_N\otimes G)\Delta(i)\right]_1\right| \\ &\leq \left|\frac{\|B\|\sqrt{mN}W_u(\lambda_1^t(A(K,G)) - \gamma^t)}{\lambda_1(A(K,G)) - \gamma}\right. \\ &\quad\left. + \frac{\|G\|\sqrt{nN}W(\lambda_1^{t+1}(A(K,G)) - \gamma^{t+1})}{\lambda_1(A(K,G)) - \gamma}\right| \\ &\leq \left(\frac{2W_u\|B\|\sqrt{mN}}{1-\rho} + \frac{2L_G W\sqrt{nN}}{1-\rho}\right)|\lambda_1(A(K,G))|^{t+1} \\ &< \frac{a}{2}|\lambda_1(A(K,G))|^{t+1}.\end{aligned} \tag{15}$$

From (14), (15) and note that $H(0) = \mathbf{0}$, we can see that

$$\left|[Z(t+1)]_1\right| \geq \left||\lambda_1(A(K,G))|^{t+1}a - \frac{a}{2}|\lambda_1(A(K,G))|^{t+1}\right| \geq \frac{a}{2}|\lambda_1(A(K,G))|^{t+1}.$$

By the invertibility of P, we know that $\begin{pmatrix} E(t) \\ \delta(t) \end{pmatrix}$ does not vanish as $t \to \infty$. This is in contradiction with that the dynamic network achieves cooperative state observation and cooperative stabilization. So, A1) and A2) hold. □

Remark 3.2. In Theorem 3.2 the communication protocol set parameter $\rho < 1$. Actually, the communication protocol parameter γ can represent the convergence speed of the coordination (for both cooperative state observation and cooperative stabilization). The smaller γ is, the faster the convergence will be. The constant ρ is an upper bound of γ, so it is a uniform upper bound of the convergence speed under any admissible communication protocol. Theorem 3.2 shows that if $(A, B, C, \mathscr{G})$ is locally cooperatable with a uniform exponential convergence speed, then $(A, B, C, \mathscr{G})$ must satisfy A1) and A2).

At present, we still don't know whether A1) and A2) are necessary conditions for $(A, B, C, \mathscr{G})$ to be locally cooperatable w. r. t. $\mathscr{H}(1, L_G)$ and $\mathscr{U}(L_K)$. However, we can show that if $(A, B, C, \mathscr{G})$ is globally cooperatable, then A1) and A2) are necessary.

Theorem 3.3. For $(A, B, C, \mathscr{G})$ and $L_G = +\infty$, $L_K = +\infty$ and $\rho = 1$, if there exist a communication protocol $H(\gamma, \alpha, \alpha_u, L, L_u, G) \in \mathscr{H}(\rho, L_G)$ and a control protocol $U(K) \in \mathscr{U}(L_K)$, such that for any $X(0) \in \mathbb{R}^{nN}$, $\hat{X}(0) \in \mathbb{R}^{nN}$, $\hat{U}(0) \in \mathbb{R}^{mN}$, the closed-loop system achieves cooperative state observation and cooperative stabilization under H and U, and $\sup_{t\geq 0}\sup_{1\leq i\leq N} \|\Delta_i(t)\|_\infty \leq W$ and $\sup_{t\geq 0}\sup_{1\leq i\leq N} \|\Delta_{u,i}(t)\|_\infty \leq W_u$, where W and W_u are positive constants independent of γ, α, α_u, L, L_u, G and K, then Assumptions A1) and A2) hold.

Remark 3.3. For the uniform quantizers $Q(\cdot)$ and $Q_u(\cdot)$, α and α_u represent the quantization precision, αL and $\alpha_u L_u$ represent the quantization range. If $\alpha, \alpha_u \to 0$ and $\alpha L, \alpha_u L_u \to \infty$, then $Q(\cdot)$ and $Q_u(\cdot)$ tend to be identity mappings. So the case with precise communication can be viewed as a limit case of the communication with limited data rate. Similarly, we can also show that Assumptions A1) and A2) are the necessary and sufficient conditions for $(A, B, C, \mathscr{G})$ to be globally cooperatable w. r. t. $\mathscr{H}(\rho, L_G)$ and $\mathscr{U}(L_K)$ for the case with precise communication.

4. CONCLUSION

In this paper we studied the cooperatability of discrete time linear multi-agent systems with unmeasurable states over digital networks. We gave a universal framework for general multi-agent systems over digital networks to describe the cooperative state observation and cooperative stabilization, which are the objectives of cooperative communication and control jointly. Under this framework, we proposed a class of quantized-observer based admissible communication protocols and a class of certainty equivalence principle based admissible control protocols, and provided necessary conditions and sufficient conditions for the multi-agent systems to be cooperatable w. r. t. the admissible communication protocol set and the admissible control protocol set.

As a preliminary research, we assume that the communication channel is both noise-free and delay-free. The case with channel noise and time delay may be the future research topics. Here, we assume the communication topology is fixed. The case with switching topologies is also interesting.

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