A Novel Probability Density Estimating Method for Relative Altitudes of Civil Aircraft in Cruising Phase

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Abstract: With the rapid development of civil aviation, the shortage of airspace resource is becoming one of the main limiting factors for air traffic transportation. Inspired by the ideas of reduced vertical separation minimum and reducing aircraft separation standard, this paper analyzes the distributing characteristics of civil aircraft tracks in cruising level space and presents a novel method for probability density estimation of civil aircraft tracks' relative altitudes in their cruising flight level spaces, and analyzes the distributing characteristics at their cruising altitude direction.

The method is based on QAR data of enough random civil aircraft tracks from a domestic airline. First, we choose altitude data of each track's main cruise phase and calculate altitude values relative to the standard cruise altitude of respective cruising flight level, we call these processes as spatial alignment and the relative altitudes as relative-cruising altitudes or relative-cruise altitudes. Secondly, we do temporal alignment and figure out the relative altitude values of these tracks in panel data form. Then, we estimate the probability density of the relative altitude values using kernel density estimate method, with the least square cross-validation method choosing asymptotically optimal smoothing parameters. Finally, we figure out different confidence track bands, namely confidence interval limits, and do analysis.

All the analysis results show that there is a great deal of airspace margin and space resource potential in each cruising levels. Hence there are a great possibility and expectation that the vertical separation minimum can be reduced again and add flight levels, which is beneficial to increase air traffic capacity and relieve traffic jams.

Keywords: QAR, panel data, relative-cruising altitude, least squares cross-validation, vertical separation

1. INTRODUCTION

With the rapid growth of civil aviation of China, the lack of airspace resource is becoming the most critical restrictive factors for air transportation (Huaqun, 2010). The airspace for aviation is becoming more and more crowded, and "airspace congestion" is gradually obvious and serious (Wen, 2010), leading increasingly more flight delays and waste passengers' much more time. Consequently, it not only increases airlines' operating costs, but also causes losses to passengers. What's worse, the increased fuel consumption due to traffic delays and diversions is making the acute aeronautical environment issues more serious.

Therefore, improving air traffic flow management and improving utilization of airspace resources have become hot topics concerned and have been studied by both the industry and airspace experts. For instance, China has implemented RVSM(reduced vertical separation minimum) in the airspace for civil aviation between 8900m(FL291) and 12500m (FL411) inclusive successfully from Nov.22,2007, which added the flight levels available from 7 levels to 13 levels, increasing our civil airspace capacity significantly. Although our studies about air traffic flow management and collaborative decision-making mechanisms start later than America and Europe, Some academics, especially HuMinghua and XuXiaohao, have carried out some useful studies on the key technology or theories, and have obtained a lot of achievements (Tian, 2009; Chen, 2009). But most of the studies as well as the achievements are macroscopic, few from microscopic aspect.

Inspired by the ideas of RVSM and reducing aircraft separation standard, this paper analyzes the civil aviation aircraft distribution in cruising level space, presenting a novel estimating method under character of panel data for cruising altitudes distribution probability density. The method is based on QAR data of enough random flights and tracks of several different types 0f aircrafts from a domestic airline. As for each track, we choose the data of its main cruise phase (we mean the main cruise phase as the longest continuous cruise phase in the same flight level), do temporal mapping-alignment and spatial alignment, then figure out their altitude values relative to lower boundary of respective cruising flight level. Afterwards, using least squares cross-validation method with kernel density estimation method, we analyze density and confidence interval limits of spatial distribution of the relative altitude data. The results show a good and promising utilization potential of level space resource of China civil aviation from statistical views.

2. QAR DATA AND PANEL DATA

Quick Access Recorder(QAR), an important record device of flight parameter, is widely used by civil airplanes. It can collect thousands of data simultaneously including most of the operation data. Data Mining Techniques based on QAR has found applications in flight technique check, security assessment, safety accident investigation, and aircraft maintenance, etc. (Li, 2011). But the research on probability density analysis of aircraft tracks is still empty.

The source of data is the altitude parameters of QAR data of 600 random flight tracks of different types of aircraft from a domestic airline. According to flight characteristics of civil aircraft during cruise phase, we choose altitude values in the main cruise phase of each track (main-cruise altitude data). We will use the data in panel data forms.

Panel data set is one that follows a given sample of individuals over time, and thus involves two dimensions: a cross-sectional dimension and a time-series dimension (Cheng, 2003). Therefore panel data is better able to identify and measure effects that are simply not detectable in pure cross-section or pure time-series data. It allows us to observe how the individual living standards change during the development process. Furthermore, panel data gives more informative data, more variability, less collinearity among the variables, more degrees of freedom and more efficiency (Badi, 2003). However, almost all applications of panel data are about econometric modeling and econometric analysis, and several about cluster analysis. Application of panel data to density estimation is still empty so far.

Since analysis of altitude distribution of aircraft tracks is based on panel data, so temporal and spatial alignment must be done before. There will be three kind of temporal-spatial alignment in this paper. The first one is only for distribution of flight levels of sample tracks; the second one and the three one are aim to estimate probability density of sample tracks in flight level space.

3. DISTRIBUTION OF MAIN CRUISING LEVELS

Before mining the potential utilization of airspace resource, we would like to have a look at the distributing characteristics of sample tracks' cruising levels in panel data form. We only need do temporal alignment here. Time linear mapping method is applied to align the altitude data of cruise phase of each track to 100 sampling times.

For example, suppose there is a track with m sampling times in its cruise phase. Let's denote their sampling times in series by t0,t1,t2,...,tm-1, and their altitude by h0,h1,h2,...,hm-1 respectively. According to temporal linear mapping technology, we map these sampling data to n(n < m)sampling times(called "aligned or forge sampling times"). Let's denote these aligned sampling times in series by T0,T1,T2,...,Tm-1, and their altitude by H0,H1,H2,...,Hm-1respectively. Then

(1)T0=t0, H0=h0;

$$(2)$$
Tn-1 =tm-1, Hn-1 =hm-1;

(3)As for aligned sampling times Ti (i=1,2,...,n-2), denote its real sampling time by tx, therefore, x=(m-1)/(n-1)*i. If p is the maximal integer that is not bigger than x, and q is the minimal integer that is not smaller than x. Then

$$T_{i} = t_{p}, H_{i} = \begin{cases} h_{p}, & p = q; \\ h_{p} + \frac{x - p}{q - p} * (h_{q} - h_{p}), p \neq q. \end{cases}$$
(1)

Using temporal linear mapping method, we align the altitude data in cruise phase of each track to 100 forged sampling times, namely choosing m as 100. Then we plot the aligned altitude of the 600 forged tracks in two dimensional form (Fig. 1) and three dimensional form of panel data (Fig. 2).



Fig. 1. Two-dimensional panel data of altitudes: Altitudes at 100 aligned forged sampling times in the main cruise phase of 600 aircraft tracks.



Fig. 2. Three-dimensional panel data of altitudes: Altitudes at 100 aligned forged sampling times in the main cruise phase of 600 aircraft tracks.

In Fig.1 and Fig.2, both x-axis represent the aligned forged sampling times of each track's main cruise phase. In Fig.1, y-axis represents altitude, and its labels are standard cruise altitudes of all flight levels from 6 km to 12 km. We can notice from Fig.1 that, altitudes of all our 600 random sample tracks' main cruising levels are between 6 km to 12

km, and there is at least one cruise track in each flight level between the altitude range spaces.

In Fig.2, not as in Fig.1, z-axis represents altitude while yaxis represents number or sequence of main cruise tracks in each flight level space. Because fuel consumption in different levels varies greatly, most flights would like operate at or closer to more economic levels. As a result, just as vividly illuminated in Fig. 2, some flight levels have too much main cruise tracks and very busy. In other words, economic cruise levels are not enough, or the more the better.

4. RELATIVE CRUISING ALTITUDES

In order to analyze the cruising distribution characteristics of aircraft tracks in level space and utilization of flight level space resource in altitude dimension in panel data form, do temporal alignment and spatial alignment to make altitude data of all tracks into one fight level space. Then estimate the density of the forged sample tracks and do analysis in statistical view.

First, do spatial alignment. Each cruise altitude data minuses the standard cruise altitude of its flight level, figuring out the relative-cruising altitude values. Then all relative-cruising tracks will be cruised in the same flight level space.

Then, do temporal alignment. Two temporal alignment schemes are provided. The first one is called continuous sampling temporal alignment scheme, directly sampling same number of relative-cruising altitude data from each track. In practice, the middle 100 continuous sampling times of relative-cruising altitude is chosen, shown in Fig. 3.





The other one is employing the temporal linear mapping method above, and we called the scheme as mapping forged temporal alignment scheme. For comparison, we map each track into 100 forged aligned sampling times, shown in Fig. 4.



Fig.4. Two-dimensional panel data of relative-cruising altitudes: Relative-cruising altitudes of 600 tracks at 100 aligned forged sampling times in the main cruise phase.

In Fig.3 and Fig.4, both x-axes represent aligned time, and y-axes represent relative-cruising altitude. Because the real time intervals of forged track points of mapping forged temporal alignment scheme are bigger continuous sampling temporal alignment scheme, thus the stability of forged tracks of the second scheme (Fig. 4) is not as good as the first scheme (Fig. 3). Important but not surprisingly, the relative-cruising altitude ranges and their distribution features of the two temporal-spatial alignment scheme are quite similar. As vividly illustrated in Fig.3 and Fig.4, all the relative-cruising altitude data distribute in the spatial range from 12 meter blow standard cruise altitude to 12 meter above standard of respective flight level, leaving the space below -12 meter and above 12 meter relative the standard cruise altitude idle. Furthermore, altitude hold performances of these 600 random sample tracks are generally good and stable. However, for the sake of analyzing distribution features of the relative-cruising altitude, we had better analyze the probability density distribution firstly.

Determination of the probability density function from which a random sample came is a basic and vital problem when making statistical analysis. There are two approaches to determine the unknown density function for a random sample. People can either choose the parametric approach, which requires the assumption that the random sample belongs to a parametric family of distributions and then estimating the unknown parameters, or the nonparametric approach, which requires no assumption about the density of the random sample. An obvious pitfall of the parametric approach is important data structure can be masked when there is no previous knowledge of the sample to assist in the choice of the parametric family of distributions. Thus, a nonparametric approach is a good choice to estimate the unknown density. In practice, the nonparametric approach is known as kernel density estimation (Abdel-Razzaq, 2010; Wang, 2009).

5. KERNEL DENSITY ESTIMATION METHOD

The kernel estimation method is an important and most popular tool in nonparametric density and distribution functions estimation. Given we have observed i.i.d. data X1, X2, ..., Xn from a random distribution with continuous, univariate unknown density f(x), then the kernel density estimate of f(x) is defined as

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right).$$
⁽²⁾

where the kernel K(x) refers any smooth non-negative function satisfies $\int_{-\infty}^{+\infty} K(x) dx = 1$, and the smoothing parameter h is a positive real number, called the bandwidth or window width. In practice, the kernel K is generally chosen to be a unimodal probability density symmetric about zero, such as standard Gaussian kernel:

$$K(x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-x^2/2\right).$$
(3)

Appropriate selection of the bandwidth is critical to the estimation process. If the bandwidth chosen is too large, the kernel density estimator is oversmoothed and key aspects of the true density may not be revealed. On the other hand, if the bandwidth is too small, then the kernel density estimator is undersmoothed (Abdel-Razzaq, 2010).

6. LEAST SQUARES CROSS-VALIDATION FOR BANDWIDTH SELECTION

In order to select the optimal asymptotic value of the bandwidth h and to measure the performance of the kernel density estimator, one must have appropriate error criterion. It is usually desirable, especially from a data analytic viewpoint, to estimate \hat{f} over the entire real line, so we need to consider an error criterion that globally measures the distance between the functions \hat{f} and f. The generally accepted error criteria or performance measures are the integrated squared error(ISE)

$$ISE(\hat{f}_h(x)) = \int \left[\hat{f}_h(x) - f(x)\right]^2 dx \quad . \tag{4}$$

or alternatively, the mean integrated squared error(MISE), i.e.

$$\mathrm{MISE}(\hat{f}_{h}(x)) = E\left[\int \left\{\hat{f}_{h}(x) - f(x)\right\}^{2} dx\right].$$
 (5)

It is evident that ISE is an unbiased estimator of MISE and asymptotically these criteria coincide (Stephan, 1992).

The problem of selecting the smoothing parameter for kernel estimation has been explored by many authors, but there is no method has yet been considered the best in every situation. Most methods are based on minimizing the ISE or the MISE. Among them, least squares cross-validation (LSCV) is a popular and widely studied for choosing the smoothing parameter. It is readily implemented heuristic and completely automatic (Byung, 1990), and it has been shown to have the attractive asymptotic property of given an answer that converges to the optimum under weak

conditions (Berwin, 1993; Charles, 1984). In addition, a major advantage of least squares cross-validation over other methods is that it is widely applicable.

LSCV takes the ISE as its criterion, and chooses the bandwidth as the value of h that minimizes the estimate of $ISE(\hat{f}_h(x))$. The idea is to consider the expansion of the $ISE(\hat{f}_h(x))$ in the following way:

$$ISE(\hat{f}_{h}(x)) = \int [\hat{f}_{h}(x) - f(x)]^{2} dx$$

$$= \int \hat{f}_{h}^{2}(x) dx - 2 \int \hat{f}_{h}(x) f(x) dx + \int f^{2}(x) dx.$$
(6)

Note that the last term does not depend on $\hat{f}_h(x)$, hence on h, so that we only need to consider the first two terms. The ideal choice of bandwidth is the one which minimizes

$$L(h) = ISE(\hat{f}_{h}(x)) - \int f^{2}(x)dx$$

= $\int \hat{f}_{h}^{2}(x)dx - 2\int \hat{f}_{h}(x)f(x)dx.$ (7)

The principle of the least squares cross-validation method is to find an estimator of L(h) from the data and minimize it over h.

As for the first term on the right-hand side in function (7) (Charles, 1984; Wang, 2009):

$$\int \hat{f}_{h}^{2}(x) dx = n^{-2} h^{-2} \sum_{i=1}^{n} \sum_{j=1}^{n} \int K \left(\frac{X_{i} - x}{h} \right) K \left(\frac{X_{j} - x}{h} \right) dx$$

$$= n^{-2} h^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} \int K \left(\frac{X_{i} - X_{j}}{h} - t \right) K(t) dx$$

$$= n^{-2} h^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} K * K \left(\frac{X_{i} - X_{j}}{h} \right).$$
(8)

where $K * K(u) = \int K(u - t)K(t)dt$ is a convolution of the kernel with itself and assuming K is symmetric.

As for the second term (Charles J. Stone, 1984; Wang Xing, 2009):

$$E\left[\int \hat{f}_{h}(x)f(x)dx\right] = \iint f(x)f(y)h^{-1}K\left(\frac{x-y}{h}\right)dxdy$$

$$= E\left[h^{-1}K\left(\frac{X-Y}{h}\right)\right]$$

$$= E\left[\frac{1}{n(n-1)h}\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}K\left(\frac{X_{i}-X_{j}}{h}\right)\right].$$
(9)

This leads to the unbiased estimate of $\int \hat{f}_h(x) f(x) dx$:

$$\frac{1}{n(n-1)h} \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} K\left(\frac{X_i - X_j}{h}\right) = n^{-1} \sum_{i=1}^{n} \hat{f}_{-i,h}(x).$$
(10)

where $\hat{f}_{-1,h}(X_i) = \frac{1}{(n-1)h} \sum_{j \neq i}^n K((x - X_j) / h)$ denotes

the kernel estimator constructed from the data without the observation X_i . Hence the method is commonly called as

the least squared cross-validation because of the so-called leave-one-out density estimator $\hat{f}_{-i,h}(X_i)$ (Simon, 2004).

Hence the LSCV based criterion can be changed from L(h) to

$$LSCV(h) = \int \hat{f}_{h}^{2}(x) - 2n^{-1} \sum_{i=1}^{n} \hat{f}_{-i,h}(X_{i}) .$$
 (11)

since "it is slightly simpler to compute, without affecting the asymptotics." When K is symmetric about zero, There is:

$$LSCV(h) = n^{-2}h^{-1}\sum_{i=1}^{n}\sum_{j=1}^{n}K * K\left(\frac{X_{i} - X_{j}}{h}\right) - 2n^{-1}\sum_{i=1}^{n}\hat{f}_{-i,h}(x_{i})$$
(12)

Before any simulations can be performed, an exact expression for LSCV(h) is needed (Abdel-Razzaq, 2010). In order to compute LSCV(h), Silverman presents an algorithm for the efficient computation of kernel density estimate method by Fourier transform method and the improvements to this algorithm suggested by Jones and Lotwick (Byung, 1990). However, Using a Gaussian kernel, LSCV(h) can be simplified as (Wang, 2009):

$$LSCV(h) = \frac{1}{2\sqrt{\pi}n^{2}h} \sum_{i=1}^{n} \sum_{j=1}^{n} \exp\left(\left(X_{i} - X_{j}\right)^{2} / \left(-4h^{2}\right)\right)$$
(13)
$$- \frac{2}{\sqrt{2\pi}n(n-1)h} \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \exp\left(\left(X_{i} - X_{j}\right)^{2} / \left(-2h^{2}\right)\right).$$

Many practitioners use the LSCV method in this version because of its intuitive definition and its practical flavor.

Before we do density estimation of the sample flight tracks in panel data form, we'd better have a test of the applicability of LSCV for this paper. The tested data source is relative-cruising altitude values at the middle sample time of main cruise phase from 300 random sample tracks. We choose the standard Gaussian kernel and calculate the appropriate bandwidth value by the LSCV algorithm, then estimate the density of the tested altitude data. The density curve estimated is shown in Fig.5 in red line. Meantime, according to the simple distributing features shown in Fig. 3 and Fig. 4, an appropriate frequency histogram of the tested altitude data is given in Fig. 5 for comparative.

The shapes and trends of the density curve and the frequency histogram in Fig.5 are quite similar, which indicates the practicability and appropriateness of the application of the Gaussian kernel and the LCSV method for this paper. As shown by Fig.3 and Fig.4, according to the shapes of the density curve and the frequency histogram in Fig.5, the relative cruising altitude range at this aligned sampling time is between -7 to 8 meter relative to standard cruise altitude of respective flight level, mostly between -2 to 6 meter.





7. DENSITY ESTIMATION OF TRACKS IN CRUISING LEVEL SPACE

Therefore, in order to analysis the distributing characteristics of cruising tracks in flight level space and the margin and potential of space source of flight levels, we would do kernel density estimation of relative-cruising tracks in panel data forms with LSCV method.

First, we do kernel probability density estimation of the relative-cruising altitude data under the continuous sampling temporal-spatial alignment scheme. It means the data source is the source of Fig.3. We select asymptotically optimal bandwidth using LSCV algorithm at each aligned time respectively, then perform kernel density estimation method with the asymptotically optimal bandwidth respectively. The probability density estimation result surface is vividly illustrated in Fig. 6. The probability density curves follow approximately normal distributions, and there shapes and basic distribution characters at all aligned times are very similar, and they are similar to the one in Fig.5 as well. Their peak points are a little above respective standard cruise altitude, and almost all relative-cruising altitude tracks range from -6 to 7 meter relative their respective standard cruise altitude in statistic view. And What's more, the density curves at different times vary so continuously and slightly that it means the altitude-keeping performance of our civil aircrafts is quite stable.

Furthermore, according to probability density estimated values, we compute and figure out the shortest confidence intervals of the relative cruising altitude at different confidence levels. We define the confidence interval as confidence track band (Fig.7). In Fig.7, the middle red bold lines is halve center track line, which means that when we observe some tracks, it is very probable that half of the relative cruising altitude values are above the halve center track line and half blow. Other six pairs lines are upper or

lower boundary lines of confidence track bands at different confidence levels, and the confidence levels are noted by the legends in Fig.7. For instance, the uppermost and lowest red lines are boundary lines of 99% confidence track bands. It means that before observing any track, we can be 98% confident that its relative cruising altitudes will range between the pair red lines. Obviously, from the view of statistical analyses, when continuously observing the airplanes cruise, overwhelming majority of them could hold their relative cruising altitude between -6 to 6 meter, not more than 12 meters.



Fig.6. Density estimated of relative-cruising altitudes at 100 aligned continuous sampling times in the main cruise phase of 600 aircraft tracks.



Fig.7. Confidence track bands of relative-cruising altitudes at 100 aligned continuous sampling times in the main cruise phase of 600 aircraft tracks.

Finally, in order to analyze the distributing characteristics of flight tracks from the whole main cruise phase view, let's estimate the probability distribution density of relativecruising altitude data aligned under the mapping forged temporal alignment scheme, namely the data source is same of Fig. 4, and then calculate the confidence track bands as well. The estimated probability density and confidence track bands are shown in Fig.8 and Fig.9.



Fig.8. Density estimated of relative-cruising altitudes at 100 aligned forged sampling times in the main cruise phase of 600 aircraft tracks.



Fig.9. Confidence track bands of relative-cruising altitudes at 100 aligned forged sampling times in the main cruise phase of 600 aircraft tracks.

Comparing Fig.6 and Fig.8, Fig.7 and Fig.9, it is shown that the stability of forged tracks of the mapping forged temporal alignment scheme is not as good as the continuous sampling temporal alignment scheme, and the range of the mapping forged temporal alignment scheme is a little bigger than that of the continuous sampling temporal alignment scheme. Even through the stabilities of these forged tracks is not as good as the continuous time aligned tracks above, the basic distributing characteristics of forged flight tracks of the two time alignment scheme are quite similar, especially their most used distributing spaces are almost the same and not more than 12 meters. It also illustrate that the sample flight tracks keep their cruise altitude very well.

7. Situation of More Other Tracks

Last but not least, even if our 600 random tracks are of different types of aircrafts, we still use another 2000 random tracks of more different types of aircrafts from another two famouse domestic airlines for analysis under mapping forged temporal alignment scheme. The relative-cruising altitudes of these 2000 new tracks are shown in Fig.10 in panel data form, the probability density estimated in Fig.11, and the confidence track bands in Fig.12. The center track line in Fig.12 is very close to the standard cruising altitude, not as in Fig.9. But the important distribution characteristics of the 2000 random tracks are very similar to those of 600 tracks' situation, and their confidence track bands are more stable.



Fig.10. Relative-cruising altitudes of 2000 tracks at 100 aligned forged sampling times in the main cruise phase.



Fig.11. Density estimated of relative-cruising altitudes at 100 aligned forged sampling times in the main cruise phase of 2000 aircraft tracks.





9. CONCLUSIONS

Novel ideas of QAR data application, panel data applications and probability density estimation are provided in this paper. The density estimation of civil flight tracks is a new kind of analysis of airspace resource utilization and a new idea to find airspace resource potential. The statistical results reveal that total altitude ranges in flight level space of almost all our civil flight tracks during their cruising phase is less than 30 meters, far less than vertical separation standard for civil aircraft. Meanwhile, the altitude keeping performance of our civil aircraft is quite stability (Cai, 2009). It means there is a good potential of flight level space source utilization of China's civil airspace.

In the condition of safety, if we could reduce civil aircraft vertical separation minimum, especially in the cruising airspace, and add more flight levels, it would increase civil airspace capacity significantly. It would not only release "airspace congestion", reduce flight delays or reroute or airholdings, but also allows controllers to plan for, and operate at or closer to the optimum vertical route profile for the particular aircraft type, which would economize on fuel for a particular flight (Paisit, 2004). Moreover, even if reducing the vertical separation standard maybe reduce the safety somehow, it would increase the airspace capacity and allow bigger portrait separation, which are good for safety in other ways. However, if we really want to reduce vertical separation minimum, there are a lot work must be done to determine how much can be reduce. The transportation safety must be guaranteed first of all.

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