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State Dependent Difference Riccati Equation based Estimation for Corkscrew Maneuver

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Abstract: Estimation of corkscrew target maneuver with unknown turning rate is considered. The modeling of the target's equations of motion takes into account the rotation of the velocity and acceleration vectors as the target maneuvers. The inclusion of the more detailed kinematic behavior of the maneuvering target creates nonlinear equations of motion. The state - position, velocity acceleration and jerk, and the angular rate of the velocity vector are estimated. This is done without inclusion of the angular rate into the state vector, but in separate equation. As the equations of motion are nonlinear the State Dependent Differential-Difference Riccati Equation based estimator (SDDRE) is implemented. Two filters are evaluated. The velocity based jerk and acceleration based snap filters are considered. It is demonstrated via simulations that the acceleration based snap filters has improved performance with respect to the velocity based jerk filter.

Keywords: Estimation algorithm, SDRE, SDDRE, estimation of nonlinear system, target tracking.

1. INTRODUCTION

The issue of estimating a maneuvering target is widely treated subject. A comprehensive survey of models and estimators is presented in [Li and Jilkov (2000), (2010),(2001),(2002),(2005)]. The simplest approach is to implement three independent Constant-Step acceleration filters (CA) or Exponentially Correlated Acceleration (ECA) filters [Fitzgerald], one filter for each coordinate.

However these filters may not achieve the required performance for corkscrew maneuver as they are not matched to these maneuvers, i.e. steady state errors are created.

For more advanced estimators it has been understood that incorporating detailed information on the target dynamics and kinematics into the estimator's equations has the potential to increase the quality of estimation. However, the inclusion of more detailed target maneuver model and the related constraints leads to nonlinear models. Thus the Kalman Filter is not directly applicable. The most common approach to deal with nonlinear systems is the Extended Kalman Filter (EKF).

The current approaches to estimation of nonlinear systems include many methods and many publications. A comprehensive survey of such methods applied to maneuvering target estimation is presented in [Li and Jilkov.(2000),(2010),(2001),(2002),(2005)]. In [Tahk and Speyer] the issue of pseudo measurements had been introduced and the Extended Kalman Filter was applied. In [Tahk and Speyer] it was pointed out that inclusion of a constraint is usually difficult to incorporate into the dynamic equation and it is much easier to incorporate them into the measurement equations.

For example: in [Marks] a multiple model approach is applied. In [Chen, Speyer and Lianos] algebraic constraint is incorporated in the state equations; and in [Dezert and Pannetier] the IMM approach is applied to estimate maneuvering target. The corkscrew maneuver estimation is dealt with in [Imado and Miwa][Ryoo, Whang and Tahk][Zarchan and Alpert] [Kim, Vaddi and Menon] [Chadwick and Zarchan] [Kim, Vaddi, Menon and Ohlmeyer, [Rusnak. and Meir] [Speyer, Kimand Tahk.] [Zarchan]. Extensive review of tracking algorithms of corkscrew-barrel roll maneuvers is presented in [Ghosh and Mukhopadhyay] where the jerk has been included into the target's state.

In this paper the corkscrew maneuver [Imado and Miwa] with unknown turning-barrel roll rate is considered. One of the main and important issues is the estimation of the angular turning rate. This is needed for achieving better matching of the estimator and for the derivation of high performance guidance law for this type of evading maneuver [Agarwal]. [Zarchan and Musoff., chapter 10] show that the algorithms that include the turning rate as part of the state are problematic, to say the least.

Two novel approaches to modelling the maneuvering target equations of motion are dealt with here:

i) Rotating velocity vector based jerk equations – VJ (Jerk is the derivative of the Acceleration);

ii) Rotating acceleration vector based snap equations – AS (Snap is the derivative of the Jerk).

In this paper the State Dependent Differential-Difference Riccati Equation (SDDRE) based estimator is applied to the nonlinear equations of motion. The SDDRE approach is very intuitive, although it is not optimal as shown for the State Dependent Algebraic Riccati Equation (SDARE) approach in [Mracek, Cloutier, and D'Souza][Shue and Agarwal][Xin and Balakrishnan][Lam, Anderson and Xin]. The optimal filter requires additional terms for optimality][Shue S and Agarwal][Xi and Balakrishnan][Lam, Anderson and Xin]

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[Xin, Balakrishnan and Ohlmeyer]. Despite the sub-optimality of the direct SDDRE approach it is claimed/known that the SDDRE based estimator is BIBO stable [Haessig and Friedland].

The SDDRE approach to estimation of piecewise coordinated turn and barrel roll target's maneuvers in 2D, based on the equation of motions of a rotating vector, has been presented in [Ryoo, Whang and Tahk] and for piecewise 2D spiralling trajectory in [Zarchan and Alpert].

The novelty in this paper is: Application of the SDDRE to estimation of a 3D corkscrew maneuver with unknown turning rate. Simulations demonstrate the performance of estimators based on the rotating velocity (VJ) and rotating acceleration (AS) vector kinematics equations of motion. It is shown that the AS based estimator performs well for 3D corkscrew target maneuver, and is able to estimate the corkscrew maneuver with unknown turning rate.

2. PROBLEM STATEMENT AND APPROACH TO THE SOLUTION

The problem considered here is the state estimation of the nonlinear stochastic system

$$\dot{x}(t) = f(x(t)) + \Gamma w(t), \quad x(t_o) = x_o, \qquad z(t) = g(x(t)) + v(t) \tag{2.1}$$

where $x(t)$ is the state vector, $z(t)$ is the measurement, $w(t)$, $v(t)$ are the white Gaussian stochastic processes representing the system driving noise and the measurement noise, respectively, $x(t_o)$ is a Gaussian random vector, and

$$\begin{aligned} &E[x_o]=\bar{x}_o, E[w(t)]=0, E[v(t)]=0, \\ &E[w(t)w(\tau)^T]= W\delta(t-\tau), E[v(t)v(\tau)^T]=V\delta(t-\tau), \\ &E[w(t)v(\tau)^T]=0, E[w(t)x_o^T]=0, E[v(t)x_o^T]=0, \\ &E[(x_o-E(x_o))(x_o-E(x_o)^T]=Q_o. \end{aligned} \tag{2.2}$$

All vectors and matrices are of appropriate dimensions.

The problem being considered here is finding the optimal estimate $\hat{x}(t)$ as a functional of $\{z(t), t_o \le t \le t_f\}$ that minimizes the quadratic criterion:

$$J = E\left[[x(t)-\hat{x}(t)]^T \Theta [x(t)-\hat{x}(t)]\right]; \quad \Theta > 0. \tag{2.3}$$

2.1. Estimators for Nonlinear System

For nonlinear systems there are several approaches. Here the SDRE/SDDRE [Mracek, Cloutier, and D'Souza] [Shue and Agarwal][Xin and Balakrishnan][Lam, Anderson, and Xin] approaches are considered. The SDRE approach is based on the dual of the SDRE based nonlinear control [Lam, Anderson, and Xin]. This approach parameterizes the state equation (2.1) into the linear structure called State Dependent Coefficient Form. This approach includes the State Dependent Algebraic Riccati Equation (SDARE) based estimation and the State Dependent Differential-Difference Riccati Equation (SDDRE) based estimation. Then for linear measurement of the state, i.e. $g(x(t))=Cx(t)$, The state equation are represented as

$$\dot{x}(t) = A_s(x(t))x(t) + \Gamma w(t), \quad x(t_o) = x_o, \qquad z(t) = Cx(t) + v(t) \tag{2.8}$$

The SDDRE based estimator is

$$\begin{aligned} &\dot{\hat{x}}(t) = A_s(\hat{x}(t))\hat{x}(t) + K(\hat{x}(t),t)[z(t) - C\hat{x}(t)], \quad \hat{x}(t_o) = \bar{x}_o, \\ &K(\hat{x}(t),t) = Q(t)CV^{-1} \\ &\dot{Q}(t) = A_s(\hat{x}(t))Q(t) + Q(t)A_s^T(\hat{x}(t)) + \Gamma W \Gamma^T \\ &\qquad - Q(t)C^T V^{-1} CQ(t), \; Q(t_o) = Q_o. \end{aligned} \tag{2.10}$$

This is a suboptimal estimator. The optimal estimator has additional terms as detailed in [Shue and Agarwal][Xin, and Balakrishnan][Lam, Anderson and Xin].

3. KINEMATIC EQUATIONS OF MANEUVERING TARGET

Comprehensive survey of modelling the behaviour of a maneuvering target can be found in [Li and Jilkov (2000),(2010),(2001),(2002),(2005)]. Here one specific case is considered. Aerodynamically controlled aircraft is assumed, i.e. velocity perpendicular to acceleration.

3.1. The Kinematics as a Function of Velocity

We assume that the target's velocity is expressed as

$$\vec{v}_T = v_T 1_v \tag{3.1}$$

where

$\vec{v}_T$ - target velocity vector [m/sec]

v_T - absolute value of the target's velocity

1_v - unit vector in the target velocity direction

3.1.1. Velocity based acceleration equations -VA

First order equations of motion based on the target's velocity (3.1) are derived. As the target is maneuvering the velocity vector is rotating. The target's acceleration is given by [Blakelock, J.H.]

$$\vec{a}_T = \frac{d\vec{v}_T}{dt} = \dot{v}_T 1_v + \vec{\omega}_{Tv} \times \vec{v}_T \tag{3.2}$$

$$\vec{\omega}_{Tv} = \frac{\vec{v}_T \times \vec{a}_T}{\|\vec{v}_T\|^2}; \tag{3.3}$$

$\vec{a}_T$ - target acceleration vector [m/sec^2]

$\vec{\omega}_{Tv}$ - angular rate of the target velocity vector [rad/sec]

3.1.2. Velocity based Jerk equations - VJ

Jerk is the derivative of the acceleration. The jerk has been included into the target's state in [Ghosh and Mukhopadhyay]. Second order equations of motion based on the targets velocity (3.1) are derived. As the target is maneuvering the velocity vector is rotating. The jerk (derivative of acceleration) is the given by [Blakelock][Asseo and Ardila].This is called here Velocity based Jerk (VJ) equations of motion. The target's jerk is [Blakelock] [Asseo and Ardila.]

$$\vec{j}_T = \frac{d\vec{a}_T}{dt} = \frac{d^2\vec{v}_T}{dt^2} = \ddot{v}_T 1_v + \dot{\vec{\omega}}_{Tv} \times \vec{v}_T + 2(\vec{\omega}_{Tv} \times \dot{v}_T 1_v) + \vec{\omega}_{Tv} \times (\vec{\omega}_{Tv} \times \vec{v}_T) \tag{3.4}$$

$\vec{j}_T$ - target's jerk vector [m/sec^3]

Substituting $\dot{v}_T 1_v$ from (3.2) gives [Asseo and Ardila]

$$\vec{j}_T = \ddot{v}_T 1_v + \dot{\vec{\omega}}_{Tv} \times \vec{v}_T + 2\vec{\omega}_{Tv} \times \frac{d\,\vec{v}_T}{dt} - \vec{\omega}_{Tv} \times (\vec{\omega}_{Tv} \times \vec{v}_T) \quad (3.5)$$
$$= \ddot{v}_T 1_v + \dot{\vec{\omega}}_{Tv} \times \vec{v}_T + 2\vec{\omega}_{Tv} \times \vec{a}_T - \vec{\omega}_{Tv} \times (\vec{\omega}_{Tv} \times \vec{v}_T)$$

3.2. The Kinematics as a Function of Acceleration

In effort to reduce the computational effort with the SDDRE based on the Velocity based Jerk (VJ) equations of motion, as can be seen in section 6, here an acceleration based equations are derived. For aerodynamically controlled aircraft, under the aforementioned assumptions (the acceleration and velocity are perpendicular) the target's velocity's angular rate is equal to the target's acceleration angular rate. Then if the target's acceleration is expressed as

$$\vec{a}_T = a_T 1_a \quad (3.6)$$

where

$\vec{a}_T$ - target acceleration vector [m/sec^3]

a_T - absolute value of the target's velocity

1_a - unit vector in the target acceleration direction (perpendicular to 1_v)

From (3.7) it is possible to derive the following kinematic relations.

3.2.1. Acceleration based Jerk equations - AJ

First order equations of motion based on the target's acceleration (3.6) are derived. As the target is maneuvering the acceleration vector is rotating. The jerk (derivative of acceleration) is the given by [Blakelock]. This is called Acceleration based Jerk (AJ) equations of motion. The target's jerk is given by

$$\vec{j}_T = \frac{d\,\vec{a}_T}{dt} = \dot{a}_T 1_a + \vec{\omega}_{Ta} \times \vec{a}_T \quad (3.7)$$

$$\vec{\omega}_{Ta} = \frac{\vec{a}_T \times \vec{j}_T}{\|\vec{a}_T\|^2} \quad (3.8)$$

where

$\vec{v}_T$ - target velocity vector [m/sec]

$\vec{j}_T$ - target jerk vector [m/sec^3]

$\vec{\omega}_{Ta}$ - angular rate of the target acceleration vector [rad/sec]

Remark: The angular rate of the target velocity vector (3.3) is not necessarily equal to the angular rate of the target acceleration vector (3.8)!

3.2.2. Acceleration Based Snap equations - AS

Snap is the derivative of the jerk. Second order equations of motion based on the targets acceleration (3.6) are derived. As the target is maneuvering the acceleration vector is rotating. The snap is the given by [Blakelock][Asseo and Ardila]. This is called here Acceleration based Snap (AS) equations of motion. The target's snap is [Blakelock,[Asseo and Ardila]

$$\vec{s}_T = \frac{d\,\vec{j}_T}{dt} = \frac{d^2 a_T}{dt^2} = \ddot{a}_T 1_v + \dot{\vec{\omega}}_{Ta} \times \vec{a}_T \quad (3.8)$$
$$+ 2(\vec{\omega}_{Ta} \times \dot{a}_T 1_v) + \vec{\omega}_{Ta} \times (\vec{\omega}_{Ta} \times \vec{a}_T)$$

$\vec{s}_T$ - target's snap vector [m/sec^3]

Substituting $\dot{a}_T 1_v$ from (4.7) gives the snap [Asseo and Ardila]

$$\vec{s}_T = \ddot{a}_T 1_v + \dot{\vec{\omega}}_{Ta} \times \vec{a}_T + 2\vec{\omega}_{Ta} \times \frac{d\,\vec{a}_T}{dt} - \vec{\omega}_{Ta} \times (\vec{\omega}_{Ta} \times \vec{a}_T) \quad (3.10)$$
$$= \ddot{a}_T 1_v + \dot{\vec{\omega}}_{Ta} \times \vec{a}_T + 2\vec{\omega}_{Ta} \times \vec{j}_T - \vec{\omega}_{Ta} \times (\vec{\omega}_{Ta} \times \vec{a}_T)$$

4. THE VARIANCE AND SPECTRUM OF THE GLINT NOISE

It is assumed that the measurement noise is the glint noise. The spectral density $V_{go}[m^2/Hz]$ of the glint is given by [Papoulis]

$$V_{go} = \sigma_g^2 T_s, \left[\frac{m^2}{Hz}\right] \quad (4.1)$$

is the one presented in [Peled-Eitan and Rusnak (2012)] where σ_g is the standard deviation of the glint noise [Barton and Ward], and T_s is the sampling interval of the frequency agile radar the spectral density of the measurement noise, v(t).

5. INCORPORATION OF THE KINEMATIC CONSTRAINT

The kinematics equations in section 3 are constraints that can be incorporated into the estimator equations. It is possible to incorporate the kinematic constraint into the state equation or measurement equation and the "unknown" quantities are interpreted as either a measurement noise or system driving noise. The approach here has the advantage that the kinematic constraints are incorporated in the system equations, the unknowns are the system driving noise and the measurements are linear.

5.1. Velocity based Kinematic Equation

5.1.1. Jerk kinematic equation - VJ

With velocity based jerk equations (3.5), the kinematics of maneuvering target is modelled as

$$\vec{j}_T = \frac{d\,\vec{a}_T}{dt} = 2\vec{\omega}_{Tv} \times \vec{a}_T - \vec{\omega}_{Tv} \times (\vec{\omega}_{Tv} \times \vec{v}_T) + w_T \quad (5.1)$$

$$w_T = \ddot{v}_T 1_v + \dot{\vec{\omega}}_{Tv} \times \vec{v}_T$$
$$\vec{\omega}_{Tv} = \frac{\vec{v}_T \times \vec{a}_T}{\|\vec{v}_T\|^2}; \quad (5.2)$$

where it is assumed that: (i) the absolute value of target velocity is almost constant, i.e. $\|\vec{v}_T\| \cong constant$; (ii the target's angular turning rate is constant, i.e. $\dot{\vec{\omega}}_T \cong 0$; and (iii) w_T represents the deviation of the actual behaviour of the target from the constant angular turning rate and constant absolute value of the velocity assumptions. We have the state space representation in the State Dependent Coefficient Form in three dimensions (notice that this is a third order differential equation)

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ z \\ \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_y^2+\omega_z^2 & 0 & 0 & -\omega_x\omega_y & -2\omega_z & 0 & -\omega_x\omega_z & 2\omega_y \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\omega_x\omega_y & 2\omega_z & 0 & \omega_x^2+\omega_z^2 & 0 & 0 & -\omega_z\omega_y & -2\omega_x \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\omega_x\omega_z & -2\omega_y & 0 & -\omega_y\omega_z & 2\omega_x & 0 & \omega_x^2+\omega_y^2 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ z \\ \dot{z} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \tag{5.3}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{msr} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ z \\ \dot{z} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

where

$$\vec{\omega}_{Tv} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \frac{\vec{v}_T \times \vec{a}_T}{\|\vec{v}_T\|^2} = \frac{1}{\dot{x}^2+\dot{y}^2+\dot{z}^2}\begin{bmatrix} \dot{y}\,\ddot{z} - \dot{z}\,\ddot{y} \\ \dot{z}\,\ddot{x} - \dot{x}\,\ddot{z} \\ \dot{x}\,\ddot{y} - \dot{y}\,\ddot{x} \end{bmatrix}; \tag{5.4}$$

where

x, y, z - target position [m]

$\dot{x}, \dot{y}, \dot{z}$ - target velocity [m/s]

$\ddot{x}, \ddot{y}, \ddot{z}$ - target acceleration [m/s^2]

a_T - target acceleration [m/sec^2]

$w_{Tj}(t)$ - target process driving noise (jerk) [m/sec^3]

$v(t)$-target-missile separation measurement noise(glint) [m]

5.2. *Acceleration based Kinematic Equation*

5.2.1. Snap kinematic equation – AS

With acceleration based snap equations (3.10) it is assumed that $\dot{\vec{\omega}} = 0$. Thus the kinematics of maneuvering target is modelled as

$$\vec{s}_T = 2\vec{\omega}_{Ta} \times \vec{j}_T - \vec{\omega}_{Ta} \times (\vec{\omega}_{Ta} \times \vec{a}_T)$$
$$w_T = \ddot{a}_T 1_v + \dot{\vec{\omega}}_{Ta} \times \vec{a}_T \tag{5.5}$$

$$\vec{\omega}_{Ta} = \frac{\vec{a}_T \times \vec{j}_T}{\|\vec{a}_T\|^2} \tag{5.6}$$

where it is assumed that (i) the absolute value of target acceleration is almost constant, i.e. $\|\vec{a}_T\| \cong constant$; (ii) the target's angular turning acceleration is constant, i.e. $\dot{\vec{\omega}}_{Ta} \cong 0$; and (iii) w_T represents the deviation of the actual behaviour of the target from the constant angular turning acceleration and constant absolute value of the acceleration assumptions.

The state space representation in the State Dependent Coefficient Form is in three dimensions is (notice that this is a fourth order differential equation)

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ \dddot{y} \\ z \\ \dot{z} \\ \ddot{z} \\ \dddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_y^2+\omega_z^2 & 0 & 0 & 0 & -\omega_x\omega_y & -2\omega_z & 0 & 0 & -\omega_x\omega_z & 2\omega_y \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_x\omega_y & 2\omega_z & 0 & 0 & \omega_x^2+\omega_z^2 & 0 & 0 & 0 & -\omega_z\omega_y & -2\omega_x \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_x\omega_z & -2\omega_y & 0 & 0 & -\omega_y\omega_z & 2\omega_x & 0 & 0 & \omega_x^2+\omega_y^2 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ \dddot{y} \\ z \\ \dot{z} \\ \ddot{z} \\ \dddot{z} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \tag{5.7}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{m} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \\ y \\ \dot{y} \\ \ddot{y} \\ y \\ z \\ \dot{z} \\ \ddot{z} \\ \dddot{z} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$\vec{\omega}_{Ta} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \frac{\vec{a}_T \times \vec{j}_T}{\|\vec{a}_T\|^2} = \frac{1}{\ddot{x}^2+\ddot{y}^2+\ddot{z}^2}\begin{bmatrix} \ddot{y}\,\dddot{z} - \ddot{z}\,\dddot{y} \\ \ddot{z}\,\dddot{x} - \ddot{x}\,\dddot{z} \\ \ddot{x}\,\dddot{y} - \ddot{y}\,\dddot{x} \end{bmatrix}; \tag{5.8}$$

$w_{Ts}(t)$ - target process driving noise (snap) [m/sec^4]

6. TARGET MANEUVER MODEL

6.1. *Corkscrew Target Maneuver*

It is assumed that the target moves in a constant direction and the corkscrew (barrel roll) trajectory is created by accelerations in the perpendicular plane.

6.2. *The trajectory*

The trajectory of the corkscrew (barrel roll) is modelled as in [Li and Jilkov(2000)][Zarchan and Musoff [Imado and Miwa]. It is

$$\xi_T = \xi_{T0} + \dot{\xi}_{T0}t + \rho_{T0}\cos(\omega_{CS}t - \varphi)$$
$$\psi_T = \psi_{T0} + \dot{\psi}_{T0}t + \rho_{T0}\sin(\omega_{CS}t - \varphi) \tag{6.1}$$
$$\zeta_T = \zeta_{T0} + \dot{\zeta}_{T0}t$$

where ω_{CS} is the corkscrew maneuver turning rate.

This trajectory is transformed to the inertial estimator's space according to

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = T_{IP}(\,pitch, yaw, roll\,)\begin{bmatrix} \xi \\ \psi \\ \zeta \end{bmatrix} \tag{6.2}$$

where $T_{IP}(\,pitch, yaw, roll\,) \in R^{3\times 3}$ is a transformation from the trajectory generation coordinates (ξ,ψ,ζ) to the inertial space (x_T,y_T,z_T) by pitch, yaw, and roll angles, and the rest of the variables are self evident.

In this example, the trajectory is obtained by multiplying the trajectory above by the transformation matrix above at some arbitrary angles in pitch, yaw and roll.

6.3. *Angular rate of the velocity and acceleration vectors*

Corkscrew maneuver propagating in the z-axis direction is considered, i.e.

$$
\begin{aligned}
x_T &= x_{T0} + \dot{x}_{T0}t + \rho_{T0}\cos(\omega_{CS}t - \varphi) \\
y_T &= y_{T0} + \dot{y}_{T0}t + \rho_{T0}\sin(\omega_{CS}t - \varphi) \\
z_T &= z_{T0} + \dot{z}_{T0}t
\end{aligned}
\tag{6.3}
$$

6.4. The angular rate of the velocity vector

For $\dot{x}_{T0} = \dot{y}_{T0} = 0$ we get

$$
\vec{\omega}_{Tv} = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \frac{\vec{v}_T \times \vec{a}_T}{\|\vec{v}_T\|^2} = \frac{1}{\dot{x}^2+\dot{y}^2+\dot{z}^2}\begin{bmatrix}\dot{y}\,\ddot{z}-\dot{z}\,\ddot{y}\\ \dot{z}\,\ddot{x}-\dot{x}\,\ddot{z}\\ \dot{x}\,\ddot{y}-\dot{y}\,\ddot{x}\end{bmatrix}
\tag{6.4}
$$

$$
= \frac{1}{[\rho_{T0}\omega_{CS}\sin(\omega_{CS}t-\varphi)]^2 + [\rho_{T0}\omega_{CS}\cos(\omega_{CS}t-\varphi)]^2 + \dot{z}_{T0}^2}\begin{bmatrix}\dot{y}\,\ddot{z}-\dot{z}\,\ddot{y}\\ \dot{z}\,\ddot{x}-\dot{x}\,\ddot{z}\\ \dot{x}\,\ddot{y}-\dot{y}\,\ddot{x}\end{bmatrix}
$$

$$
= \frac{1}{\omega_{CS}^2\rho_{T0}^2+\dot{z}_{T0}^2}\begin{bmatrix}-\dot{z}_{T0}\,\rho_{T0}\omega_{CS}^2\sin(\omega_{CS}t-\varphi)\\ -\dot{z}_{T0}\,\rho_{T0}\omega_{CS}^2\cos(\omega_{CS}t-\varphi)\\ \rho_{T0}^2\omega_{CS}^3\end{bmatrix}
$$

This is a constant modulus rotating vector. Its direction in the space is not constant. That is $\dot{\vec{\omega}}_T \neq 0$. This representation is not consistent with the assumption of constant angular acceleration (5.1) of the VJ based estimator and eventually is the cause of increased estimation errors.

6.5. The angular rate of the acceleration vector

We have

$$
\vec{\omega}_{Ta} = \frac{\vec{a}_T \times \vec{j}_T}{\|\vec{a}_T\|^2} = \frac{1}{\ddot{x}^2+\ddot{y}^2+\ddot{z}^2}\begin{bmatrix}\ddot{y}\dddot{z}-\ddot{z}\dddot{y}\\ \ddot{z}\dddot{x}-\ddot{x}\dddot{z}\\ \ddot{x}\dddot{y}-\ddot{y}\dddot{x}\end{bmatrix} = \frac{1}{\ddot{x}^2+\ddot{y}^2+\ddot{z}^2}\begin{bmatrix}0\\ 0\\ \ddot{x}\dddot{y}-\ddot{y}\dddot{x}\end{bmatrix}
\tag{6.5}
$$

$$
= \frac{1}{\rho_{T0}^2}\begin{bmatrix}0\\ 0\\ \ddot{x}\dddot{y}-\ddot{y}\dddot{x}\end{bmatrix} = \frac{1}{\omega_{CS}^4\rho_{T0}^2}\begin{bmatrix}0\\ 0\\ \rho_{T0}^2\omega_{CS}^5\end{bmatrix} = \begin{bmatrix}0\\ 0\\ \omega_{CS}\end{bmatrix}
$$

This is a constant direction and constant modulus vector. Its direction in the space is constant. This representation is consistent with the assumption of constant angular acceleration (5.5) of the AS based estimator.

6.6. Discussion + Rational

One can clearly see that the directions of the angular velocity vector and the angular acceleration vectors are different! They are equal only if the circular motion is planar, i.e. no motion in the z direction. In looking for representation of the trajectory motion with unknown parameters the representation for which the unknown parameters are constant is preferred as for this case the estimator is not required to increase its bandwidth to account for the dynamics of this unknown parameter/variable.

7. SIMULATION RESULTS

In this section the performance of the Velocity based Jerk (VJ) (5.1) and the Acceleration based Snap (AS) equation based filters (5.5) are compared via simulations. The discrete estimator has been implemented.

$$W_{process\,VJ} = 10[(m/s^3)^2/Hz];\; W_{process\,AS} = 10[(m/s^4)^2/Hz];$$

The sampling interval is $T_s = 100\,ms$ and the measurement noise level is σ_g =3m.

7.1. Tilted corkscrew trajectory

The initial conditions and the trajectories' parameters are

$$\xi_T = 0[m], \psi_T = 0[m], \zeta_T = 0[m], \dot{\xi}_{T0} = 40[m/s], \dot{\psi}_{T0} = 0[m/s], \dot{\zeta}_{T0} = 40[m/s],$$
$$a_T = 8[g], \omega_{CS} = 2[rad/s], T_{IP} = T_{IP}[30,0,0],\; [angles\;\; in\;\; \deg]$$

Figure 7.1 presents a tilted corkscrew trajectory used in this example. Figures 7.2 present the deterministic and stochastic tracking error of tilted corkscrew target maneuver by the VJ and AS based estimators.

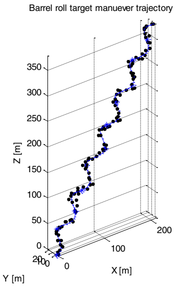
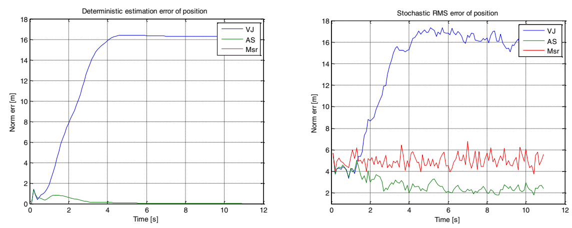


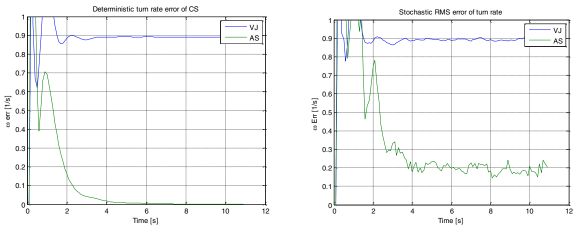
Figure 7.1: Tilted corkscrew trajectory in the inertial space, real and measured position.



(a) (b)

Figure 7.2: Deterministic(a) and stochastic(b) tracking error of tilted corkscrew target maneuver by the VJ and AS based estimators.

Figures 7.3 present deterministic and stochastic turning rate estimation error of tilted corkscrew target maneuver by the VJ and AS based estimators. The supremacy of the AS based estimators is clearly seen from these figures. Figure 7.4 presents the position estimation RMS errors of tilted corkscrew target maneuver. Figure 7.4 presents the Position(x), velocity (dx), acceleration(d2x) and jerk(d3x) estimation RMS errors of tilted corkscrew target maneuver. The performance of the VJ and AS based filters is clearly demonstrated.



(a) (b)

Figure 7.3: Deterministic(a) and Stochastic (b) turning rate estimation error of tilted corkscrew target maneuver by the VJ and AS based estimators.

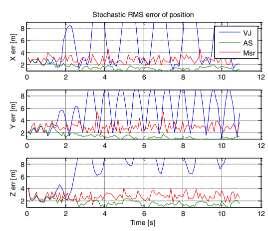


Figure 7.4: Position estimation RMS errors of tilted corkscrew target maneuver.

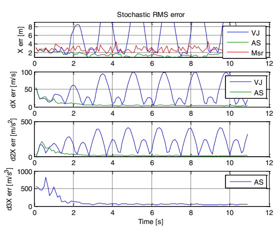


Figure 7.5: The X-Position(x), velocity(dx), acceleration(d2x) and jerk(d3x) estimation RMS errors of tilted corkscrew target maneuver.

From the results above one can clearly see the superiority of the acceleration based snap (AS) filters.

Unlike other approaches, e.g. the IMM filter in [Peled, Moran and Rusnak], the performance is independent of the tilt of the trajectory.

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