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Model Reference Switching Quasi-LPV Control of a Four Wheeled Omnidirectional Robot ⋆

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Abstract: In this paper, the problem of trajectory tracking for a four wheeled omnidirectional robot is solved in the inertial fixed coordinate system. The solution relies on a reference model approach, where the resulting nonlinear error model is brought to a quasi-Linear Parameter Varying (LPV) form suitable for designing an LPV controller using Linear Matrix Inequalities (LMI)-based techniques. In particular, the controller is obtained within the switching LPV framework. The effectiveness of the proposed approach is shown through simulation results.

Keywords: Linear parameter-varying systems, Control of switched systems, Tracking, Mobile robots.

1. INTRODUCTION

Omni-directional mobile robots are gaining popularity due to their enhanced mobility with respect to traditional robots (Oliveira et al., 2009). Holonomic robots are interesting because they offer advantages in manoeuvrability and effectiveness, even though at the expense of increased mechanical and control complexities (Oliveira et al., 2008). These robots use some kind of variation of the Mecanum wheels proposed by Diegel et al. (2002) and Salih et al. (2006). A robot with three or more motorized wheels of this kind can have almost independent tangential, normal and angular velocities. In order to increase the performance of these robots, there have been some efforts in developing a dynamic model, see, e.g., (Campion et al., 1996), (Conceicao et al., 2006), (Khosla, 1989), (Williams et al., 2002).

Different techniques have been applied in order to solve the control problem for omni-directional mobile robots. The solution proposed by Muir and Neuman (1987) relies on a kinematic control based on actuated inverse and sensed forward solutions. More recently, Purwin and D'Andrea (2006) have proposed an algorithm to calculate near-optimal minimum time trajectories, based on a relaxed optimal control problem. Rojas and Förster (2006) have used one PID controller for each motor in order to avoid problems arising from slipping wheels. In Liu et al. (2008), the robot controller consists of an outer-loop kinematics controller and an inner-loop dynamics controller, which are both designed using the Trajectory Linearization Control (TLC) method, based on a nonlinear robot dynamic model. Indiveri (2009) has formulated the trajectory tracking and pose regulation problems as a guidance control problem. Finally, a model-based PI-fuzzy control and an adaptive controller based on multi-input fuzzy rules emulated networks have been proposed in Hashemi et al. (2011) and Treesatayapun (2011), respectively.

In the last decades, the Linear Parameter Varying (LPV) paradigm has become a standard formalism in systems and control, for analysis, controller synthesis and system identification (Shamma, 2012). This class of systems is important because gain-scheduling control of nonlinear systems can be performed using an extension of linear techniques, by embedding the system nonlinearities in the varying parameters that depend on some endogenous signal, e.g. system states (in this case, the system is referred to as *quasi-LPV*, to make a distinction with respect to *pure* LPV systems, where the varying parameters only depend on exogenous signals).

Some applications of the LPV control techniques to mobile robots can be found in the literature of the last decade. Tsourdos et al. (2003) have proposed a fuzzy LPV controller that guarantees the global stability of the closed loop system over the entire operating range of the fuzzy model. Inoue et al. (2009) use a kinematic-based control to obtain desired velocities that form the input errors for the proposed quasi-LPV H_∞ controller used to attenuate the effect of external disturbances. LeBel and Rodrigues (2008) have considered the problem of path following, and designed a piecewise-affine parameter-varying steering control law, that is used in combination with a backstepping-type approach which, including the vehicle dynamics, guarantees the convergence of the robot forward and rotational velocities to the desired values. Notice that none of these solutions uses a controller, designed within the LPV framework, for solving the trajectory tracking problem in the inertial fixed coordinate system. All the listed solutions use either a combination of two controllers (the kinematic-based and the dynamic-based), or a control law obtained in the robot local coordinate system.

In this paper, a solution for the trajectory tracking problem in the inertial fixed coordinate system is proposed for a four wheeled omnidirectional robot. This solution relies on the use of a reference model that describes the desired trajectory, an idea that is well-established in the LTI framework (Landau, 1979), and has recently been extended to cope with the con-

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trol of LPV systems (Abdullah and Zribi, 2009). The resulting nonlinear error model is brought to a quasi-LPV form suitable for designing an LPV controller using a Linear Matrix Inequalities (LMI)-based approach. In particular, it is shown that, if polytopic techniques are used to reduce the number of LMI constraints from infinite to finite, there could not exist a solution within the standard LPV framework. Hence, the switching LPV framework is considered (Lu and Wu, 2004), (He et al., 2010).

The paper is structured as follows. Section 2 introduces the dynamic model of the four wheeled omnidirectional robot, and the corresponding reference model. The resulting error model is brought to a quasi-LPV representation. In Section 3, the reference inputs calculation for a circular trajectory tracking is discussed. The error feedback controller design using switching LPV techniques is presented in Section 4. Simulation results are shown in Section 5. Finally, the main conclusions and the possible future work are summarized in Section 6.

2. QUASI-LPV MODELING OF THE FOUR WHEELED OMNIDIRECTIONAL ROBOT

The dynamic model of the four wheeled omnidirectional robot (see Fig. 1) relates the wheel inputs and robot velocities with the corresponding accelerations, taking into account the traction, viscous friction and Coulomb friction forces. It is given by the following set of differential equations (Oliveira et al., 2009):

$$\dot{x} = v_x \tag{1}$$

$$\begin{aligned}\dot{v}_x = &\left(A_{11}c_\theta^2 + A_{22}s_\theta^2\right)v_x + \left((A_{11}-A_{22})s_\theta c_\theta - \omega\right)v_y \\ &+K_{11}c_\theta sign\left(v_x c_\theta + v_y s_\theta\right) - B_{21}s_\theta u_0 + B_{12}c_\theta u_1 \\ &-K_{22}s_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) - B_{23}s_\theta u_2 + B_{14}c_\theta u_3\end{aligned} \tag{2}$$

$$\dot{y} = v_y \tag{3}$$

$$\begin{aligned}\dot{v}_y = &\left((A_{11}-A_{22})s_\theta c_\theta + \omega\right)v_x + \left(A_{11}s_\theta^2 + A_{22}c_\theta^2\right)v_y \\ &+K_{11}s_\theta sign\left(v_x c_\theta + v_y s_\theta\right) + B_{21}c_\theta u_0 + B_{12}s_\theta u_1 \\ &+K_{22}c_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) + B_{23}c_\theta u_2 + B_{14}s_\theta u_3\end{aligned} \tag{4}$$

$$\dot{\theta} = \omega \tag{5}$$

$$\dot{\omega} = A_{33}\omega + B_{31}u_0 + B_{32}u_1 + B_{33}u_2 + B_{34}u_3 + K_{33}sign(\omega) \tag{6}$$

where (x,y) is the robot position, θ is the angle with respect to the defined front of robot ($s_\theta \triangleq \sin\theta$ and $c_\theta \triangleq \cos\theta$), v_x, v_y and ω are the corresponding linear/angular velocities, and u_0, u_1, u_2 and u_3 the motor voltage applied to the wheel 1, 2, 3 and 4, respectively. The coefficients A_{ii}, B_{ij}, K_{ii}, $i = 1,2,3$, $j = 1,2,3,4$, are defined as follows [2] :

$$A_{11} = \frac{2K_t^2 l^2}{r^2 RM} - \frac{B_v}{M} \quad A_{22} = \frac{2K_t^2 l^2}{r^2 RM} - \frac{B_{vr}}{M} \quad A_{33} = -\frac{4d^2 K_t^2 l^2}{r^2 RJ} - \frac{B_\omega}{J}$$

$$B_{12} = B_{23} = -\frac{lK_t}{rRM} \quad B_{14} = B_{21} = \frac{lK_t}{rRM} \quad B_{31} = B_{32} = B_{33} = B_{34} = \frac{lK_t d}{rRJ}$$

$$K_{11} = -\frac{C_v}{M} \quad K_{22} = -\frac{C_{vn}}{M} \quad K_{33} = -\frac{C_\omega}{J}$$

By introducing the following reference model:

$$\dot{x}_r = v_x^r \tag{7}$$

$$\begin{aligned}\dot{v}_x^r = &\left(A_{11}c_\theta^2 + A_{22}s_\theta^2\right)v_x^r + \left((A_{11}-A_{22})s_\theta c_\theta - \omega\right)v_y^r \\ &+K_{11}c_\theta sign\left(v_x c_\theta + v_y s_\theta\right) - B_{21}s_\theta u_0^r + B_{12}c_\theta u_1^r \\ &-K_{22}s_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) - B_{23}s_\theta u_2^r + B_{14}c_\theta u_3^r\end{aligned} \tag{8}$$

$$\dot{y}_r = v_y^r \tag{9}$$

[2] For a description of the system parameters, as well as the values used in the simulations taken from Oliveira et al. (2009), see Table 1.

Table 1. System parameters description and values

Param.	Description	Value
K_t	Motor torque constant	0.0259 $[Vs/rad]$
l	Gearbox reduction	5
r	Wheel radius	0.0325 $[m]$
R	Motor resistor	4.3111 $[\Omega]$
M	Mass	2.34 $[kg]$
d	Distance between wheels and robot center	0.089 $[m]$
J	Inertia moment	0.0228 $[kgm^2]$
B_v	Front viscous friction coefficient	0.4978 $[Ns/m]$
B_{vn}	Orthogonal viscous friction coefficient	0.6763 $[Ns/m]$
B_ω	Angular viscous friction coefficient	0.0141 $[Nms/rad]$
C_v	Front Coulomb friction coefficient	1.8738 $[N]$
C_{vn}	Orthogonal Coulomb friction coefficient	2.2198 $[N]$
C_ω	Angular Coulomb friction coefficient	0.1385 $[Nm]$

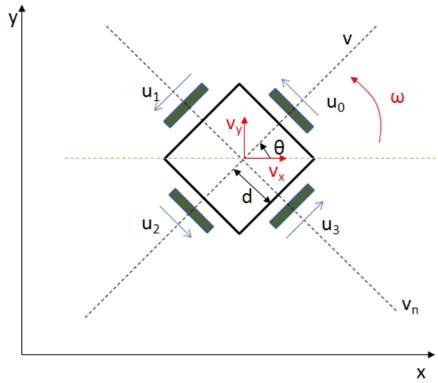

Fig. 1. Four wheeled omnidirectional mobile robot.

$$\begin{aligned}\dot{v}_y^r = &\left((A_{11}-A_{22})s_\theta c_\theta + \omega\right)v_x^r + \left(A_{11}s_\theta^2 + A_{22}c_\theta^2\right)v_y^r + \\ &+K_{11}s_\theta sign\left(v_x c_\theta + v_y s_\theta\right) + B_{21}c_\theta u_0^r + B_{12}s_\theta u_1^r \\ &+K_{22}c_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) + B_{23}c_\theta u_2^r + B_{14}s_\theta u_3^r\end{aligned} \tag{10}$$

$$\dot{\theta}_r = \omega_r \tag{11}$$

$$\dot{\omega}_r = A_{33}\omega_r + B_{31}u_0^r + B_{32}u_1^r + B_{33}u_2^r + B_{34}u_3^r + K_{33}sign(\omega) \tag{12}$$

where (x_r, y_r) is the reference vehicle position, θ_r is its angle, v_x^r, v_y^r and ω_r are the corresponding linear/angular velocities, and $u_0^r, u_1^r, u_2^r, u_3^r$ are the reference inputs (feedforward actions), then, if the tracking errors $e_1 \triangleq x_r - x$, $e_2 \triangleq v_x^r - v_x$, $e_3 \triangleq y_r - y$, $e_4 \triangleq v_y^r - v_y$, $e_5 \triangleq \theta_r - \theta$, $e_6 \triangleq \omega_r - \omega$, and the new inputs $\Delta u_i \triangleq u_i^r - u_i$, $i = 1,2,3,4$, are defined, the error model for the four wheeled omnidirectional mobile robot can be obtained from (1)-(12), and brought to a quasi-LPV representation, as follows:

$$\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\\ \dot{e}_5\\ \dot{e}_6\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & \vartheta_1 & 0 & \vartheta_2 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & \vartheta_3 & 0 & A_{11}+A_{22}-\vartheta_1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & A_{33}\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6\end{pmatrix} + \begin{pmatrix}0 & 0 & 0 & 0\\ -B_{21}\vartheta_4 & B_{12}\vartheta_5 & -B_{23}\vartheta_4 & B_{14}\vartheta_5\\ 0 & 0 & 0 & 0\\ B_{21}\vartheta_5 & B_{12}\vartheta_4 & B_{23}\vartheta_5 & B_{14}\vartheta_4\\ 0 & 0 & 0 & 0\\ B_{31} & B_{32} & B_{33} & B_{34}\end{pmatrix}\begin{pmatrix}\Delta u_0\\ \Delta u_1\\ \Delta u_2\\ \Delta u_3\end{pmatrix} \tag{13}$$

where the vector of varying parameters is:

$$\vartheta(t) = \begin{pmatrix} \vartheta_1(t) \\ \vartheta_2(t) \\ \vartheta_3(t) \\ \vartheta_4(t) \\ \vartheta_5(t) \end{pmatrix} = \begin{pmatrix} A_{11}\cos^2(\theta) + A_{22}\sin^2(\theta) \\ (A_{11} - A_{22})\sin(\theta)\cos(\theta) - \omega \\ (A_{11} - A_{22})\sin(\theta)\cos(\theta) + \omega \\ \sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

3. REFERENCE INPUTS CALCULATION FOR A CIRCULAR TRAJECTORY TRACKING

To make the robot track a desired trajectory, proper values of u_0^r, u_1^r, u_2^r, u_3^r should be fed to the reference model, such that its state equals the one corresponding to the desired trajectory. In this paper, a circular trajectory is chosen, defined as follows:

$$x_r(t) = \rho \cos(\theta_r(t)) \tag{14}$$

$$y_r(t) = \rho \sin(\theta_r(t)) \tag{15}$$

$$\theta_r(t) = \frac{2\pi t}{T} \tag{16}$$

where ρ is the circle radius and T is the desired revolution period around the circle center. Taking the derivative of (14)-(16), and taking into account (7), (9) and (11), respectively, the following is obtained:

$$\dot{x}_r(t) = -\rho \frac{2\pi}{T} \sin \frac{2\pi t}{T} = v_x^r(t) \tag{17}$$

$$\dot{y}_r(t) = \rho \frac{2\pi}{T} \cos \frac{2\pi t}{T} = v_y^r(t) \tag{18}$$

$$\dot{\theta}_r(t) = \frac{2\pi}{T} = \omega_r(t) \tag{19}$$

Then, another differentiation of (17)-(19) leads to:

$$\dot{v}_x^r(t) = -\left(\frac{2\pi}{T}\right)^2 \rho \cos \frac{2\pi t}{T} \tag{20}$$

$$\dot{v}_y^r(t) = -\left(\frac{2\pi}{T}\right)^2 \rho \sin \frac{2\pi t}{T} \tag{21}$$

$$\dot{\omega}_r(t) = 0 \tag{22}$$

and, by properly replacing (17)-(22) into (7)-(12) we obtain:

$$A_{ref}(t) \begin{pmatrix} u_0^r(t) \\ u_1^r(t) \\ u_2^r(t) \\ u_3^r(t) \end{pmatrix} = B_{ref}(t) \tag{23}$$

with:

$$A_{ref}(t) = \begin{pmatrix} -B_{21}\vartheta_4(t) & B_{12}\vartheta_5(t) & -B_{23}\vartheta_4(t) & B_{14}\vartheta_5(t) \\ B_{21}\vartheta_5(t) & B_{12}\vartheta_4(t) & B_{23}\vartheta_5(t) & B_{14}\vartheta_4(t) \\ B_{31} & B_{32} & B_{33} & B_{34} \end{pmatrix} \tag{24}$$

$$B_{ref}(t) = \left(\beta_{ref1}(t) \;\; \beta_{ref2}(t) \;\; \beta_{ref3}(t) \right)^T \tag{25}$$

$$\begin{aligned} \beta_{ref1}(t) = {} & \rho \frac{2\pi}{T} \left(\sin\left(\frac{2\pi t}{T}\right) \vartheta_1(t) - \cos\left(\frac{2\pi t}{T}\right) \left(\vartheta_2(t) + \frac{2\pi}{T}\right) \right) \\ & -K_{11}\vartheta_5(t) sign(v_x\vartheta_5(t) + v_y\vartheta_4(t)) \\ & -K_{22}\vartheta_4(t) sign(v_y\vartheta_5(t) - v_x\vartheta_4(t)) \end{aligned}$$

$$\begin{aligned} \beta_{ref2}(t) = {} & \rho \frac{2\pi}{T} \left(\sin\left(\frac{2\pi t}{T}\right) \left(\vartheta_3(t) - \frac{2\pi}{T}\right) - \cos\left(\frac{2\pi t}{T}\right) \vartheta_1(t) \right) \\ & -K_{11}\vartheta_4(t) sign(v_x\vartheta_5(t) + v_y\vartheta_4(t)) \\ & -K_{22}\vartheta_5(t) sign(v_y\vartheta_5(t) - v_x\vartheta_4(t)) \end{aligned}$$

$$\beta_{ref3}(t) = -A_{33}\frac{2\pi}{T} - K_{33} sign(\omega(t))$$

Finally, the reference model inputs $u_i^{ref}(t)$, $i = 1, 2, 3, 4$, are obtained as:

$$\begin{pmatrix} u_0^r(t) \\ u_1^r(t) \\ u_2^r(t) \\ u_3^r(t) \end{pmatrix} = A_{ref}^{\dagger}(t) B_{ref}(t) \tag{26}$$

where $A_{ref}^{\dagger}$ denotes the pseudoinverse of A_{ref}.

Remark: The obtained values $u_i^r(t)$, $i = 1, 2, 3, 4$ depend on the specifications, defined by the radius ρ and revolution period T of the desired circular trajectory (14)-(16). Special care should be put in choosing ρ and T, such that the resulting reference inputs do not cause the motors to work near/in their saturation region.

4. ERROR FEEDBACK CONTROLLER DESIGN USING SWITCHING LPV TECHNIQUES

Consider the following (quasi-)LPV error system:

$$\dot{e}(t) = A(\vartheta(t))\, e(t) + B(\vartheta(t))\, \Delta u(t) \tag{27}$$

where $e \in \mathbb{R}^{n_e}$ is the error vector, $\Delta u \in \mathbb{R}^{n_u}$ is the input vector, and $A(\vartheta(t))$, $B(\vartheta(t))$ are varying matrices of appropriate dimensions and $\vartheta \in \Theta \subset \mathbb{R}^{n_\theta}$ is the vector of varying parameters. The system is controlled through an error-feedback control law:

$$\Delta u(t) = K(\vartheta(t))\, e(t) \tag{28}$$

and it is wished to solve the design problem of finding an error-feedback gain matrix $K(\vartheta(t))$ such that the resulting closed-loop error system is stable with poles placed in some desired region of the complex plane[3].

In this paper, both stability and pole clustering are analyzed within the quadratic Lyapunov framework, where the specifications are assured by the use of a single quadratic Lyapunov function. Despite the introduction of conservativeness with respect to other existing approaches, where the Lyapunov function is allowed to be parameter-varying, the quadratic approach has undeniable advantages in terms of computational complexity.

In particular, the (quasi-)LPV error system (27) with the error-feedback control law (28) is quadratically stable if and only if there exist $X_s = X_s^T > 0$ and $K(\vartheta(t))$ such that (Packard and Becker, 1992):

$$(A(\vartheta) + B(\vartheta)K(\vartheta))X_s + X_s(A(\vartheta) + B(\vartheta)K(\vartheta))^T < 0 \tag{29}$$

$\forall \vartheta \in \Theta$. On the other hand, pole clustering is based on the results obtained by Chilali and Gahinet (1996), where subsets $\mathscr{D}$ of the complex plane, referred to as *LMI regions*, are defined as:

$$\mathscr{D} = \{z \in \mathbb{C} : f_{\mathscr{D}}(z) < 0\} \tag{30}$$

where $f_{\mathscr{D}}$ is the *characteristic function*, defined as:

$$f_{\mathscr{D}}(z) = \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl} z + \beta_{lk}\bar{z}]_{1 \le k,l \le m} \tag{31}$$

where $\alpha = \alpha^T \in \mathbb{R}^{m\times m}$ and $\beta \in \mathbb{R}^{m\times m}$. Hence, the (quasi-)LPV error system (27) with error-feedback control law (28) has poles in $\mathscr{D}$ if there exist $X_{\mathscr{D}} = X_{\mathscr{D}}^T > 0$ and $K(\vartheta(t))$ such that:

$$\left[\alpha_{kl}X_{\mathscr{D}} + \beta_{kl}(A(\vartheta) + B(\vartheta)K(\vartheta))X_{\mathscr{D}} + \beta_{lk}X_{\mathscr{D}}(A(\vartheta) + B(\vartheta)K(\vartheta))^T\right]_{1\le k,l\le m} < 0 \tag{32}$$

$\forall \vartheta \in \Theta$. The main difficulty with using (29) and (32) is that they impose an infinite number of constraints. In order to reduce this number to finite, a polytopic approximation of (27)-(28) is considered, as follows:

$$A(\vartheta(t)) = \sum_{i=1}^{N} \gamma_i(\vartheta(t)) A_i \quad \gamma_i(\vartheta) \ge 0, \quad \sum_{i=1}^{N} \gamma_i(\vartheta) = 1 \quad \forall \vartheta \in \Theta \tag{33}$$

[3] According to Ghersin and Sanchez-Peña (2002), and with a little abuse of language, the poles of an LPV system are defined as the set of all the poles of the LTI systems obtained by freezing $\vartheta(t)$ to all its possible values $\vartheta^* \in \Theta$.

$$B(\vartheta(t)) = \sum_{w=1}^{W} \delta_w(\vartheta(t)) B_w \quad \delta_w(\vartheta) \geq 0, \quad \sum_{w=1}^{W} \delta_w(\vartheta) = 1 \quad \forall \vartheta \in \Theta \tag{34}$$

$$K(\vartheta(t)) = \sum_{i=1}^{N} \gamma_i(\vartheta(t)) K_i \tag{35}$$

where each combination (A_i, B_w), $i = 1,\ldots,N$, $w = 1,\ldots,W$ is called *vertex system* and is controlled through the vertex controller K_i. Then, quadratic stability and pole clustering can be assessed through the following conditions, obtained from (29)-(32) using a common Lyapunov matrix $X = X_s = X_{\mathscr{D}} > 0$:

$$(A_i + B_w K_i) X + X (A_i + B_w K_i)^T < 0 \tag{36}$$

$$\left[\alpha_{kl} X + \beta_{kl} (A_i + B_w K_i) X + \beta_{lk} X (A_i + B_w K_i)^T\right]_{1\leq k,l\leq m} < 0 \tag{37}$$

with $i = 1,\ldots,N$ and $w = 1,\ldots,W$. Conditions (36) and (37) are Bilinear Matrix Inequalities (BMIs) that can be brought to Linear Matrix Inequalities (LMIs) form through the change of variable $\Gamma_i \triangleq K_i X$:

$$(A_i X + B_w \Gamma_i) + (A_i X + B_w \Gamma_i)^T < 0 \tag{38}$$

$$\left[\alpha_{kl} X + \beta_{kl} (A_i X + B_w \Gamma_i) + \beta_{lk} (A_i X + B_w \Gamma_i)^T\right]_{1\leq k,l\leq m} < 0 \tag{39}$$

that can be solved using available software, e.g. the YALMIP toolbox (Löfberg, 2004) with SeDuMi solver (Sturm, 1999).

However, when the polytopic LPV conditions (38)-(39) are applied to some polytopic approximation of the four wheeled omnidirectional mobile robot quasi-LPV model (13), a solution could not exist due to the loss of controllability occurring for $\vartheta_4 = \vartheta_5 = 0$, values for which the input matrix becomes:

$$B_{\vartheta_4=\vartheta_5=0} = \begin{pmatrix} 0_{5\times1} & 0_{5\times1} & 0_{5\times1} & 0_{5\times1} \\ B_{31} & B_{32} & B_{33} & B_{34} \end{pmatrix} \tag{40}$$

Due to the fact that the set described by the polytopic approximation (34) is convex, it is straightforward that any polytopic approximation of the admissible values for $\vartheta_4(t) = \sin\theta(t)$ and $\vartheta_5(t) = \cos\theta(t)$ will contain the origin, that is, the singularity (40) of the input matrix B (see the dash-dot black line in Fig. 2). In order to solve this issue, the solution to the design problem is searched within the switching LPV framework, where the overall system behavior is given by an interaction between different LPV systems through discrete switching events, which can depend on states or time. Similarly, the overall controller is obtained from different LPV controllers that are switched when discrete events occur.

More specifically, it is assumed that (33)-(35) are modified including a switching part, as follows:

$$A(\vartheta(t)) = \begin{cases} \sum_{i=1}^{N_1} \gamma_i^{(1)}(\vartheta(t)) A_i^{(1)}, \ \gamma_i^{(1)}(\vartheta) \geq 0, \ \sum_{i=1}^{N_1} \gamma_i^{(1)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_1 \\ \vdots \\ \sum_{i=1}^{N_r} \gamma_i^{(r)}(\vartheta(t)) A_i^{(r)}, \ \gamma_i^{(r)}(\vartheta) \geq 0, \ \sum_{i=1}^{N_r} \gamma_i^{(r)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_r \\ \vdots \\ \sum_{i=1}^{N_R} \gamma_i^{(R)}(\vartheta(t)) A_i^{(R)}, \ \gamma_i^{(R)}(\vartheta) \geq 0, \ \sum_{i=1}^{N_R} \gamma_i^{(R)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_R \end{cases} \tag{41}$$

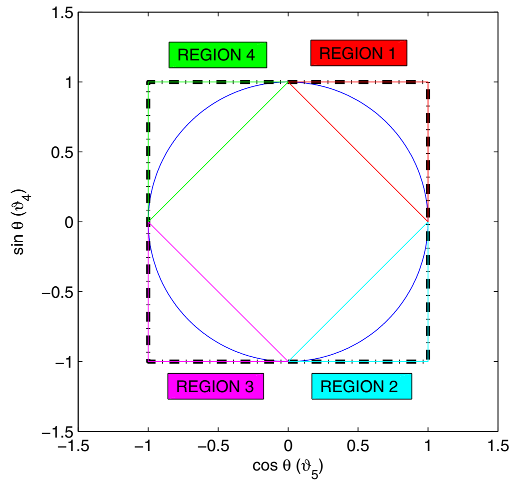

Fig. 2. Polytopic LPV and Polytopic Switching LPV approximations of the scheduling variables $\vartheta_4(t)$ and $\vartheta_5(t)$.

$$B(\vartheta(t)) = \begin{cases} \sum_{w=1}^{W_1} \delta_w^{(1)}(\vartheta(t)) B_w^{(1)}, \ \delta_w^{(1)}(\vartheta) \geq 0, \ \sum_{w=1}^{W_1} \delta_w^{(1)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_1 \\ \vdots \\ \sum_{w=1}^{W_r} \delta_w^{(r)}(\vartheta(t)) B_w^{(r)}, \ \delta_w^{(r)}(\vartheta) \geq 0, \ \sum_{w=1}^{W_r} \delta_w^{(r)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_r \\ \vdots \\ \sum_{w=1}^{W_R} \delta_w^{(R)}(\vartheta(t)) B_w^{(R)}, \ \delta_w^{(R)}(\vartheta) \geq 0, \ \sum_{w=1}^{W_R} \delta_w^{(R)}(\vartheta) = 1 \ \forall \vartheta \in \Theta_R \end{cases} \tag{42}$$

$$K(\vartheta(t)) = \begin{cases} \sum_{i=1}^{N_1} \gamma_i^{(1)}(\vartheta(t)) K_i^{(1)}, \quad \forall \vartheta \in \Theta_1 \\ \vdots \\ \sum_{i=1}^{N_r} \gamma_i^{(r)}(\vartheta(t)) K_i^{(r)}, \quad \forall \vartheta \in \Theta_r \\ \vdots \\ \sum_{i=1}^{N_R} \gamma_i^{(R)}(\vartheta(t)) K_i^{(R)}, \quad \forall \vartheta \in \Theta_R \end{cases} \tag{43}$$

where $\vartheta_1,\ldots,\vartheta_R$ are subsets of the varying parameter space Θ, such that $\Theta = \Theta_1 \cup \ldots \cup \Theta_r \cup \ldots \cup \Theta_R$. In each subset ϑ_r, $r = 1,\ldots,R$, the system is described by a polytopic combination of vertex systems. The controller (28) with gain (43) assures that the error system (27) with state and input matrix as in (41) and (42), respectively, is quadratically stable and has poles in $\mathscr{D}$ if there exist $X = X^T > 0$ and $\Gamma_i^{(r)}$, $i = 1,\ldots,N_r$, $r = 1,\ldots,R$, such that [4]:

$$\left(A_i^{(r)} X + B_w^{(r)} \Gamma_i^{(r)}\right) + \left(A_i^{(r)} X + B_w^{(r)} \Gamma_i^{(r)}\right)^T < 0 \tag{44}$$

$$\left[\alpha_{kl} X + \beta_{kl}\left(A_i^{(r)} X + B_w^{(r)} \Gamma_i^{(r)}\right) + \beta_{lk}\left(A_i^{(r)} X + B_w^{(r)} \Gamma_i^{(r)}\right)^T\right]_{1\leq k,l\leq m} < 0 \tag{45}$$

with $i = 1,\ldots,N_R$, $w = 1,\ldots,W_R$ and $r = 1,\ldots,R$.

Using (41)-(45), the problem arising due to the loss of controllability for $\vartheta_4 = \vartheta_5 = 0$ can be avoided by splitting the subset of the parameter space generated by ϑ_4 and ϑ_5 in more regions,

[4] This result is a particular case of the one obtained in He et al. (2010), where a common parameter-dependent Lyapunov function has been used for control design of switched LPV systems. In this paper, a common fixed Lyapunov function is used instead, since it has proved to be enough for stabilizing the four wheeled omnidirectional mobile robot and placing its poles in the desired LMI region $\mathscr{D}$.

such that in each region the resulting polytopic approximation does not include the origin. In particular, in this work, the quadrants have been considered as regions, with $\theta = k\pi/2$, $k \in \mathbb{N}$ being the switching condition (see Fig. 2). Then, a triangular polytopic approximation has been used in each region, as shown in Fig. 2.

Remark 4.1. In the case of quasi-LPV systems obtained from a nonlinear system, the closed loop system could be unstable for some operating conditions despite the feasibility of the design conditions. A rigorous analysis of the stability should also take into account the region of attraction estimates as in Bruzelius et al. (2003).

5. RESULTS

The overall polytopic approximation (41)-(42) of the four wheeled omnidirectional mobile robot quasi-LPV model (13) has been obtained by considering:

$$\vartheta_1 \in [\underline{\vartheta}_1, \overline{\vartheta}_1] = [\min(A_{11}, A_{22}), \max(A_{11}, A_{22})]$$

$$\vartheta_2 \in [\underline{\vartheta}_2, \overline{\vartheta}_2] = \left[\min_\theta((A_{11}-A_{22})s_\theta c_\theta) - \overline{\omega}, \max_\theta((A_{11}-A_{22})s_\theta c_\theta) - \underline{\omega}\right]$$

$$\vartheta_3 \in [\underline{\vartheta}_3, \overline{\vartheta}_3] = \left[\min_\theta((A_{11}-A_{22})s_\theta c_\theta) + \underline{\omega}, \max_\theta((A_{11}-A_{22})s_\theta c_\theta) + \overline{\omega}\right]$$

with:

$$\underline{\omega} = -\overline{\omega} = \frac{\left(B_{31}u_0^{\max} + B_{32}u_1^{\max} + B_{33}u_2^{\max} + B_{34}u_3^{\max} + K_{33}\right)}{A_{33}}$$

where $u_i^{max} = 12V$, $i = 1, \ldots, 4$ denotes the maximum input voltage that can be applied to the i^{th} motor, that is assumed to be limited by symmetric constant saturation limits, $u_i \in [-u_i^{max}, u_i^{max}]$.

The controller has been designed using (44) and (45), to assure stability and pole clustering in:

$$\mathscr{D} = \{z \in \mathbb{C} : \mathrm{Re}(z) < -0.1\}$$

The results shown in this paper refer to a simulation which lasts $20\,s$, where the four wheeled mobile robot is driven from the initial state:

$$(x(0), v_x(0), y(0), v_y(0), \theta(0), \omega(0))^T = 0_{6\times 1}$$

to the desired trajectory, defined as in (14)-(16) with $\rho = 2$ and $T = 20\,s$. The desired trajectory has been generated by the reference model (7)-(12) using the reference inputs calculated as described in Section 3, and starting from the initial reference state:

$$\left(x_r(0), v_x^r(0), y_r(0), v_y^r(0), \theta_r(0), \omega_r(0)\right)^T = (\rho, 0, 0, 2\pi\rho/T, 0, 2\pi/T)^T$$

Fig. 3 shows the tracking of the desired circular trajectory in the $(x-y)$ plane. It can be seen that the robot (blue line) reaches asymptotically the reference vehicle trajectory (red line) that coincides with the desired one (black line). All the tracking errors go to zero, as depicted in Fig. 4. Finally, in Fig. 5, the control inputs are shown. It can be seen that, except in the very beginning of the simulation, the control inputs are such that all the motors are working in their linear region.

Remark: It has been noticed that, whereas the real system state is too different from the reference vehicle one, the effect of saturations could be such that the system becomes unstable. Hence, on one hand, special attention should be paid in planning the experiment (e.g. choosing the reference trajectory and the robot initial condition), while on the other hand some future line of research would be to include some mechanism that enforces the system stability despite the actuator saturations.

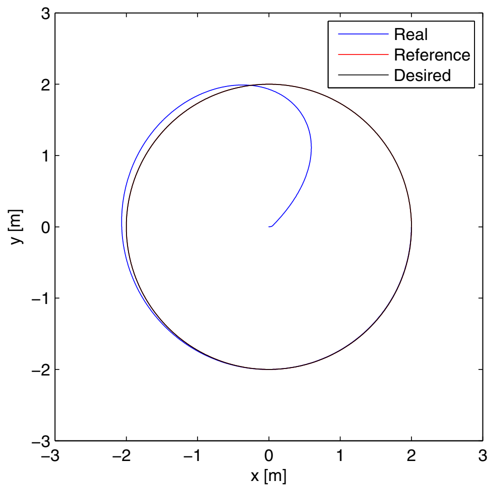

Fig. 3. Switching LPV control of the four wheeled omnidirectional mobile robot: tracking of the desired circular trajectory.

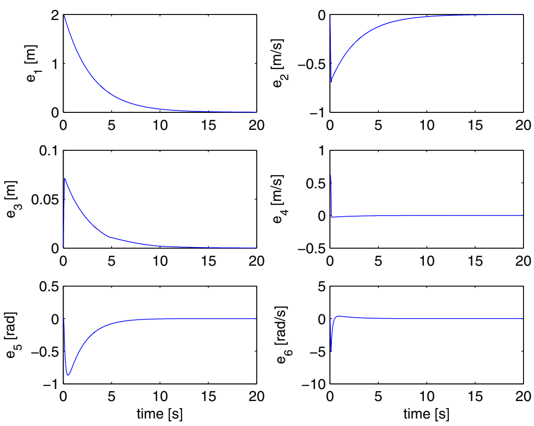

Fig. 4. Switching LPV control of the four wheeled omnidirectional mobile robot: tracking errors.

6. CONCLUSIONS AND FUTURE WORK

In this paper, the problem of controlling a four wheeled omnidirectional mobile robot such that it tracks a desired trajectory has been solved. The proposed solution relies on the use of a reference model that describes the desired trajectory. The resulting nonlinear error model is brought to a quasi-LPV form that is used for designing a switching LPV controller using LMI-based techniques. The results obtained in simulation environment have demonstrated the effectiveness of the proposed technique.

Future work will follow different directions. Since it has been noticed that saturations can play an important role, degrading the system performance and even leading the system to instability, some mechanisms will be included in order to improve the control system effectiveness against actuator saturations. Another line of investigation is the application of the proposed technique to the real setup, taking into account the errors due to the model uncertainty and the sensor noise. Finally, the four wheeled omnidirectional robot is a system characterized by actuator redundancy, characteristic that could make it an interesting testbed for testing Fault Tolerant Control (FTC) techniques against actuator breakdown or faults.

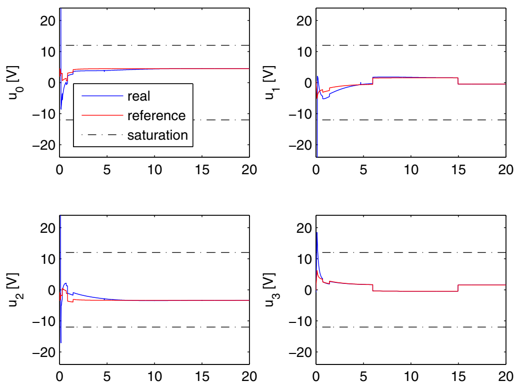

Fig. 5. Switching LPV control of the four wheeled omnidirectional mobile robot: control inputs.

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