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Adaptive H_∞ Consensus Control of Multi-Agent Systems by Utilizing Neural Network Approximators

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Abstract: Design methods of adaptive H_∞ consensus control of multi-agent systems composed of the first-order and the second-order regression models and nonlinear terms by utilizing neural network approximators, are presented in this paper. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, imperfect knowledge of the leader, and approximate and algorithmic errors in the neural network estimation schemes are regarded as external disturbances to the process.

1. INTRODUCTION

Among plenty of cooperative control problems of multi-agent systems, distributed consensus tracking of multi-agent systems with limited communication networks, has been a basic and important topic, and various research results have been reported for various processes and under various conditions (Olfati-Saber et al. [2007], Ren et al. [2007], Cao and Ren [2011], Wen et al. [2012]). In those research works, adaptive control or sliding mode control methodologies were also proposed in order to deal with uncertainties of agents, and stability of control systems was assured via Lyapunov function analysis. Furthermore, robustness properties of the control schemes were also discussed. However, those results are restricted to simple linear processes, and so much attention does not have been paid on control performance such as optimal property or transient performance in those approaches.

The purpose of the paper is to present design methods of adaptive H_∞ consensus control of multi-agent systems composed of the first-order and the second-order regression models and nonlinear terms based on the notion of inverse optimality (Krstić and Deng [1998], Miyasato [2000]). This is an extension of the work (Miyasato [2013]) to nonlinear regression models, and the neural network approximators are introduced to estimate nonlinear parametric elements in the agents. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, imperfect knowledge of the leader, and approximate and algorithmic errors in the neural network estimation schemes are regarded as external disturbances to the process.

2. MULTI-AGENT SYSTEM AND INFORMATION NETWORK

First, mathematical preliminaries on information network graph of multi-agent systems are summarized (Ren et al. [2007], Cao and Ren [2011]). We consider a weighted undirected graph $\mathcal{G} = (\mathcal{V},\, \mathcal{E},\, A)$ as a model of interaction among agents. $\mathcal{V} = \{1, \cdots, N\}$ is a node set, which corresponds to a set of agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set. An edge $(i, j) \in \mathcal{E}$ indicates that the agent i and j can communicate with each other. Associated with $\mathcal{E}$, we introduce a weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{N\times N}$, and the entry a_{ij} of it is defined such as $a_{ij} = a_{ji} > 0$ (when $(i,\, j) \in \mathcal{E}$) and $a_{ij} = a_{ji} = 0$ (otherwise). A path is a sequence of edges in the form (i_1, i_2), (i_2, i_3), $\cdots$ $(i_j \in \mathcal{V})$, and the undirected graph is called connected, if there is always an undirected path between every pair of distinct nodes. For the adjacency matrix $A = [a_{ij}]$, the Laplacian matrix $L = [l_{ij}] \in \mathbf{R}^{N\times N}$ is defined by $l_{ii} = \sum_{j=1,\, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ $(i \neq j)$. The Laplacian matrix is symmetric and positive-semidefinite, and furthermore, has a simple 0 eigenvalue with the associated eigenvector $\mathbf{1} = [1 \cdots 1]^{\mathsf{T}}$, and all other eigenvalues are positive, if the corresponding undirected graph is connected.

In this manuscript, we consider a consensus control problem of leader-follower type, and x_0 is a leader which each agent $i \in \mathcal{V}$ (a follower) should follow. For the leader and the followers, a_{i0} is defined such as $a_{i0} > 0$ (when leader's information is available to follower i), and $a_{i0} = 0$ (otherwise), and from a_{i0} and L, the matrix $M \in \mathbf{R}^{N\times N}$ is defined by $M = L + \mathrm{diag}\,(a_{10} \cdots a_{N0})$. M is symmetric and positive definite, if 1. at least one a_{i0} $(1 \le i \le N)$ is positive, and 2. the graph $\mathcal{G}$ is connected (Cao and Ren [2011]). Hereafter, we assume those assumptions 1 and 2.

3. ADAPTIVE H_∞ CONSENSUS CONTOL FOR FIRST-ORDER MODELS

3.1 Problem Statement

We consider a multi-agent system composed of the first-order regression models with nonlinear terms described as follows $(i = 1, \cdots, N)$:

$$\dot{x}_i(t) = X_i(t)\theta_i + F_i(x_i(t)) + B_i u_i(t), \tag{1}$$

where $x_i \in \mathbf{R}^n$ is a state, $u_i \in \mathbf{R}^n$ is an input, $\theta_i \in \mathbf{R}^l$ is an unknown parameter vector, and $X_i \in \mathbf{R}^{n\times l}$ is a regressor matrix composed of x_i and its structure is known a priori. It is assumed that X_i is bounded for bounded x_i. $F_i(x_i) \in \mathbf{R}^n$ is an unknown nonlinear term, and $B_i \in \mathbf{R}^{n\times n}$ is an unknown matrix of the form

$$B_i = \mathrm{diag}\,(b_{i1}, \cdots, b_{in}), \tag{2}$$

and the sign of b_{ij} is known a priori. Hereafter, it is assumed that $b_{ij} > 0$ without loss of generality. The control objective is to achieve consensus tracking of the leader-follower type such as $x_i \to x_j$, $x_i \to x_0$ $(i, j = 1, \cdots, N)$.

3.2 Representation of Nonlinear Term

In this paper, it is assumed that $F_i(x_i)$ is approximated by a three-layered neural network (a nonlinear parametric model) as follows:

$$\begin{aligned} F_i(x_i) &= \begin{bmatrix} W_{i1}^{\mathsf T} S(V_{i1}^{\mathsf T}\bar{x}_i) + \mu_{i11}(x_i) \\ \vdots \\ W_{in}^{\mathsf T} S(V_{in}^{\mathsf T}\bar{x}_i) + \mu_{i1n}(x_i) \end{bmatrix} \\ &\equiv W_i^{\mathsf T} S(V_i^{\mathsf T}\bar{x}_i) + \mu_{i1}(x_i) \ \in \mathbf{R}^n, \end{aligned} \tag{3}$$

$$\bar{x}_i = [x_i^{\mathsf T}, 1]^{\mathsf T} \in \mathbf{R}^{n+1}, \tag{4}$$

$$W_{ij} = [w_{ij1}, \cdots, w_{ijm}]^{\mathsf T} \ \in \mathbf{R}^m, \quad (1 \le j \le n), \tag{5}$$

$$\begin{aligned} &V_{ij} = [v_{ij1}, \cdots, v_{ijm}] \in \mathbf{R}^{(n+1)\times m}, \\ &v_{ijk} \ \in \mathbf{R}^{n+1}, \quad (1 \le j \le n, \ 1 \le k \le m), \end{aligned} \tag{6}$$

$$S(V_{ij}^{\mathsf T}\bar{x}_i) = [s(v_{ij1}^{\mathsf T}\bar{x}_i), \cdots, s(v_{ijm}^{\mathsf T}\bar{x}_i)]^{\mathsf T} \ \in \mathbf{R}^m, \tag{7}$$

$$s(v^{\mathsf T}\bar{x}) = \frac{1}{1+\exp\{-\gamma^*(v^{\mathsf T}\bar{x})\}}, \qquad (\gamma^* > 0), \tag{8}$$

$$W_i = \begin{bmatrix} W_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_{in} \end{bmatrix} \in \mathbf{R}^{mn\times n}, \tag{9}$$

$$S(V_i^{\mathsf T}\bar{x}_i) = [S(V_{i1}^{\mathsf T}\bar{x}_i)^{\mathsf T}, \cdots, S(V_{in}^{\mathsf T}\bar{x}_i)^{\mathsf T}]^{\mathsf T} \ \in \mathbf{R}^{mn}, \tag{10}$$

$$\mu_{i1}(x_i) = [\mu_{i11}(x_i), \cdots, \mu_{i1n}(x_i)]^{\mathsf T} \ \in \mathbf{R}^n, \tag{11}$$

where V_{ij} and W_{ij} are layer weights of the j-th neural network for the i-th agent, and m is a number of cells of each neural network. $s(v^{\mathsf T}\bar{z})$ is a sigmoid function, and $\mu_{i1}(x_i)$ is a vector of an approximation error for $F_i(x_i)$.

3.3 Neural Network Approximator

Based on the fact that any continuous function over a compact set can be approximated by a three-layered neural network with an arbitrary small approximate error (Funahashi [1989]), the following assumption is introduced.

Assumption 1. There exist layer weights V_{ij} and W_{ij} satisfying the following relations.

$$|\mu_{i1j}(x_i)| \le d_{i1j}\psi_{ij}(x_i), \quad (1 \le j \le n), \tag{12}$$

where d_{i1j} are unknown positive constants, and $\psi_{ij}(x_i)$ are known positive functions.

The estimates of the layer weights V_{ij} and W_{ij} are denoted by $\hat{V}_{ij}$ and $\hat{W}_{ij}$, respectively. Then, the neural network estimation error $\hat{W}_{ij}^{\mathsf T} S(\hat{V}_{ij}^{\mathsf T}\bar{x}_i) - W_{ij}^T S(V_{ij}^T\bar{x}_i)$ is evaluated in the following lemma (Zhang et al. [1999]).

Lemma 2. For the three-layered neural network, the estimation error is evaluated as follows:

$$\begin{aligned} &\hat{W}_{ij}^{\mathsf T} S(\hat{V}_{ij}^{\mathsf T}\bar{x}_i) - W_{ij}^{\mathsf T} S(V_{ij}^{\mathsf T}\bar{x}_i) \\ &\quad = \tilde{W}_{ij}^{\mathsf T}(\hat{S}_{ij} - \hat{S}_{ij}^{'}\hat{V}_{ij}^{\mathsf T}\bar{x}_i) + \hat{W}_{ij}^{\mathsf T}\hat{S}_{ij}^{'}\tilde{V}_{ij}^{\mathsf T}\bar{x}_i + \mu_{i2j}, \end{aligned} \tag{13}$$

$$\begin{aligned} |\mu_{i2j}| \le{} & \|V_{ij}\| \cdot \|\bar{x}_i\hat{W}_{ij}^{\mathsf T}\hat{S}_{ij}^{'}\| \\ & + \|W_{ij}\| \cdot \|\hat{S}_{ij}^{'}\hat{V}_{ij}^{\mathsf T}\bar{x}_i\| + |W_{ij}|_1, \end{aligned} \tag{14}$$

$$\tilde{W}_{ij} = \hat{W}_{ij} - W_{ij}, \quad \tilde{V}_{ij} = \hat{V}_{ij} - V_{ij}, \tag{15}$$

$$\hat{S}_{ij} = S(\hat{V}_{ij}^{\mathsf T}\bar{x}_i), \tag{16}$$

$$\hat{S}_{ij}^{'} = \mathrm{diag}(\hat{s}_{ij1}^{'}, \cdots, \hat{s}_{ijm}^{'}), \tag{17}$$

$$\hat{s}_{ijk}^{'} = s^{'}(\hat{v}_{ijk}^{\mathsf T}\bar{x}_i) = \left[\frac{ds(z)}{dz}\right]_{z=\hat{v}_{ijk}^{\mathsf T}\bar{x}_i}. \tag{18}$$

For convenience' sake, $\tilde{W}_{ij}^{\mathsf T}(\hat{S}_{ij} - \hat{S}_{ij}^{'}\hat{V}_{ij}^{\mathsf T}\bar{x}_i)$ and $\hat{W}_{ij}^{\mathsf T}\hat{S}_{ij}^{'}\tilde{V}_{ij}^{\mathsf T}\bar{x}_i$ in (13) are rewritten into the following regression forms.

$$\tilde{W}_{ij}^{\mathsf T}(\hat{S}_{ij} - \hat{S}_{ij}^{'}\hat{V}_{ij}^{\mathsf T}\bar{x}_i) = \tilde{W}_{ij}^{\mathsf T}\omega_{ij0}, \tag{19}$$

$$\hat{W}_{ij}^{\mathsf T}\hat{S}_{ij}^{'}\tilde{V}_{ij}^{\mathsf T}\bar{x}_i = \sum_{k=1}^{m}\hat{w}_{ijk}\hat{s}_{ijk}^{'}\tilde{v}_{ijk}^{\mathsf T}\bar{x}_i = \sum_{k=1}^{m}\tilde{v}_{ijk}^{\mathsf T}\omega_{ijk}, \tag{20}$$

$$\omega_{ij0} = \hat{S}_{ij} - \hat{S}_{ij}^{'}\hat{V}_{ij}^{\mathsf T}\bar{x}_i, \qquad \omega_{ijk} = (\hat{w}_{ijk}\hat{s}_{ijk}^{'})\bar{x}_i, \tag{21}$$

$$\tilde{v}_{ijk} = \hat{v}_{ijk} - v_{ijk}. \tag{22}$$

Then the overall representation of (13) is given by

$$\begin{aligned} &\hat{W}_i^{\mathsf T} S(\hat{V}_i^{\mathsf T}\bar{x}_i) - W_i^{\mathsf T} S(V_i^{\mathsf T}\bar{x}_i) \\ &\quad = \Omega_i\tilde{\Phi}_i + \mu_{i2}, \end{aligned} \tag{23}$$

$$\Phi_i = [\Phi_{i1}^{\mathsf T}, \cdots, \Phi_{in}^{\mathsf T}]^{\mathsf T}, \quad \left(\tilde{\Phi}_i = \hat{\Phi}_i - \Phi_i\right), \tag{24}$$

$$\Phi_{ij} = [W_{ij}^{\mathsf T}, v_{ij1}^{\mathsf T}, \cdots, v_{ijm}^{\mathsf T}]^{\mathsf T}, \tag{25}$$

$$\Omega_i = \text{block diag}[\Omega_{i1}, \cdots, \Omega_{in}], \tag{26}$$

$$\Omega_{ij} = [\omega_{ij0}^{\mathsf T}, \omega_{ij1}^{\mathsf T}, \cdots, \omega_{ijm}^{\mathsf T}], \tag{27}$$

$$\mu_{i2} = [\mu_{i21}, \cdots, \mu_{i2n}]^{\mathsf T}, \tag{28}$$

$$\hat{S}_i = S(\hat{V}_i^{\mathsf T}\bar{x}_i) = [S(\hat{V}_{i1}^{\mathsf T}\bar{x}_i)^{\mathsf T}, \cdots, S(\hat{V}_{in}^{\mathsf T}\bar{x}_i)^{\mathsf T}]^{\mathsf T}, \tag{29}$$

$$\hat{S}_i^{'} = \begin{bmatrix} \hat{S}_{i1}^{'} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{S}_{in}^{'} \end{bmatrix}, \tag{30}$$

$$\hat{V}_i = [\hat{V}_{i1}, \cdots, \hat{V}_{in}]. \tag{31}$$

Also, the left-hand sides of (12) and (14) are summarized into the following forms, respectively.

$$\begin{bmatrix} d_{i11}\psi_{i1} \\ \vdots \\ d_{i1n}\psi_{in} \end{bmatrix} = \begin{bmatrix} \psi_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \psi_{in} \end{bmatrix} \begin{bmatrix} d_{i11} \\ \vdots \\ d_{i1n} \end{bmatrix} \equiv \Psi_{i1}D_{i1}, \tag{32}$$

$$\begin{aligned} &\begin{bmatrix} \|V_{i1}\| \cdot \|\bar{x}_i\hat{W}_{i1}^{\mathsf T}\hat{S}_{i1}^{'}\| + \|W_{i1}\| \cdot \|\hat{S}_{i1}^{'}\hat{V}_{i1}^{\mathsf T}\bar{x}_i\| + |W_{i1}|_1 \\ \vdots \\ \|V_{in}\| \cdot \|\bar{x}_i\hat{W}_{in}^{\mathsf T}\hat{S}_{in}^{'}\| + \|W_{in}\| \cdot \|\hat{S}_{in}^{'}\hat{V}_{in}^{\mathsf T}\bar{x}_i\| + |W_{in}|_1 \end{bmatrix} \\ &\quad = \begin{bmatrix} \Psi_{i21} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Psi_{i2n} \end{bmatrix} \begin{bmatrix} D_{i21} \\ \vdots \\ D_{i2n} \end{bmatrix} \equiv \Psi_{i2}D_{i2}, \end{aligned} \tag{33}$$

$$\Psi_{i2j} = \left[\|\bar{x}_i \hat{W}_{ij}^{\mathsf{T}} \hat{S}'_{ij}\|, \, \|\hat{S}'_{ij} \hat{V}_{ij}^{\mathsf{T}} \bar{x}_i\|, \, 1\right], \tag{34}$$

$$D_{i2j} = [\|V_{ij}\|, \, \|W_{ij}\|, \, |W_{ij}|_1]^{\mathsf{T}}. \tag{35}$$

3.4 Control Law and Error Equation

Associated with the information network graph $\mathcal{G}$, we employ the following control law.

$$\begin{aligned} u_i(t) = \hat{P}_i(t) \Bigg[& -X_i(t)\hat{\theta}_i(t) - \hat{W}_i^{\mathsf{T}} S(\hat{V}_i^{\mathsf{T}} \bar{x}_i) \\ & -\alpha \sum_{\substack{j=0 \\ j \neq i}}^{N} a_{ij}\{x_i(t) - x_j(t)\} + n_{i0}\dot{x}_0(t) \Bigg] + v_i(t) \\ \equiv {} & \hat{P}_i(t) u_{i0}(t) + v_i(t), \end{aligned} \tag{36}$$

where a_{ij} $(1 \le i \le N, \, 0 \le j \le N)$ is defined as the entry of the adjacency matrix A and M, and $\alpha > 0$ is a design parameter. $\hat{(\cdot)}$ is denoted as a current estimate of $(\cdot)$, and P_i is defined by

$$P_i = \mathrm{diag}\,(p_{i1}, \cdots, p_{in}), \quad p_{ij} = 1/b_{ij}. \tag{37}$$

Concerned with a_{i0}, n_{i0} is defined as follows:

$$n_{i0} = \begin{cases} 1 : a_{i0} > 0 \\ 0 : \text{otherwise.} \end{cases} \tag{38}$$

Furthermore, v_i is a stabilizing signal to be determined later based on H_∞ control criterion. A tracking error between the leader x_0 and the follower x_i is defined by

$$\tilde{x}_i(t) \equiv x_i(t) - x_0(t) \tag{39}$$

and the substitution of (36) and (39) into (1) yields

$$\begin{aligned} \dot{\tilde{x}}_i(t) = {} & X_i(t)\theta_i + F_i(x_i(t)) + B_i u_i(t) - \dot{x}_0(t) \\ & -X_i(t)\tilde{\theta}_i - \Omega_i(t)\tilde{\Phi}_i + U_{i0}(t) B_i \tilde{p}_i \\ & +\mu_{i1} - \mu_{i2} + B_i v_i(t) \\ & +\alpha \left\{ -(l_{ii} + a_{i0})\tilde{x}_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^{N} l_{ij}\tilde{x}_j(t) \right\} \\ & +(n_{i0} - 1)\dot{x}_0(t), \end{aligned} \tag{40}$$

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i, \quad \tilde{p}_i = \hat{p}_i - p_i, \tag{41}$$

$$U_{i0} = \mathrm{diag}\,(u_{i01}, \cdots, u_{i0n}), \tag{42}$$

$$u_{i0} = [u_{i01}, \cdots, u_{i0n}]^{\mathsf{T}}, \tag{43}$$

$$p_i = [p_{i1}, \cdots, p_{in}]^{\mathsf{T}}. \tag{44}$$

Then, the total representation of the multi-agent system is given as follows ($\otimes$ denotes Kronecker product) :

$$\begin{aligned} \dot{\tilde{x}}(t) = {} & -X(t)\tilde{\theta} - \Omega(t)\tilde{\Phi} + U_0(t) B \tilde{p} - \alpha\,(M \otimes I)\,\tilde{x}(t) \\ & +\{(N_0 - \mathbf{1}) \otimes I\}\,\dot{x}_0(t) + \mu_1 - \mu_2 + Bv(t), \end{aligned} \tag{45}$$

$$\tilde{x} = [\tilde{x}_1^{\mathsf{T}}, \cdots, \tilde{x}_N^{\mathsf{T}}]^{\mathsf{T}}, \tag{46}$$

$$X = \text{block diag}\,(X_1, \cdots, X_N), \tag{47}$$

$$\Omega = \text{block diag}\,(\Omega_1, \cdots, \Omega_N), \tag{48}$$

$$\theta = [\theta_1^{\mathsf{T}}, \cdots, \theta_N^{\mathsf{T}}]^{\mathsf{T}}, \quad (\tilde{\theta} = \hat{\theta} - \theta), \tag{49}$$

$$\Phi = [\Phi_1^{\mathsf{T}}, \cdots, \Phi_N^{\mathsf{T}}]^{\mathsf{T}}, \quad (\tilde{\Phi} = \hat{\Phi} - \Phi), \tag{50}$$

$$U_0 = \text{block diag}\,(U_{10}, \cdots, U_{N0}), \tag{51}$$

$$B = \text{block diag}\,(B_1, \cdots, B_N), \tag{52}$$

$$p = [p_1^{\mathsf{T}}, \cdots, p_N^{\mathsf{T}}]^{\mathsf{T}}, \quad (\tilde{p} = \hat{p} - p), \tag{53}$$

$$N_0 = [n_{10}, \cdots, n_{N0}]^{\mathsf{T}}, \tag{54}$$

$$\mathbf{1} = [1, \cdots, 1]^{\mathsf{T}}, \tag{55}$$

$$v = [v_1^{\mathsf{T}}, \cdots, v_N^{\mathsf{T}}]^{\mathsf{T}} \tag{56}$$

$$\mu_1 = [\mu_{11}^{\mathsf{T}}, \cdots, \mu_{N1}^{\mathsf{T}}]^{\mathsf{T}}, \quad \mu_2 = [\mu_{12}^{\mathsf{T}}, \cdots, \mu_{N2}^{\mathsf{T}}]^{\mathsf{T}}. \tag{57}$$

3.5 Adaptive H_∞ Consensus Control for First-Order Models

A positive function W_0 is defined by

$$\begin{aligned} W_0(t) = {} & \frac{1}{2}\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,\tilde{x}(t) \\ & +\frac{1}{2}\left\{\hat{b}(t) - b\right\}^{\mathsf{T}} \Gamma_1^{-1} \left\{\hat{b}(t) - b\right\}, \end{aligned} \tag{58}$$

$$(\Gamma_1 = \Gamma_1^{\mathsf{T}} > 0),$$

$$b = [b_1^{\mathsf{T}}, \cdots, b_N^{\mathsf{T}}]^{\mathsf{T}}, \; b_i = [b_{i1}, \cdots, b_{iN}]^{\mathsf{T}}. \tag{59}$$

The tuning law of $\hat{b}$ is determined such as

$$\dot{\hat{b}}(t) = \Pr\left\{\Gamma_1 V(t)^{\mathsf{T}}\,(M \otimes I)\,\tilde{x}(t)\right\}, \tag{60}$$

$$V = \text{block diag}\,(V_1, \cdots, V_N), \tag{61}$$

$$V_i = \mathrm{diag}\,(v_{i1}, \cdots, v_{in}), \tag{62}$$

$$v_i = [v_{i1}, \cdots, v_{in}]^{\mathsf{T}}, \tag{63}$$

where $\Pr(\cdot)$ are projection operations in which tuning parameters are constrained to bounded regions deduced from upper-bounds and lower-bounds of each element of b (Ioannou and Sun [1996]). Then, the time derivative of W_0 along its trajectory is given as follows:

$$\begin{aligned} \dot{W}_0(t) \le {} & -\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,X(t)\tilde{\theta}(t) \\ & -\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,\Omega(t)\tilde{\Phi}(t) \\ & +\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,U_0(t)B\tilde{p}(t) \\ & -\alpha\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)^2\,\tilde{x}(t) \\ & +\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,\{(N_0 - \mathbf{1}) \otimes I\}\,\dot{x}_0(t) \\ & +\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,\hat{B}(t)v(t) \\ & +\tilde{x}(t)^{\mathsf{T}}\,(M \otimes I)\,(\mu_1 - \mu_2). \end{aligned} \tag{64}$$

From the evaluation of $\dot{W}_0$ (64), we introduce the next virtual system.

$$\dot{\tilde{x}} = f + \sum_{i=1}^{6} g_{1i} d_i + g_2 v, \tag{65}$$

$$f = -\alpha\,(M \otimes I)\,\tilde{x}, \tag{66}$$

$$\begin{aligned} & g_{11} = X, \; g_{12} = \Omega, \; g_{13} = U_0, \; g_{14} = I, \\ & g_{15} = \Psi_1, \; g_{16} = \Psi_2, \; g_2 = \hat{B}, \end{aligned} \tag{67}$$

$$\begin{aligned} & d_1 = -\tilde{\theta}, \; d_2 = -\tilde{\Phi}, \; d_3 = B\tilde{p}, \\ & d_4 = \{(N_0 - \mathbf{1}) \otimes I\}\,\dot{x}_0, \; d_5 = D_1, \; d_6 = D_2, \end{aligned} \tag{68}$$

$$\begin{aligned} & \Psi_1 = \text{block diag}[\Psi_{11}, \cdots, \Psi_{N1}], \\ & \Psi_2 = \text{block diag}[\Psi_{12}, \cdots, \Psi_{N2}], \end{aligned} \tag{69}$$

$$\begin{aligned} & D_1 = [D_{11}^{\mathsf{T}}, \cdots, D_{N1}^{\mathsf{T}}]^{\mathsf{T}}, \\ & D_2 = [D_{12}^{\mathsf{T}}, \cdots, D_{N2}^{\mathsf{T}}]^{\mathsf{T}}, \end{aligned} \tag{70}$$

where $d_1 \sim d_6$ are regarded as external disturbances to the process. Especially, $d_1 \sim d_3$ are estimation errors of the tuning parameters, and d_4 is concerned with imperfect knowledge of the leader. d_5 and d_6 correspond to approximate and algorithmic errors respectively included in the neural network estimation schemes. We are to stabilize the virtual system via a control input v by utilizing H_∞ criterion for those external disturbances $d_1 \sim d_6$ (Krstić and Deng [1998], Miyasato [2000]). For that purpose, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation and its solution V_0.

$$\mathcal{L}_f V_0 + \frac{1}{4}\left\{\sum_{i=1}^{6}\frac{\|\mathcal{L}_{g_{1i}}V_0\|^2}{\gamma_i^2} - (\mathcal{L}_{g_2}V_0)R^{-1}(\mathcal{L}_{g_2}V_0)^\mathsf{T}\right\} + q = 0, \tag{71}$$

$$V_0 = \frac{1}{2}\tilde{x}^\mathsf{T}(M\otimes I)\tilde{x}, \tag{72}$$

where q and R are a positive function and a positive definite matrix respectively, and those are derived from HJI equation based on inverse optimality for the given solution V_0 and the positive constants $\gamma_1 \sim \gamma_6$. The substitution of the solution V_0 (72) into HJI equation (71) yields

$$\begin{aligned}&-\alpha\tilde{x}^\mathsf{T}(M\otimes I)^2\tilde{x} + \frac{1}{4}\tilde{x}^\mathsf{T}(M\otimes I)\left\{\frac{XX^\mathsf{T}}{\gamma_1^2} + \frac{\Omega\Omega^\mathsf{T}}{\gamma_2^2}\right.\\ &\left. + \frac{U_0U_0^\mathsf{T}}{\gamma_3^2} + \frac{I}{\gamma_4^2} + \frac{\Psi_1\Psi_1^\mathsf{T}}{\gamma_5^2} + \frac{\Psi_2\Psi_2^\mathsf{T}}{\gamma_6^2} - \hat{B}R^{-1}\hat{B}^\mathsf{T}\right\}\cdot\\ &\cdot(M\otimes I)\tilde{x} + q = 0.\end{aligned} \tag{73}$$

Then, R and q are obtained such as

$$\begin{aligned}R = &\left(\frac{\hat{B}^{-1}XX^\mathsf{T}\hat{B}^{-\mathsf{T}}}{\gamma_1^2} + \frac{\hat{B}^{-1}\Omega\Omega^\mathsf{T}\hat{B}^{-\mathsf{T}}}{\gamma_2^2}\right.\\ &+ \frac{\hat{B}^{-1}U_0U_0^\mathsf{T}\hat{B}^{-\mathsf{T}}}{\gamma_3^2} + \frac{\hat{B}^{-1}\hat{B}^{-\mathsf{T}}}{\gamma_4^2}\\ &\left. + \frac{\hat{B}^{-1}\Psi_1\Psi_1^\mathsf{T}\hat{B}^{-\mathsf{T}}}{\gamma_5^2} + \frac{\hat{B}^{-1}\Psi_2\Psi_2^\mathsf{T}\hat{B}^{-\mathsf{T}}}{\gamma_6^2} + K\right)^{-1},\end{aligned} \tag{74}$$

$$\begin{aligned}q = &\alpha\tilde{x}^\mathsf{T}(M\otimes I)^2\tilde{x}\\ &+ \frac{1}{4}\tilde{x}^\mathsf{T}(M\otimes I)\hat{B}K\hat{B}^\mathsf{T}(M\otimes I)\tilde{x},\end{aligned} \tag{75}$$

where K is a diagonal positive definite matrix (a design parameter). From R, v is derived as a solution of the corresponding H_∞ control problem as follows:

$$v = -\frac{1}{2}R^{-1}(\mathcal{L}_{g_2}V_0)^\mathsf{T} = -\frac{1}{2}R^{-1}\hat{B}^\mathsf{T}(M\otimes I)\tilde{x}. \tag{76}$$

Then, by evaluating the time derivative of W_0,

$$\begin{aligned}\dot{W}_0 \le &-q - v^\mathsf{T}Rv\\ &+ \left(v + \frac{1}{2}R^{-1}\hat{B}^\mathsf{T}(M\otimes I)\tilde{x}\right)^\mathsf{T} R\cdot\\ &\cdot\left(v + \frac{1}{2}R^{-1}\hat{B}^\mathsf{T}(M\otimes I)\tilde{x}\right)\\ &+\gamma_1^2\|d_1\|^2 - \gamma_1^2\left\|d_1 - \frac{X^\mathsf{T}(M\otimes I)\tilde{x}}{2\gamma_1^2}\right\|^2\\ &+\gamma_2^2\|d_1\|^2 - \gamma_2^2\left\|d_2 - \frac{\Omega^\mathsf{T}(M\otimes I)\tilde{x}}{2\gamma_2^2}\right\|^2\\ &+\gamma_3^2\|d_2\|^2 - \gamma_3^2\left\|d_3 - \frac{U_0^\mathsf{T}(M\otimes I)\tilde{x}}{2\gamma_3^2}\right\|^2\\ &+\gamma_4^2\|d_4\|^2 - \gamma_4^2\left\|d_4 - \frac{(M\otimes I)\tilde{x}}{2\gamma_4^2}\right\|^2\\ &+\gamma_5^2\|d_5\|^2 - \gamma_5^2\left\|d_5 - \frac{\Psi_1^\mathsf{T}|(M\otimes I)\tilde{x}|_*}{2\gamma_5^2}\right\|^2\\ &+\gamma_6^2\|d_6\|^2 - \gamma_6^2\left\|d_6 - \frac{\Psi_2^\mathsf{T}|(M\otimes I)\tilde{x}|_*}{2\gamma_6^2}\right\|^2,\end{aligned} \tag{77}$$

$$|z|_* \equiv [|z_1|,\cdots,|z_m|]^\mathsf{T}, \quad (z = [z_1,\cdots,z_m]^\mathsf{T}), \tag{78}$$

we obtain the next theorem.

Theorem 3. The partial adaptive control system (36), (60), (76) is uniformly bounded for arbitrary bounded design parameters $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$, and v is a sub-optimal control input which minimizes the upper bound on the cost functional J.

$$J(t) \equiv \sup_{d_1\sim d_6\in\mathcal{L}_2}\left[\int_0^t\{q + v^\mathsf{T}Rv\}d\tau + W_0(t) - \sum_{i=1}^{6}\gamma_i^2\int_0^t\|d_i\|^2 d\tau\right]. \tag{79}$$

Also we have the next inequality.

$$\int_0^t\{q + v^\mathsf{T}Rv\}d\tau + W_0(t) \le \sum_{i=1}^{6}\gamma_i^2\int_0^t\|d_i\|^2 d\tau + W_0(0). \tag{80}$$

Theorem 3 denotes the properties of the partial adaptive control system (36), (60), (76), where the tunings of $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$ are not necessarily required. Furthermore, the L_2-gain property between $\sqrt{q + v^\mathsf{T}Rv}$ and $d_1 \sim d_6$ is prescribed by the design parameters $\gamma_1 \sim \gamma_6$, and it indicates that the boundedness of the control systems is assured for arbitrary bounded system parameters $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$.

Next, the tuning laws of $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$ are determined as follows:

$$\begin{cases}\dot{\hat{\theta}}(t) = \Pr\left\{\Gamma_2 X(t)^\mathsf{T}(M\otimes I)\tilde{x}(t)\right\},\\ \dot{\hat{\Phi}}(t) = \Pr\left\{\Gamma_3\Omega(t)^\mathsf{T}(M\otimes I)\tilde{x}(t)\right\},\\ \dot{\hat{p}}(t) = \Pr\left\{-\Gamma_4 U_0(t)^\mathsf{T}(M\otimes I)\tilde{x}(t)\right\},\end{cases} \tag{81}$$

$$(\Gamma_2 = \Gamma_2^\mathsf{T} > 0,\ \Gamma_3 = \Gamma_3^\mathsf{T} > 0,\ \Gamma_4 = \Gamma_4^\mathsf{T} > 0),$$

where Γ_4 is especially chosen as a diagonal matrix. $\Pr(\cdot)$ are projection operations in which tuning parameters $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$ are constrained to bounded regions deduced from upper-bounds of $\hat{\theta}$, $\hat{\Phi}_i$ and upper-bounds and lower-bounds of each element of p, respectively (Ioannou and Sun [1996]). A positive function W is defined by

$$\begin{aligned}W(t) = &\frac{1}{2}\tilde{x}(t)^\mathsf{T}(M\otimes I)\tilde{x}(t)\\ &+\frac{1}{2}\left\{\hat{b}(t) - b\right\}^\mathsf{T}\Gamma_1^{-1}\left\{\hat{b}(t) - b\right\}\\ &+\frac{1}{2}\left\{\hat{\theta}(t) - \theta\right\}^\mathsf{T}\Gamma_2^{-1}\left\{\hat{\theta}(t) - \theta\right\}\end{aligned}$$

$$+\frac{1}{2}\left\{\hat{\Phi}(t)-\Phi\right\}^{\mathsf{T}}\Gamma_3^{-1}\left\{\hat{\Phi}(t)-\Phi\right\}$$
$$+\frac{1}{2}\left\{\hat{p}(t)-p\right\}^{\mathsf{T}}B\,\Gamma_4^{-1}\left\{\hat{p}(t)-p\right\}. \tag{82}$$

From the time derivative of W along its trajectory,

$$\begin{aligned}\dot{W}(t) \le& -\alpha\tilde{x}(t)^{\mathsf{T}}(M\otimes I)^2\tilde{x}(t)-\frac{1}{4\gamma_4^2}\tilde{x}(t)^{\mathsf{T}}(M\otimes I)^2\tilde{x}(t)\\ &-\frac{1}{4\gamma_5^2}\tilde{x}(t)^{\mathsf{T}}(M\otimes I)\,\Psi_1\Psi_1^{\mathsf{T}}(M\otimes I)\,\tilde{x}(t)\\ &-\frac{1}{4\gamma_6^2}\tilde{x}(t)^{\mathsf{T}}(M\otimes I)\,\Psi_2\Psi_2^{\mathsf{T}}(M\otimes I)\,\tilde{x}(t)\\ &-\frac{1}{2}\tilde{x}(t)^{\mathsf{T}}(M\otimes I)\,\hat{B}(t)K\hat{B}(t)^{\mathsf{T}}(M\otimes I)\,\tilde{x}(t)\\ &+\gamma_4^2\|d_4\|^2+\gamma_5^2\|d_5\|^2+\gamma_6^2\|d_6\|^2,\end{aligned} \tag{83}$$

we obtain the following theorem.

Theorem 4. The total adaptive control system (36), (60), (76), (81) is uniformly bounded, and if $\dot{x}_0(t)=0$ or the information of the leader $\dot{x}_0$ is available for all followers ($\{(N_0-\mathbf{1})\otimes I\}\,\dot{x}_0=0$), then it follows that

$$\lim_{T\to\infty}\sup\frac{1}{T}\int_0^T\|\tilde{x}(t)\|^2dt \le \text{const}\cdot\sum_{i=5}^{6}\gamma_i^2. \tag{84}$$

Otherwise, when $\dot{x}_0(t)\neq 0$ and the information of $\dot{x}_0$ is not available for all followers ($\{(N_0-\mathbf{1})\otimes I\}\,\dot{x}_0\neq 0$), then the next relation holds.

$$\lim_{T\to\infty}\sup\frac{1}{T}\int_0^T\|\tilde{x}(t)\|^2dt \le \text{const}\cdot\sum_{i=4}^{6}\gamma_i^2. \tag{85}$$

4. ADAPTIVE H_∞ CONSEINSUS CONTROL FOR SECOND-ORDER MODEL

4.1 Problem Statement

Next, we consider a multi-agent systems composed of the second-order regression models with nonlinear terms described as follows $(i=1,\cdots,N)$:

$$\ddot{x}_i(t)=X_i(t)\theta_i+F_i(x_i(t),\dot{x}_i(t))+B_iu_i(t), \tag{86}$$

where x_i, u_i, θ_i, $F_i(x_i,\dot{x}_i)$, X_i are defined similarly to the previous case, and the form of B_i is the same as the former one. X_i is a regressor matrix composed of x_i and $\dot{x}_i$, and is bounded for bounded x_i and $\dot{x}_i$. The communication structure among agents is prescribed by the information network graph $\mathcal{G}$. The control objective is to achieve consensus tracking of the leader-follower type together with velocity tracking such as $x_i\to x_j$, $\dot{x}_i\to\dot{x}_j$, $x_i\to x_0$, $\dot{x}_i\to\dot{x}_0$ $(i,\,j=1,\cdots,N)$.

4.2 Representation of Nonlinear Term

Similarly to the first-order case, it is assumed that $F_i(x_i,\dot{x}_i)$ is approximated by a three-layered neural network (a nonlinear parametric model) as follows:

$$F_i(x_i,\dot{x}_i)=\begin{bmatrix} W_{i1}^{\mathsf{T}}S(V_{i1}^{\mathsf{T}}\bar{z}_i)+\mu_{i11}(z_i)\\ \vdots\\ W_{in}^{\mathsf{T}}S(V_{in}^{\mathsf{T}}\bar{z}_i)+\mu_{i1n}(z_i)\end{bmatrix}$$
$$\equiv W_i^{\mathsf{T}}S(V_i^{\mathsf{T}}\bar{z}_i)+\mu_{i1}(z_i)\ \in\mathbf{R}^n, \tag{87}$$
$$\bar{z}_i=[x_i^{\mathsf{T}},\,\dot{x}_i^{\mathsf{T}},\,1]^{\mathsf{T}}\in\mathbf{R}^{2n+1}, \tag{88}$$

where notations are the same as the previous one except for $\bar{z}_i$ instead of $\bar{x}_i$.

4.3 Control Law and Error Equation

Associated with the information network graph, we utilize the following control law.

$$\begin{aligned}u_i(t)=&\hat{P}_i(t)\Bigg[-X_i(t)\hat{\theta}_i(t)-\hat{W}_i^{\mathsf{T}}S(\hat{V}_i^{\mathsf{T}}\bar{z}_i))\\ &-\sum_{\substack{j=0\\ j\neq i}}^{N}a_{ij}\{x_i(t)-x_j(t)\}\\ &-\alpha\sum_{\substack{j=0\\ j\neq i}}^{N}a_{ij}\{\dot{x}_i(t)-\dot{x}_j(t)\}+n_{i0}\ddot{x}_0(t)\Bigg]\\ &+v_i(t)\\ \equiv&\hat{P}_i(t)u_{i0}(t)+v_i(t),\end{aligned} \tag{89}$$

where the definitions of a_{ij} $(1\le i\le N,\ 0\le j\le N)$, $\alpha>0$, P_i, n_{i0}, v_i are the same as the previous case. A consensus tracking error $\tilde{x}_i$ is denoted by (39), and the substitution of (89) and (39) into (86) yields the total representation of the multi-agent system such as

$$\begin{aligned}\ddot{\tilde{x}}(t)=&-X(t)\tilde{\theta}-\Omega(t)\tilde{\Phi}+U_0(t)B\tilde{p}-\ (M\otimes I)\,\tilde{x}(t)\\ &-\alpha\,(M\otimes I)\,\dot{\tilde{x}}(t)+\ \{(N_0-\mathbf{1})\otimes I\}\,\ddot{x}_0(t)\\ &+\mu_1-\mu_2+Bv(t),\end{aligned} \tag{90}$$

where the definitions of $\tilde{x}$, X, θ, Ω, U_0, B, p, N, $\mathbf{1}$, v, $\otimes$ are the same as the previous ones.

4.4 Adaptive H_∞ Consensus Control for Second-Order Models

For the matrix M and the positive constants α, γ, the matrices P and Q are defined such as

$$P=\begin{bmatrix}\frac{1}{2}M^2 & \frac{\gamma}{2}M\\ \frac{\gamma}{2}M & \frac{1}{2}M\end{bmatrix},\ Q=\begin{bmatrix}\gamma^2M^2 & \frac{\alpha\gamma}{2}M^2\\ \frac{\alpha\gamma}{2}M^2 & \alpha M^2-\gamma M\end{bmatrix}. \tag{91}$$

It can be shown that P and Q are both positive definite, if γ satisfies the next condition (Cao and Ren [2011]).

$$0<\gamma<\min\left\{\sqrt{\lambda_{\min}(M)},\ \frac{4\alpha\lambda_{\min}(M)}{4+\alpha^2\lambda_{\min}(M)}\right\}. \tag{92}$$

Hereafter, it is assumed that γ satisfies (92). Utilizing the positive definite P, a positive function W_0 is defined by

$$W_0(t)=\tilde{z}(t)^{\mathsf{T}}(P\otimes I)\,\tilde{z}(t)$$
$$+\frac{1}{2}\left\{\hat{b}(t)-b\right\}^{\mathsf{T}}\Gamma_1^{-1}\left\{\hat{b}(t)-b\right\}, \tag{93}$$
$$\tilde{z}=[\tilde{x}^{\mathsf{T}},\,\dot{\tilde{x}}^{\mathsf{T}}]^{\mathsf{T}}, \tag{94}$$
$$(\Gamma_1=\Gamma_1^{\mathsf{T}}>0),$$

where b is defined similarly to the previous case. The tuning law of $\hat{b}$ is chosen such as

$$\dot{\hat{b}}(t) = \Pr\left\{\Gamma_1 V(t)^{\mathsf{T}} (M \otimes I)\, \tilde{s}(t)\right\}, \tag{95}$$

$$\tilde{s}(t) \equiv \dot{\tilde{x}}(t) + \gamma \tilde{x}(t), \tag{96}$$

where V, $\Pr(\cdot)$ are the same as the previous ones. Then, the time derivative of W_0 is given by

$$\begin{aligned}\dot{W}_0(t) \le & -\tilde{s}(t)^{\mathsf{T}} (M \otimes I)\, X(t)\tilde{\theta}(t) \\ & -\tilde{x}(t)^{\mathsf{T}} (M \otimes I)\, \Omega(t)\tilde{\Phi}(t) \\ & +\tilde{s}(t)^{\mathsf{T}} (M \otimes I)\, U_0(t) B\tilde{p}(t) \\ & -\tilde{z}(t)^{\mathsf{T}} (Q \otimes I)\, \tilde{z}(t) \\ & +\tilde{s}(t)^{\mathsf{T}} (M \otimes I) \{(N_0 - \mathbf{1}) \otimes I\}\, \ddot{x}_0(t) \\ & +\tilde{s}(t)^{\mathsf{T}} (M \otimes I)\, \hat{B}(t) v(t) \\ & +\tilde{x}(t)^{\mathsf{T}} (M \otimes I)\, (\mu_1 - \mu_2).\end{aligned} \tag{97}$$

From (97), we introduce the next virtual system.

$$\dot{\tilde{z}} = f + \sum_{i=1}^{6} g_{1i} d_i + g_2 v, \tag{98}$$

$$f = \begin{bmatrix} 0 & I \\ -(M \otimes I) & -\alpha (M \otimes I) \end{bmatrix} \tilde{z}, \tag{99}$$

$$\begin{aligned} & g_{11} = \begin{bmatrix} 0 \\ X \end{bmatrix},\ g_{12} = \begin{bmatrix} 0 \\ \Omega \end{bmatrix},\ g_{13} = \begin{bmatrix} 0 \\ U_0 \end{bmatrix}, \\ & g_{14} = \begin{bmatrix} 0 \\ I \end{bmatrix},\ g_{15} = \begin{bmatrix} 0 \\ \Psi_1 \end{bmatrix},\ g_{16} = \begin{bmatrix} 0 \\ \Psi_2 \end{bmatrix}, \\ & g_2 = \begin{bmatrix} 0 \\ \hat{B} \end{bmatrix}. \end{aligned} \tag{100}$$

The definitions $d_1 \sim d_6$ are the same as the previous case. We are to stabilize the virtual system via a control input v by utilizing H_∞ criterion, where $d_1 \sim d_6$ are regarded as external disturbances to the process. Then, by repeating the similar discussions to the first-order case, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation (71) and its solution V_0 (101).

$$V_0 = \tilde{z}^{\mathsf{T}} (P \otimes I)\, \tilde{z}. \tag{101}$$

Then similarly to the first-order case, for R (74) and q defined such as

$$q = \tilde{z}^{\mathsf{T}} (Q \otimes I)\, \tilde{z} + \frac{1}{4} \tilde{s}^{\mathsf{T}} (M \otimes I)\, \hat{B} K \hat{B}^{\mathsf{T}} (M \otimes I)\, \tilde{s}, \tag{102}$$

and for v deduced from R such as

$$v = -\frac{1}{2} R^{-1} (\mathcal{L}_{g_2} V_0)^{\mathsf{T}} = -\frac{1}{2} R^{-1} \hat{B}^{\mathsf{T}} (M \otimes I)\, \tilde{s}, \tag{103}$$

we obtain the next theorem.

Theorem 5. The partial adaptive control system (89), (95), (103) is uniformly bounded for arbitrary bounded design parameters $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$, and and v is a sub-optimal control input which minimizes the upper bound on the cost functional J (79), where W_0 and q are newly defined by (93), (102). Also we have the inequality (80) for the new W_0 and q.

Next, the tuning laws of $\hat{\theta}$, $\hat{\Phi}$, $\hat{p}$ are determined as follows:

$$\begin{cases} \dot{\hat{\theta}}(t) = \Pr\left\{\Gamma_2 X(t)^{\mathsf{T}} (M \otimes I)\, \tilde{s}(t)\right\}, \\ \dot{\hat{\Phi}}(t) = \Pr\left\{\Gamma_3 \Omega(t)^{\mathsf{T}} (M \otimes I)\, \tilde{s}(t)\right\}, \\ \dot{\hat{p}}(t) = \Pr\left\{-\Gamma_4 U_0(t)^{\mathsf{T}} (M \otimes I)\, \tilde{s}(t)\right\}, \end{cases} \tag{104}$$

where the definition of $\Pr(\cdot)$ is the same as the previous one. Then, similarly to the previous, we obtain the following theorem.

Theorem 6. The total adaptive control system (89), (95), (103), (104) is uniformly bounded, and if $\ddot{x}_0(t) = 0$ or the information of the leader $\ddot{x}_0$ is available for all followers ($\{(N_0 - \mathbf{1}) \otimes I\}\, \ddot{x}_0 = 0$), then it follows that

$$\lim_{T\to\infty} \sup \frac{1}{T} \int_0^T \|\tilde{z}(t)\|^2 dt \le \text{const} \cdot \sum_{i=5}^{6} \gamma_i^2. \tag{105}$$

Otherwise, when $\ddot{x}_0(t) \ne 0$ and the information of $\ddot{x}_0$ is not available for all followers ($\{(N_0 - \mathbf{1}) \otimes I\}\, \ddot{x}_0 \ne 0$), then the next relation holds.

$$\lim_{T\to\infty} \sup \frac{1}{T} \int_0^T \|\tilde{z}(t)\|^2 dt \le \text{const} \cdot \sum_{i=4}^{6} \gamma_i^2. \tag{106}$$

5. CONCLUDING REMARKS

Design methods of adaptive H_∞ consensus control of multi-agent systems composed of the first-order and the second-order regression models with nonlinear terms have been presented in this paper. The neural network approximators are introduced to estimate nonlinear parametric elements in the agents. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters, imperfect knowledge of the leader, and approximate and algorithmic errors in the neural network estimation schemes are regarded as external disturbances to the process. It is shown that the desirable consensus tracking is achieved approximately via adaptation schemes and L_2-gain design parameters.

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