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Sliding Mode versus Parallel Compensator Based Control under Measurement Noise ⋆

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Abstract: Two relay systems with SISO plants, the first with sliding mode control and the second with parallel compensator, are considered. The influence of measurement noise, under presence of excitations in the form of the reference value and plant output disturbance, on the quality of control is considered. First, it is shown that in the case of the system with sliding mode control the measurement noise may destroy the operation of the system and thus may lead to the loss of its robustness. Second, it is shown that in the case of no measurement noise and under appropriate choice of the parameters, both systems have comparable dynamics. Finally, it is shown that contrary to the system with sliding mode control, the system with parallel compensator copes quite well with measurement noise, also in the case of the plants with nonminimum phase zeros.

Keywords: Sliding mode control; parallel compensator; relay systems; measurement noise.

1. INTRODUCTION

The control systems which use sliding mode technique have now good theoretical elaborations (see classical books (Slotine and Lee, 1991), (Utkin, 1992)), as well as successful practical applications (e.g. commonly used voltage regulation of car alternators). This kind of systems operates well both with linear and nonlinear plants.

It is a common view that the systems with sliding mode control are very robust, so they operate well even in the case of large and rapid parameter changes, as well as for significant plant output disturbances. However with the switching action of the relay, there is connected the so called chattering effect, which sometimes is not accepted by users and/or actuators. Therefore chattering decrease is interesting from application point of view.

In the present paper, not the problem of chattering decrease, but the influence of measurement noise on the quality of control is researched. Only the classical relay systems with sliding mode control are considered, without accounting the continuous time part of the control resulting from the model of the plant. This is caused by the fact that the control generated by the relay system is responsible for the robustness of the system (by the way, only the relay with hysteresis used in the place of the sign function, is accurate; the sign function, used by most of the authors, cannot operate with the continuous-time plant which has the relative degree equal to one).

Really, these systems without measurement noise are very robust. But it is shown here, that existence of the measurement noise may completely destroy their operation.

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To show the possibility of improvement of operation next, the relay system with parallel compensator, introduced in (Gessing, 2004), (Gessing, 2007) and based on the idea of Smith predictor (Smith, 1958) is considered.

One can note that there are several modifications of the parallel compensator approach. One of them is parallel feedforward (Bar-kana, 1987), (Kaufman et al., 1998), used also to adaptive control systems. Another approach was presented in (Deng et al., 1999) and in its references, where the minimum phase plants with structured uncertainty were considered.

In the present paper it is shown, that under appropriate design of the parallel compensator both the systems: with sliding mode control and with parallel compensator, for the same plant without noise, have comparable dynamics. Additionally, it becomes that in contrary to the system with sliding mode control, the system with parallel compensator copes quite well with measurement noise.

2. SLIDING MODE CONTROL

The block diagram of the system with sliding mode control and the characteristic of the relay is shown in Fig. 1a and 1b, respectively. Here the signals u, y, r, $e = r - y$, d, m_n are the input, output of the SISO plant, reference value, error, output disturbance and measurement noise, respectively. In this section it will be assumed that the sliding mode control is based on fast switching of the relay, so that the generated high frequency harmonics appearing in the signal u are filtered by the dynamics of the plant and the output y depends mainly on the averaged value of the input u. The fast switching is obtained owing to the small hysteresis of the relay and usage of appropriate derivatives resulting from an polynomial $C(s)$. The derivatives cause the rapid change of the slope of the signal e^* at the instant of stepwise change of the signal u. The choice of $C(s)$ will be described in details further on.

In this section we assume that $d = 0$ and $m_n = 0$.

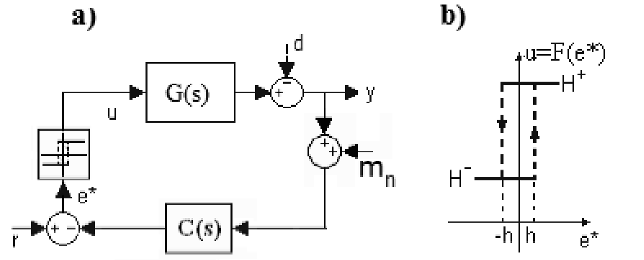

Fig. 1. a) The system with sliding mode control; b) characteristic of the relay.

Consider the linear SISO plant described by the transfer function (TF)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{L(s)}{M(s)} \tag{1}$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the plant output $y(t)$ and input $u(t)$, respectively, while $L(s)$ and $M(s)$ are known polynomials of m-th and n-th degree, respectively, $m < n$; $l = n - m$ is the relative degree of the TF $G(s)$. Assume that the TF $G(s)$ has the minimum phase zeros.

Block $C(s)$ from Fig. 1a is determined by the following Hurwitz polynomial

$$C(s) = c_0 s^{l-1} + c_1 s^{l-2} + ... + c_{l-2}s + 1, \tag{2}$$

Note that, under slowly varying r (and in an appropriate region), for the hysteresis of the relay $h \to 0$ there appears fast switching and we have $e^* \to 0$. Since then, the relay operates on the vertical segments of its characteristic, it may be replaced by the linear amplifier with high gain k ($k \to \infty$). Additionally, under $d(t) = 0$, $m_n(t) = 0$ and stable system, from the fulfilled then dependencies $e^* \approx 0$, $E^*(s) = R(s) - C(s)Y(s)$, (where $E^*(s)$ and $R(s)$ are the Laplace transforms of the functions $e^*(t)$ and $r(t)$) it results that

$$Y(s) \approx \frac{1}{C(s)} R(s). \tag{3}$$

Then it looks that $Y(s)$ is independent upon parameters of the plant. Therefore the system is very robust. Additionally, for appropriately chosen $C(s)$ the system has transients of a good quality. It may be chosen for instance $C(s) = (Ts + 1)^{l-1}$, with multiple root $s_1 = -1/T$, where $T > 0$ is possibly small time constant, which gives fast decay of the transient. However, one can note that the choice of $C(s)$ must assure stability of the system, therefore $C(s)$ and $Y(s)$ are dependent, in some degree, upon plant parameters.

Because high frequency harmonics resulting from switching are filtered by the dynamics of the plant, in further description the signals like $y(t)$, $e^*(t)$, $u(t,)$, etc. will take into account only the slower changes resulting from filtering, without higher harmonics.

The stability of the resulting closed loop (CL) system may be easy analyzed and results from the following

Lemma 1. The characteristic equation of the CL system shown in Fig. 1, described by

$$M(s) + kC(s)L(s) = 0 \tag{4}$$

for $k \to \infty$ takes the form

$$C(s)L(s) = 0, \tag{5}$$

while one root s_n of the equation (4) tends to $-\infty$. If the zeros of the polynomial $L(s)$ are minimum phase and $C(s) = (Ts + 1)^{l-1}$, with multiple root $s_1 = -1/T$, where $T > 0$, then the system is stable. □

Proof is obvious. The one root $s_n \to -\infty$ because: first - the finite roots s_i fulfill the equation (5) and second - for great values of $|s_n| \to \infty$ only the two terms of the equation (4) with highest powers n and $(n-1)$ of s play the dominant role. Since both these terms have positive coefficients then only the great, negative value of s_n may fulfill the equation (4). □

From Lemma 1 it results that the linear system with high gain k (and the analyzed system with sliding mode control), may be stable then and only then, when the plant TF $G(s)$ has minimum phase zeros.

In implementation, the higher order derivatives resulting from (2) may be approximated by substituting $s/(1 + s\tau)$, in the place of the operator s, in (2). Here $\tau > 0$ denotes a very small time constant (significantly smaller than T).

There is common view about great robustness of the systems with sliding mode control. But it should be stressed that this view is valid when measurement noise m_n disappears. Further on, it will be shown that these systems are very sensitive to measurement noise m_n and under presence of $m_n \neq 0$ their robustness collapses .

3. THE SYSTEM WITH MEASUREMENT NOISE

In the relay system with sliding mode control usually the hysteresis h of the relay is very small, to decrease the output fluctuations resulting from the relay switching. Therefore it may be a great influence of the measurement noise on the quality of control. In the literature usually this view is not sufficiently exactly noted, thought there are some exceptions e.g. (Yeh and Chen, 2011). This view will be illustrated in the two following examples.

3.1 Example 1

Consider the system with sliding mode control shown in Fig. 1 with linear plant described by the transfer function (TF)

$$G(s) = \frac{0.4s + 1}{s^3 + 2.7657s^2 + 3.9691s + 4.0854} \tag{6}$$

and the sliding surface described by

$$C(s) = 0.3s + 1 \tag{7}$$

Additionally it is $H = 10$, $h = 0.03$. Simulations were performed for $r(t) = \mathbf{1}(t)$, $d(t) = 0.5 \cdot \mathbf{1}(t-5)$ ($\mathbf{1}(t) = 1$, for $t \geq 1$ and $\mathbf{1}(t) = 0$, for $t < 0$) and for two different cases: 1. $m_n = 0$ and 2. $m_n \neq 0$. For the second case, the noise m_n was obtained from MATLAB Uniform in the interval $[-1, 1]$ Random Number Generator, with sampling period

0.001, the output signal of which was passed through the filter described by the TF $1/(s+1)$. The plot of the measurement noise is shown in Fig. 2b. The results of simulations for the cases $m_n = 0$ and $m_n \neq 0$ are shown in Fig. 2a by solid line and dotted line, respectively. It is seen that the measurement noise decreases the accuracy of the control system.

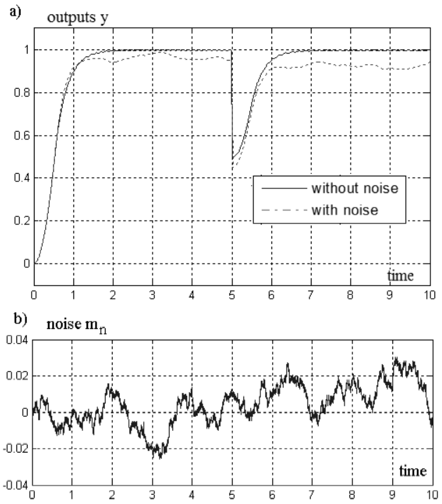

Fig. 2. a) Step responses for the system from Example 1, in the case: without noise (solid line) and with noise (dotted line); b) measurement noise.

One may suppose that if in the TF $C(s)$ it appears the second order derivative the decrease of the control quality will be significantly larger. This is confirmed by the following example.

3.2 Example 2

Now, consider the system shown in Fig. 1 with linear plant described by the transfer function (TF)

$$G(s) = \frac{0.5s+1}{s^4+3.5s^3+6s^2+7s+3} \tag{8}$$

and the sliding surface described by

$$C(s) = 0.09s^2 + 0.6s + 1 \tag{9}$$

The relay has the parameters $H = 15$, $h = 0.03$. Simulations were performed for $r(t) = \mathbf{1}(t)$, $d(t) = 0.5 \cdot \mathbf{1}(t-5)$ and as previously, for two different cases: 1. $m_n = 0$ and 2. $m_n \neq 0$. where the nonzero noise was the same as in the Example 1. The results of simulations for the cases $m_n = 0$ and $m_n \neq 0$ are shown in Fig. 3 with solid and dotted line, respectively. It is seen that the system does not operate under appearance of the measurement noise.

From both the examples it is seen that in the case of no noise the step responses are very good with short settling time. Additionally in accordance with the common view both the systems are very robust, i.e. they are not sensitive even to the large plant parameter changes as well as to some disturbances. However if the measurement noise is present then the good properties of both the systems disappear. Especially it is exactly seen in the case of the system from Example 2.

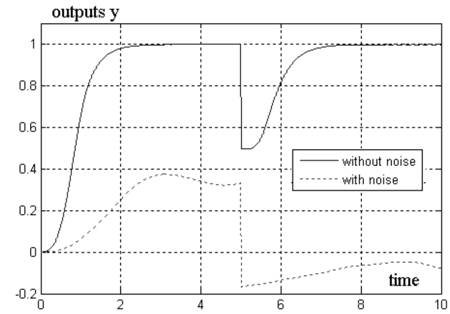

Fig. 3. a) Step responses for the system from Example 2, in the case: without noise (solid line) and with noise (dotted line).

4. RELAY SYSTEM WITH PARALLEL COMPENSATOR

In this section the idea of the system with parallel compensator introduced in (Gessing, 2007) will be remained. Notation applied here is the same as in the previous sections. In this section, during description of the system, it will be assumed that $d = 0$ and $m_n = 0$.

Consider the linear plant described by the TF (1) Assume now that the plant is stable, that is its poles p_i, $i = 1, 2, ..., n$ have negative real parts i.e. $Rep_i < 0$.

In the case of difficult plant (e.g nonminimum phase, and/or with higher order dynamics, as well as with pure time delay), when it is difficult to design the regulator assuring an appropriate accuracy, the relay system with parallel compensator shown in Fig. 4a may give good results.

Characteristic of the relay is shown in Fig. 1b. Further on, it will be shown that this system significantly better copes with the measurement noise than the systems with sliding mode control. Of course the same properties has the equivalent continuous system in which the relay is replaced by the amplifier with high gain and appropriate saturation (Gessing, 2007).

The parallel compensator is described by the TF

$$G_c(s) = \frac{Y_c(s)}{U(s)} = G_1(s) - G(s) \tag{10}$$

and its idea, as it was noted in (Gessing, 2007) is similar to that of the Smith predictor. Here $Y_c(s)$ is the Laplace transform of the output y_c of the compensator, while $G_1(s)$ is the TF which will be appropriately chosen.

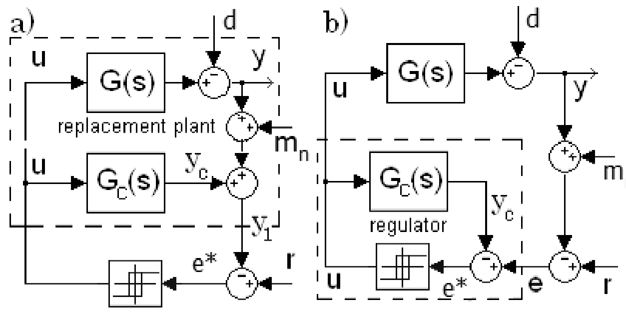

Fig. 4. The equivalent block diagrams of the system with parallel compensator.

Note that in the proposed structure shown in Fig. 4a the TF $G_r(s)$ of the replacement plant outlined by the dashed line is described by

$$G_r(s) = \frac{Y_1(s)}{U(s)} = G(s) + G_c(s) = \\ = G(s) + G_1(s) - G(s) = G_1(s) \quad (11)$$

Of course, to implement a closed loop (CL) stable system with the reference signal r determining the demanded output y the TF $G_1(s)$ should fulfill some demands.

In the case of regulation when $r = const$ the error in a constant steady state is mainly interesting, therefore for some constant steady state values it should be

$$y_c = 0, \quad y_1 = y, \quad e^* = r - y_1 = r - y \quad (12)$$

The latter condition will be fulfilled if

$$G_1(0) = G(0) \quad (13)$$

In the case of tracking of the varying reference signal r with the frequencies ω belonging to some working frequency band

$$\omega \in [0, \omega_{mx}] \quad (14)$$

the demand (12) should be at least approximately fulfilled for frequencies (14), which gives (Gessing, 2004)

$$G_1(j\omega) \approx G(\omega) \quad \text{for} \quad \omega \in [0, \omega_{mx}] \quad (15)$$

The further considerations are limited to the case of regulation.

Similarly as for the system with sliding mode control, the relay appearing in the CL system shown in Fig. 4a, with appropriately chosen H and $h \to 0$ may be treated approximately as the linear amplifier with high gain k.

The CL system should be stable and should have an appropriate quality of the transients. Let as choose the TF

$$G_1(s) = \frac{L_1(s)}{M_1(s)}, \quad (16)$$

where $L_1(s)$ and $M_1(s)$ are polynomials of $n-1$ and n degree, respectively, i.e. the relative degree l of the TF $G_1(s)$ is equal to one. Then the stability of the CL system results from the following

Lemma 2. The characteristic equation of the CL system shown in Fig. 4, described by

$$M_1(s) + kL_1(s) = 0 \quad (17)$$

for $k \to \infty$ takes the form

$$L_1(s) = 0 \quad (18)$$

and one root s_n of the equation (17) tends to $-\infty$. If the zeros of the polynomial $L_1(s)$ are minimum phase then the system is stable. □

The proof results directly from (11) and Lemma 1. Of course the Lemma 2 is strictly valid if in the place of the relay the linear amplifier with $k \to \infty$ appears.

4.1 Approximate Description of the CL System

The equivalent block diagram of the system from Fig. 4a is shown in Fig. 4b. Note that the part of the system outlined by the dashed line contains the elements of the regulator based on the parallel compensator. For the relay replaced by the linear amplifier with high gain $k \to \infty$, the regulator in the system is described by the following TF

$$C(s) = \frac{U(s)}{E(s)} = \frac{k}{1 + kG_c(s)} \approx \frac{1}{G_c(s)} \quad (19)$$

Note that if the TF $G_1(s)$ has the relative degree equal to one, then the TF (19) of the regulator has usually the degree of denominator polynomial smaller by one from that of the numerator. Strictly speaking this is valid when $k \to \infty$ i.e. when the fast switching of the relay with small hysteresis appear.

Accounting (19) we obtain the following formula describing the CL system

$$\frac{Y(s)}{U(s)} \approx \frac{G(s)/G_c(s)}{1 + G(s)/G_c(s)} = \frac{G(s)}{G_c(s) + G(s)} = \frac{G(s)}{G_1(s)} \quad (20)$$

The formula (20) may be used for designing the TF $G_1(s)$. One such a possibility will be discussed in the next subsection.

4.2 Design of the Replacement Plant Transfer Function

One way of designing $G_1(s)$ is to choose

$$M_1(s) = M(s) \quad (21)$$

$$L_1(s) = l(1 + sT)^{n-1}, \quad l = L(0) \quad (22)$$

so the condition (13) is fulfilled.

For choosing the parameter T the design based on the linear approach, e.g on frequency response, may be utilized (Gessing, 2007).

Accounting (1), (16), (21) in (20) we obtain for the CL system

$$\frac{Y(s)}{R(s)} \approx \frac{L(s)}{L_1(s)} \quad (23)$$

Thus the numerator of TF (23) contains the polynomial appearing in the numerator of (1), while the denominator of (23) contains the polynomial appearing in the numerator of (16). From these considerations it results that in the considered case the choice of $L_1(s)$ influences the dynamics of the researched CL system, essentially.

Really, the characteristic equation of the CL system is determined by (18) and its roots influence the velocity of decay of the transient response. Therefore we try to choose $L_1(s)$ in the form (22) containing the multiple root $s_i = -1/T, \ i = 1, 2, ...n-1$. Of course, to obtain fast transient, we should choose a possibly small time constant T, for which the equation (18) and the whole system have good transients.

From the previous considerations it results the following

Corollary 1. If the polynomial $L_1(s)$ is chosen such that

$$L_1(s) = C(s)L(s) \quad (24)$$

then, under $m_n = 0$, the time responses for the same excitations $r(t)$ and $d(t)$ of the system with sliding mode control and with parallel compensator should be very close one to other. If additionally $d(t) = 0$ then both the systems are described by the dependence

$$\frac{Y(s)}{R(s)} \approx \frac{1}{C(s)} \tag{25}$$

□

Really, both the systems have the same characteristic equation which results from (5) and (18), therefore the dynamics of their transients is comparable. If $m_n = 0$ and $d(t) = 0$ then the dependence (25) for the system with sliding mode control results from (3), while for the system with parallel compensator – from dependencies (23) and (24).

If the formula (24) is fulfilled and $m_n = 0$ then we will say that both the systems have comparable dynamics.

5. ILLUSTRATING EXAMPLES

5.1 Example 3

Consider the system with parallel compensator shown in Fig.4. Assume that the plant described by (6) and the relay is the same as in the Example 1. Assume that $m_n = 0$, and also $r(t)$ and $d(t)$ are the same as in Example 1. In accordance with the Corollary 1, accounting (6) and (7) assume that $L_1(s) = (0.3s + 1)(0.4s + 1)$. Then both the systems: with sliding mode control with parameters described in Example 1 and with parallel compensator considered here have comparable dynamics. The time responses of both the systems are shown in Fig. 5.

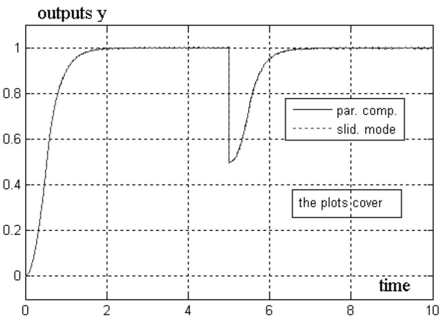

Fig. 5. The time responses of the system with parallel compensator (solid line) and of the system with sliding mode control (dotted line) (data from Examples 1, 3).

It is seen, that in accordance with the Corollary 1 the time responses of both the systems cover.

In the Fig. 6 there are shown the results of simulations for the system with parallel compensator and the plant (6). The time responses $y(t)$ for the same excitations in the case $m_n = 0$ (solid line) and $m_n \neq 0$ (dotted line) are shown.

It is shown that the system with parallel compensator copes well with measurement noise. The difference between the case when $m_n = 0$ and $m_n \neq 0$ is insignificant.

5.2 Example 4

Consider now the system with parallel compensator shown in Fig.4. Assume that the plant described by the TF (8) and also the relay as well as $r(t)$ and $d(t)$ are the same as in Example 2. Assume that $m_n = 0$. Accounting (8), (9) and Corollary 1 assume that $L_1(s) = (0.09s^2 + 0.6s + 1)(0.5s + 1)$. The time responses of the system with sliding mode control with parameters described in Example 2 and of the system with parallel compensator and comparable dynamics are shown in Fig. 7. It is shown, that in accordance with the Corollary 1 the time responses for both the systems almost cover.

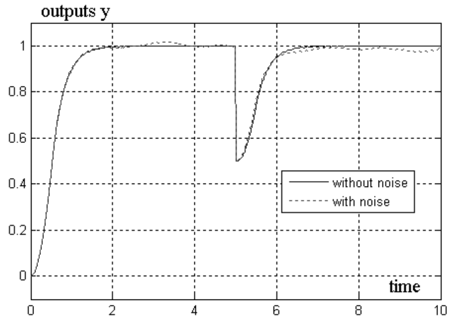

Fig. 6. The time responses of the system with parallel compensator and plant (6) with $m_n = 0$ (solid line) and $m_n \neq 0$ (dotted line) (data from Example 3).

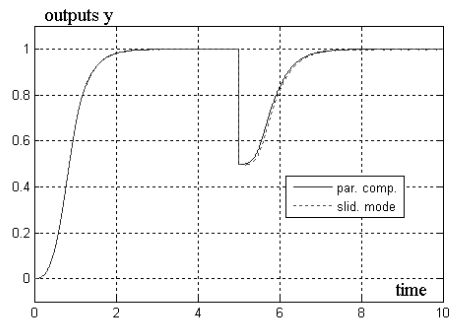

Fig. 7. The time responses of the system with parallel compensator (solid line) and of the system with sliding mode control (dotted line) (data from Examples 2, 4).

In the Fig. 8 there are shown the results of simulations for the system with parallel compensator and plant (8). The time responses $y(t)$ for the same excitations in the case $m_n = 0$ (solid line) and $m_n \neq 0$ (dotted line) are shown.

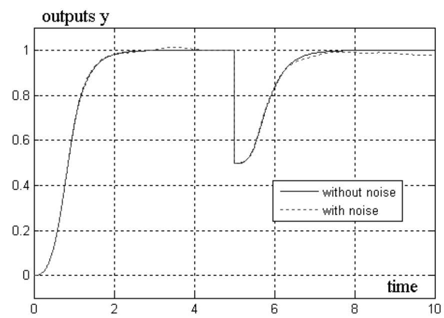

Fig. 8. The time responses of the system with parallel compensator and plant (8) with $m_n = 0$ (solid line) and $m_n \neq 0$ (dotted line) (data from Example 4).

Also now, the difference between the case when $m_n = 0$ and $m_n \neq 0$ is insignificant.

5.3 Example 5

Consider now the plant with nonminimum phase zero described by the TF

$$G(s) = \frac{-0.5s + 1}{s^4 + 3.5s^3 + 6s^2 + 7s + 3}. \quad (26)$$

Assume that the parallel compensator polynomial $L_1(s)$ is now described by

$$L_1(s) = 0.027s^3 + 0.27s^2 + 0.9s + 1 \quad (27)$$

then $T = 0.3$ and the transients will be somewhat faster than in Example 4. The remaining data applied in simulations are the same as in Example 4.

The results of simulations in the form of the time responses of the plant outputs y, for the same excitations ($r(t)$ and $d(t)$) are shown in Fig. 9. By the solid line the time responses $y(t)$ of the plant output are shown for the case $m_n = 0$, while by the dotted line - for the case $m_n \neq 0$.

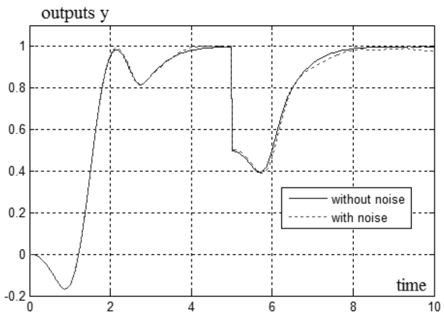

Fig. 9. The time responses of the system with parallel compensator and nonminimum phase plant (26) for $m_n = 0$ (solid line) and $m_n \neq 0$ (dotted line) (data from Example 5).

It is seen that both the time responses of $y(t)$ are close one to other, which means that the system well operates under existence of measurement noise. As the result of existence of the one nonminimum phase zero both the time responses have undershot which is characteristic for the systems with nonminimum phase plant. Because the system is rather fast the undershot is relatively high. For smaller speed of the transients (e.g. when T=0.5) the undershot would be smaller.

Of course the system with sliding mode control cannot operate with this nonminimum phase plant.

6. FINAL CONCLUSIONS

It is a common view that the relay systems with sliding mode control are very robust, i. e. they are insensitive to large and relatively fast parameter changes, as well as to the output disturbances. However usually, it is not stressed that this view is valid only then, when in the system the measurement noise disappear. In the present paper it is noted that appearance of measurement noise completely changes the properties of these systems, i.e. they stop to be robust. Due to small hysteresis of the relay, applied to decrease the output oscillations resulting from switching, even small measurement noise may destroy robustness, especially then when the higher than first order derivatives are used in sliding surface description. This observation has been confirmed by the performed simulations.

It is shown that the relay systems with parallel compensator, designed in accordance with (Gessing, 2007), applied to the stable plants with minimum phase zeros, may have almost the same dynamic properties as the systems with sliding mode control. This means that for the same reference value and output disturbance both the systems have almost the same time responses of the plant output. We may say that both the systems have comparable dynamics. However one must remember that generally the different assumptions must be made for the plants in both the systems. Namely, for the system with sliding mode control, the plant should have minimum phase zeros, but may be stable or non-stable, while for the system with parallel compensator the plant must be stable, but may have minimum or nonminimum phase zeros. Then, both the systems with comparable dynamics may be created only for the stable plants with minimum phase zeros.

It becomes that the systems with parallel compensator and comparable dynamics to that with sliding mode control, cope very well with measurement noise. This is caused by the fact that no derivatives are used in parallel compensator, while the plant plays the role of the law-pass filter. The relay systems with parallel compensator, with different difficult (e.g. with nonminimum phase zeros, or with delay) but stable plants, also cope well with measurement noise.

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