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Adaptive Fault-Tolerant Control for Time-Delay Nonlinear Systems with Stochastic Actuator Failures ⋆

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Abstract: In [18], a class of nonlinear systems with stochastic actuator failures have been studied. Stochastic functions related to Markovian variables have been introduced to describe the failure scaling factors for each actuators. In this paper we consider the kind of systems involving state-delay, which is much more practical, and adaptive fault-tolerant control problem has been studied. Two main challenges arise: how to establish the infinitesimal generator for systems involving multi-Markovian variables and state-delay simultaneously; then how to handle the delayed terms and the extra transition rate related ones appearing in the wisely designed Lyapunov-Krasovskii functional. By proposing an adaptive fault-tolerant control scheme, the existence and uniqueness of the solution process to the closed-loop system is guaranteed and the boundedness in probability of all the closed-loop signals can be ensured. An example is worked out to illustrate the effectiveness of the proposed scheme.

Keywords: Fault tolerant control; Markovian jumping actuator failures; Time-delay; Nonlinear system; Adaptive control.

1. INTRODUCTION

Time-delay phenomena are prevalent in various engineering systems such as communication-based systems, biological reactors, rolling mills, aerospace systems, etc. [1] [2] and the existence of time-delay is often a significant source of the degradation of system performance and even a cause of instability of closed-loop systems. During the past few decades, considerable efforts have been made in the stability analysis and robust stabilization control for linear and nonlinear time-delay systems; see, for examples, [3]-[6] and the references therein. Most of the results are obtained in the form of linear matrix inequality (LMI). Recently, adaptive methods are employed to deal with the uncertainties in time-delay systems and backstepping technique is adopted to design the stabilization or tracking controller constructively for a class of low-triangle nonlinear time-delay systems [5] [7] [8].

Meanwhile, actuator failure phenomenon is commonly encountered in many practical systems, which may deteriorate the control performance of systems, cause the instability of closed-loop systems, or even worse, result in catastrophic accidents. Due to their practical and theoretical importance, the fault tolerant control (FTC) problem for nonlinear systems has been one of main concerns of researchers in the field of automation and control, and fruitful results have been obtained during the past decades; see, for examples, [9]-[15] and references therein, just to name a few. In particular, adaptive-based FTC scheme has received much attention for its capability to deal with system uncertainties as well as variations caused by actuator failures simultaneously. It is worth noting that all the aforementioned literatures focus on the finite number of actuator failure case, that is, once an actuator fails, it will stay at the faulty mode during its rest operation. However, [16] shows that the patterns, times and modes of actuator failures are practically stochastic and for given adequate historical data, the abrupt failures can be modeled as Markovian process. Furthermore, the status of each actuator is mostly governed by an independent Markovian process, that is, each actuator may fail at any sampling time independently of the others. Most recently, infinite number of actuator failure case has been taken into account in [17]. In [18], a class of nonlinear uncertain systems with stochastic actuator failures and unknown parameters have been considered, in which the stochastic functions related to Markovian variables are introduced to denote the failure scaling factors for each actuator and the adaptive failure compensation problem has been studied without time-delay.

In this paper, the adaptive FTC problem for a class of nonlinear systems involving Markovian jumping actuator failures and state-delay has been taken into consideration. The considered system possesses the following three characteristics: First, stochastic functions related to Markovian variables are adopted to denote the failure scaling factors for each actuators and multi-Markovian variables are involved corresponding to the different actuators; Second, state-delay is taken into account, that is, the future state of the system under consideration is not only dependent of the present states but also the past ones; Third, different from the finite number of actuator failure case considering in most existing literatures, here the total number of actu-

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ator failure may be infinite. Two main challenges arise due to the existing multi-Markovian variables and state-delay:

(1) establishing the infinitesimal generator for nonlinear systems involving multi-Markovian variables and state-delay simultaneously;
(2) dealing with the delayed terms and the extra transition rate related ones appearing in the wisely designed Lyapunov-Krasovskii functional.

The rest part of the paper is organized as follows. Section 2 establishes some necessary preliminary results for nonlinear systems with multi-Markovian variables and state-delay. Section 3 presents the considered system model. Then, by employing the backstepping method, adaptive FTC strategy and stability analysis are given in section 4 and 5 respectively. In section 6, an example is provided to illustrate the effectiveness of the proposed scheme. Finally, the paper is concluded in section 7.

Notations. The following notations are used throughout the paper: $\mathbb{R}$ represents the set of real numbers, $\mathbb{R}_+$ denotes the set of nonnegative real numbers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ denote, respectively, n-dimensional real space and $n \times r$-dimensional real matrix space. $\|\cdot\|$ is the Euclidean norm. Let (Ω, F, P) be a complete probability space. $P(\cdot)$ means the probability, and $E(\cdot)$ denotes the expectation. $\mathcal{C}^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable bounded $\mathcal{C}([-\tau,0];\mathbb{R}^n)$-valued random variables $\xi = \{\xi(t) : -\tau \le t \le 0\}$. $\mathcal{C}^i$ stands for the set of all functions with continuous ith-order partial derivative. $\mathcal{C}^{2,1}(\mathbb{R}^n \times [-\tau,\infty) \times S^m;\mathbb{R}_+)$ represents the family of all nonnegative functions $V(x(t), t, \mathbf{r}(t))$ on $(\mathbb{R}^n \times [-\tau,\infty) \times S^m)$ which are $\mathcal{C}^2$ in x and $\mathcal{C}^1$ in t, and $S = \{1, 2, \cdots, N\}$ is a finite set. The notation $a \wedge b$ means taking the minimum of a and b. We sometimes drop the arguments of functions without causing confusion.

2. PRELIMINARY RESULTS

Consider the following nonlinear system with Markovian jump parameters and time-delay

$$\dot{x}(t) = f(x(t), x(t-\tau), t, \mathbf{r}(t)) \quad (1)$$

on $t > 0$, where $x(t) \in \mathbb{R}^n$ is the state with initial data $\{x(t) : -\tau \le t \le 0\} = \xi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, $\tau > 0$ is the time delay, and $f : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S^m \to \mathbb{R}^n$ is locally Lipschitz in $(x(t), x(t-\tau))$ uniformly in t which vanishes at $(x(t), x(t-\tau)) = (0, 0)$. $\mathbf{r}(t) = (r_1(t), r_2(t), \cdots, r_m(t))$ is a Markovian vector and for $i = 1, 2, \cdots, m$, $r_i(t)$ is a right-continuous homogeneous irreducible Markovian process on the probability space taking values in a finite set $S = \{1, 2, \cdots, N\}$.

Firstly, the multi-Markovian variables and time-delay based infinitesimal generator is derived and presented as in Lemma 1, which will play a key role in the development of a state-feedback controller for the system (1).

Lemma 1. For a function $V \in \mathcal{C}^{2,1}(\mathbb{R}^n \times [-\tau,\infty) \times S^m;\mathbb{R}_+)$, the infinitesimal generator $\mathcal{L}V$ from $\mathbb{R}^n \times \mathbb{R}^n \times [-\tau,\infty) \times S^m$ to $\mathbb{R}$ at the specified mode $(p_1, p_2, \cdots, p_m) \in S^m$ subject to (1) is given as below.

$$\begin{aligned}&\mathcal{L}V(x, y, t, p_1, p_2, \cdots, p_m)\\ =& V_t(x, t, p_1, p_2, \cdots, p_m)\\ &+ V_x(x, t, p_1, p_2, \cdots, p_m) f(x, y, t, p_1, p_2, \cdots, p_m)\\ &+ \sum_{k=1}^{m}\sum_{q_k=1}^{N} \gamma_{p_k q_k} V(x, t, p_1, p_2, \cdots, q_k, \cdots, p_m)\end{aligned}$$

where $V_t(x, t, p_1, \cdots, p_m) = \frac{\partial V(x,t,p_1,\cdots,p_m)}{\partial t}$, $V_x(x, t, p_1, \cdots, p_m) = \big(\frac{\partial V(x,t,p_1,p_2,\cdots,p_m)}{\partial x_1}, \cdots, \frac{\partial V(x,t,p_1,p_2,\cdots,p_m)}{\partial x_n}\big)$, $\gamma_{p_k q_k} > 0$ is the transition rate from mode p_k to mode q_k if $p_k \ne q_k$, and $\gamma_{p_k p_k} = -\sum_{q_k=1, q_k\ne p_k}^{N} \gamma_{p_k q_k}$.

Proof: The proof is following the similar line as in [18], and is thus omitted. ■

Remark 2. Noting that function f is not only related to argument x, but also argument y, that is, $x(t-\tau)$ for system (1), therefore, it should be emphasized that the function V has three arguments $(x, t, \mathbf{r}(t))$ defined on $\mathbb{R}^n \times [-\tau,\infty) \times S^m$, whereas the infinitesimal generator $\mathcal{L}V$ has four arguments $(x, y, t, \mathbf{r}(t))$ defined on $\mathbb{R}^n \times \mathbb{R}^n \times [-\tau,\infty) \times S^m$.

Then, based on the infinitesimal generator $\mathcal{L}V$ established in Lemma 1, two useful properties are given as follows.

Property 3.

$$\begin{aligned}&E\big[V(x, \tau_2, \mathbf{r}(\tau_2)) - V(x, \tau_1, \mathbf{r}(\tau_1))\big]\\ =&E\int_{\tau_1}^{\tau_2} \mathcal{L}V(x, y, t, \mathbf{r}(t))dt\end{aligned}$$

as long as $V(x, t, \mathbf{r}(t))$ and $\mathcal{L}V(x, y, t, \mathbf{r}(t))$ are bounded a.s. on $t \in [\tau_1, \tau_2]$, where τ_1, τ_2 are bounded stopping times and $0 \le \tau_1 \le \tau_2$.

Property 4.

$$\begin{aligned}&E\big(\mathcal{L}V(x, y, t, \mathbf{r}(t))\big)\\ =&\sum_{p_1=1}^{N}\cdots\sum_{p_m=1}^{N} E\big(\mathcal{L}V(x, y, t, p_1, \cdots, p_m)\big)\pi_{p_1}\cdots\pi_{p_m}\end{aligned}$$

as long as the expectations involved exist and are bounded a.s., where $\pi_i = (\pi_{1_i}, \pi_{2_i}, \cdots, \pi_{N_i})$ is the stationary distribution of the ith Markovian variable $r_i(t)$, that is, for each $t \ge 0$, $\pi_{p_i} = P(r_i(t) = p)$, $p \in S$. Clearly $\sum_{p=1}^{N} \pi_{p_i} = 1$ and $\pi_{p_i} > 0$.

The following lemma presents a sufficient condition to ensure the existence and uniqueness of the solution for the system (1).

Lemma 5. Assume that f is locally Lipschitz in $(x(t), x(t-\tau))$ for all $t \ge 0$. For any $l > 0$, define the stopping time $\eta_l = \inf\{t : t > 0, \|x(t)\| \ge l\}$. Assume that there is a function $V(x, t, \mathbf{r}(t)) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times [-\tau,\infty) \times S^m;\mathbb{R}_+)$ and parameters k and $K \ge 0$ such that

$$\lim_{\|x\|\to\infty} \inf_{t\ge 0} V(x, t, \mathbf{r}(t)) = \infty \quad (2)$$

and

$$EV(x(\eta_l \wedge t), \eta_l \wedge t, \mathbf{r}(\eta_l \wedge t)) \le Ke^{k(\eta_l\wedge t)} \quad (3)$$

hold.

Then, for any initial data $\{x(t) : -\tau \le t \le 0\} = \xi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ and $\mathbf{r}(0) \in S^m$, system (1) has a unique solution $x(t; \xi, \mathbf{r}(0))$.

Proof: It is noted that η_l is increasing along l so it has a limit $\eta_\infty = \lim_{l\to\infty} \eta_l$. By the known existence-and-uniqueness Theorem 7.12 in [20], the locally Lipschitz condition guarantees that there exists a unique maximal local solution to system (1) for $t \in [-\tau, \eta_\infty)$, where η_∞ is the explosion time. Therefore, in order to prove that the maximal local solution is in fact a unique global solution, we only need to show that $P\{\eta_\infty = \infty\} = 1$.

According to the formula for expectation, it yields

$$P\{\eta_l \le t\} \le \frac{EV(x(\eta_l \wedge t), \eta_l \wedge t, \mathbf{r}(\eta_l \wedge t))}{\inf\limits_{\|x\|>l, t\ge 0, p_i \in S} V(x, t, p_1, \cdots, p_m)}.$$

Letting $l \to \infty$ and using (2) and (3), we can obtain $P\{\eta_\infty \le t\} = 0$. Since t is arbitrary, it has

$$P\{\eta_\infty = \infty\} = 1.$$

The proof is therefore complete. ■

In order to analyze the stability of closed-loop systems, the notion of stability is introduced as follows which can be regarded as an generalization of the counterpart without time delay in [21].

Definition 6. A stochastic process $x(t; \xi, \mathbf{r}(0))$ is said to be bounded in probability if for all $\xi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ and $\mathbf{r}(0) \in S^m$, the stochastic variables $\|x(t; \xi, \mathbf{r}(0))\|$ are bounded in probability uniformly in t, i.e.

$$\lim_{l\to\infty} \sup_{t>0} P\Big\{\|x(t; \xi, \mathbf{r}(0))\| > l\Big\} = 0. \tag{4}$$

Moreover, the criterion about boundedness in probability for nonlinear systems with multi-Markovian variables and time-delay is presented as below.

Theorem 7. Assume that system (1) has a unique solution in almost surely sense for $t \in [-\tau, \infty)$ and there exist a positive function $V \in \mathcal{C}^{2,1}(\mathbb{R}^n \times [-\tau, \infty) \times S^m; \mathbb{R}_+)$ and $d_c > 0$ such that

$$EV\Big(x, t, \mathbf{r}(t)\Big) \le d_c \tag{5}$$

and (2) hold.
Then, for any initial data $\{x(t) : -\tau \le t \le 0\} = \xi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ and $\mathbf{r}(0) \in S^m$, the solution $x(t; \xi, \mathbf{r}(0))$ of system (1) is bounded in probability.

Proof: By the generalized Chebyshev's inequality, i.e. Lemma 1.4 in [21], and from (5), it follows that

$$\begin{aligned} P\Big\{\|x(t; \xi, \mathbf{r}(0))\| > l\Big\} \le& \frac{EV(x, t, \mathbf{r}(t))}{\inf\limits_{\|x\|>l, t\ge 0} V(x, t, \mathbf{r}(t))} \\ \le& \frac{d_c}{\inf\limits_{\|x\|>l, t\ge 0} V(x, t, \mathbf{r}(t))}, \end{aligned} \tag{6}$$

which together with (2), indicates that (4) holds. ■

3. SYSTEM MODEL

Consider a class of time-delay nonlinear systems with m actuators described by

$$\begin{aligned} \dot{x}_i(t) = x_{i+1}(t) + f_i(\bar{x}_i(t)) + \varphi_i(\bar{x}_i(t-\tau)), \\ i = 1, 2, \cdots, n-1 \end{aligned} \tag{7a}$$

$$\dot{x}_n(t) = f_n(x(t)) + \varphi_n(x(t-\tau)) + \sum_{j=1}^{m} g_j(r_j(t)) u_j(t) \tag{7b}$$

where for $i = 1, 2, \cdots, n$, $x_i(t) \in \mathbb{R}$ is the state variable with initial data $x_i(t) = \xi_i(t)$ for $-\tau \le t \le 0$ and $r_j(0) \in S$, $\bar{x}_i(t) = [x_1(t), x_2(t), \cdots, x_i(t)]^T$ is the state vector, $\bar{x}_i(t-\tau) = [x_1(t-\tau), x_2(t-\tau), \cdots, x_i(t-\tau)]^T$ is the delayed state vector, $\tau > 0$ is a constant time delay, $u_j(t) \in \mathbb{R}$ for $j = 1, 2, \cdots, m$, is the jth input of the system, $f_i \in \mathbb{R}$ and $\varphi_i \in \mathbb{R}$ are locally Lipschitz continuous functions, which vanish at $x = 0$, $r_j(t)$ is a Markovian variable which has been described in section 2. In addition, $g_j(r_j(t)) \in [0, 1]$ is a stochastic function related to Markovian variable $r_j(t)$ and may change at any time triggered by abrupt actuator failures.

In our paper, the adopted stochastic function $g_j(r_j(t))$ denotes the jth actuator failure scaling factor which can be classified into the following three cases.

Case 1. $g_j(r_j(t)) = 1 \Rightarrow$ the jth actuator works normally in the failure-free case;

Case 2. $g_j(r_j(t)) = 0 \Rightarrow$ the jth actuator has failed completely;

Case 3. $g_j(r_j(t)) \in (0\ 1) \Rightarrow$ there is partial loss of effectiveness of the jth actuator.

Before proposing the control strategy, we introduce the following assumptions for system (7a)-(7b).

Assumption 8. The m actuators cannot fail completely simultaneously which indicates that at least one actuator remains activeness. Denoting $h = 1/\sum\limits_{j=1}^{m} g_j(r_j(t))$, we assume that h is bounded.

Assumption 9. For $i = 1, 2, \cdots, n$, there exists a positive constant a_j such that

$$|\varphi_i(\bar{x}_i(t-\tau))| \le \sum_{j=1}^{n_i} a_j |x_1(t-\tau)|^j$$

where n_i is a positive integer.

4. CONTROLLER DESIGN

In this section, we will deal with a state feedback stabilizing problem for the time-delay nonlinear system (7a)-(7b) subject to stochastic actuator failures.

Firstly, introduce the following transformations

$$\begin{aligned} &z_1(x_1(t), \mathbf{r}(t)) = x_1(t), \\ &z_i(\bar{x}_i(t), \mathbf{r}(t)) = x_i(t) - \alpha_{i-1}(\bar{x}_{i-1}(t), \mathbf{r}(t)),\ i = 2, \cdots, n \end{aligned} \tag{8}$$

where $\alpha_{i-1}(\bar{x}_{i-1}(t), \mathbf{r}(t))$ for $i = 2, \cdots, n$, is a virtual controller which will be determined later.

Construct a Lyapunov-Krasovskii functional as

$$\begin{aligned} V(x(t), \mathbf{r}(t)) =& \frac{1}{2}\sum_{i=1}^{n} z_i^2(\bar{x}_i(t), \mathbf{r}(t)) + \frac{1}{2h}\tilde{h}^T \Gamma_h^{-1} \tilde{h} \\ &+ \int_{t-\tau}^{t} e^{\lambda(s-t)} q(s) ds \end{aligned} \tag{9}$$

where $\tilde{h} = h - \hat{h}$, $\hat{h}$ is the estimate of h given in Assumption 8, Γ_h is a positive definite matrix, λ is a positive constant and $q(s)$ is a positive function yet to be determined.

Without loss of generality, here and hereafter it is assumed that: at instant t, $r_i(t) = p_i$ with $p_i \in S$, $i = 1, 2, \cdots, m$.

Now we will investigate $\mathcal{L}V$ at the mode of $\mathbf{r}(t) = (p_1, p_2, \cdots, p_m)$.

For the sake of simplicity, the following notations are introduced.

$$z_{i,\mathbf{p}} = z_i(\bar{x}_i(t), p_1, \cdots, p_m), \quad \alpha_{i,\mathbf{p}} = \alpha_i(\bar{x}_i(t), p_1, \cdots, p_m),$$

$$h_{\mathbf{p}} = h(p_1, \cdots, p_m), \qquad \hat{h}_{\mathbf{p}} = \hat{h}(p_1, \cdots, p_m).$$

By Lemma 1, the infinitesimal generator of the functional $V(x(t), \mathbf{r}(t))$ is shown as

$$\begin{aligned}
&\mathcal{L}V(x(t), x(t-\tau), p_1, p_2, \cdots, p_m) \\
&= z_{1,\mathbf{p}}\Big(z_{2,\mathbf{p}} + \alpha_{1,\mathbf{p}} + f_1 + \varphi_1\Big) + \sum_{i=2}^{n-1} z_{i,\mathbf{p}}\Big[z_{i+1,\mathbf{p}} + \alpha_{i,\mathbf{p}} + f_i \\
&\quad + \varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j}(x_{j+1} + f_j + \varphi_j)\Big] + z_{n,\mathbf{p}}\Big[f_n + \varphi_n \\
&\quad + \sum_{j=1}^{m} g_j(p_j)u_j(t) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1,\mathbf{p}}}{\partial x_j}(x_{j+1} + f_j + \varphi_j)\Big] \\
&\quad - \frac{1}{h_{\mathbf{p}}}\tilde{h}_{\mathbf{p}}^T \Gamma_h^{-1} \dot{\hat{h}}_{\mathbf{p}} + q(t) - e^{-\lambda\tau} q(t-\tau) - \lambda \int_{t-\tau}^{t} e^{\lambda(s-t)} q(s) ds \\
&\quad + \sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\Big(x(t), p_1, p_2, \cdots, q_k, \cdots, p_m\Big). \qquad (10)
\end{aligned}$$

Considering the existing delayed terms $\varphi_i(\bar{x}_i(t-\tau))$ in (10), we apply the Young's inequality [22] and obtain the following inequalities

$$\begin{aligned}
&\sum_{i=1}^{n} z_{i,\mathbf{p}} \varphi_i(\bar{x}_i(t-\tau)) \\
&\le \sum_{i=1}^{n} \frac{1}{2}\varepsilon_i^2 z_{i,\mathbf{p}}^2 + \sum_{i=1}^{n} \frac{1}{2\varepsilon_i^2} \varphi_i^2(\bar{x}_i(t-\tau)) \\
&\le \sum_{i=1}^{n} \frac{1}{2}\varepsilon_i^2 z_{i,\mathbf{p}}^2 + \sum_{i=1}^{n} \frac{1}{2\varepsilon_i^2} n_i \sum_{j=1}^{n_i} a_j^2 |x_1(t-\tau)|^{2j}, \qquad (11)
\end{aligned}$$

$$\begin{aligned}
&-\sum_{i=2}^{n} z_{i,\mathbf{p}} \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j} \varphi_j(\bar{x}_j(t-\tau)) \\
&\le \sum_{i=2}^{n} z_{i,\mathbf{p}}^2 \sum_{j=1}^{i-1} \frac{1}{2}\varepsilon_{ij}^2 \Big(\frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j}\Big)^2 + \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}^2} \varphi_j^2(\bar{x}_j(t-\tau)) \\
&\le \sum_{i=2}^{n} z_{i,\mathbf{p}}^2 \sum_{j=1}^{i-1} \frac{1}{2}\varepsilon_{ij}^2 \Big(\frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j}\Big)^2 \\
&\quad + \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}^2} n_j \sum_{k=1}^{n_j} a_k^2 |x_1(t-\tau)|^{2k}, \qquad (12)
\end{aligned}$$

where $\varepsilon_i, \varepsilon_{ij}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n-1$ are positive constants to be chosen by the designer.

Substitute the inequalities (11), (12) into (10), it yields

$$\begin{aligned}
&\mathcal{L}V\Big(x(t), x(t-\tau), p_1, p_2, \cdots, p_m\Big) \\
&\le z_{1,\mathbf{p}}\Big(\alpha_{1,\mathbf{p}} + \Xi_1\Big) + \sum_{i=2}^{n-1} z_{i,\mathbf{p}}\Big(\alpha_{i,\mathbf{p}} + \Xi_i\Big) + z_{n,\mathbf{p}}\Big(\sum_{j=1}^{m} g_j(p_j)u_j(t) \\
&\quad + \Xi_n\Big) + z_{1,\mathbf{p}}(t-\tau) \sum_{i=1}^{n} \frac{1}{2\varepsilon_i^2} n_i \sum_{j=1}^{n_i} a_j^2 z_{1,\mathbf{p}}^{2j-1}(t-\tau) \\
&\quad + z_{1,\mathbf{p}}(t-\tau) \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}^2} n_j \sum_{k=1}^{n_j} a_k^2 z_{1,\mathbf{p}}^{2k-1}(t-\tau) \\
&\quad - \frac{1}{h_{\mathbf{p}}}\tilde{h}_{\mathbf{p}}^T \Gamma_h^{-1} \dot{\hat{h}}_{\mathbf{p}} + q(t) - e^{-\lambda\tau} q(t-\tau) - \lambda \int_{t-\tau}^{t} e^{\lambda(s-t)} q(s) ds \\
&\quad + \sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\Big(x(t), p_1, p_2, \cdots, q_k, \cdots, p_m\Big) \qquad (13)
\end{aligned}$$

where

$$\begin{aligned}
\Xi_1 &= f_1 + \frac{1}{2}\varepsilon_1^2 z_{1,\mathbf{p}}, \\
\Xi_i &= z_{i-1,\mathbf{p}} + f_i + \frac{1}{2}\varepsilon_i^2 z_{i,\mathbf{p}} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j}(x_{j+1} + f_j) \\
&\quad + z_{i,\mathbf{p}} \sum_{j=1}^{i-1} \frac{1}{2}\varepsilon_{ij}^2 \Big(\frac{\partial \alpha_{i-1,\mathbf{p}}}{\partial x_j}\Big)^2, \quad i = 2, \cdots, n-1 \\
\Xi_n &= z_{n-1,\mathbf{p}} + f_n + \frac{1}{2}\varepsilon_n^2 z_{n,\mathbf{p}} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1,\mathbf{p}}}{\partial x_j}(x_{j+1} + f_j) \\
&\quad + z_{n,\mathbf{p}} \sum_{j=1}^{n-1} \frac{1}{2}\varepsilon_{nj}^2 \Big(\frac{\partial \alpha_{n-1,\mathbf{p}}}{\partial x_j}\Big)^2.
\end{aligned}$$

Then, we design the positive function $q(t)$ to compensate for the delayed terms in (13). To this end, define $q(t)$ as

$$\begin{aligned}
q(t) = e^{\lambda\tau}\Big[&\sum_{i=1}^{n} \frac{1}{2\varepsilon_i^2} n_i \sum_{j=1}^{n_i} a_j^2 z_{1,\mathbf{p}}^{2j}(t) \\
&+ \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}^2} n_j \sum_{k=1}^{n_j} a_k^2 z_{1,\mathbf{p}}^{2k}(t)\Big]. \qquad (14)
\end{aligned}$$

Thus, the virtual controllers, the actual controller and the parameter update law are obtained as follows.

$$\begin{aligned}
\alpha_{1,\mathbf{p}} &= -c_1 z_{1,\mathbf{p}} - \Xi_1 - e^{\lambda\tau} \sum_{i=1}^{n} \frac{1}{2\varepsilon_i^2} n_i \sum_{j=1}^{n_i} a_j^2 z_{1,\mathbf{p}}^{2j-1} \\
&\quad - e^{\lambda\tau} \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}^2} n_j \sum_{k=1}^{n_j} a_k^2 z_{1,\mathbf{p}}^{2k-1}, \\
\alpha_{i,\mathbf{p}} &= -c_i z_{i,\mathbf{p}} - \Xi_i, \ i = 2, \cdots, n-1 \qquad (15) \\
u_j &= \hat{h}_{\mathbf{p}}(-c_n z_{n,\mathbf{p}} - \Xi_n), \ j = 1, 2, \cdots, m \qquad (16) \\
\dot{\hat{h}}_{\mathbf{p}} &= \Gamma_h z_{n,\mathbf{p}}(c_n z_{n,\mathbf{p}} + \Xi_n) - 2c_h \hat{h}_{\mathbf{p}}, \qquad (17)
\end{aligned}$$

where $c_1, c_2, \cdots, c_n, c_h$ are positive constants.

Substitute (14)-(17) into (13), it yields

$$\begin{aligned}
&\mathcal{L}V\Big(x(t), x(t-\tau), p_1, p_2, \cdots, p_m\Big) \\
&\le -\sum_{i=1}^{n} c_i z_{i,\mathbf{p}}^2 + 2c_h \frac{1}{h_{\mathbf{p}}}\tilde{h}_{\mathbf{p}}^T \Gamma_h^{-1} \hat{h}_{\mathbf{p}} - \lambda \int_{t-\tau}^{t} e^{\lambda(s-t)} q(s) ds \\
&\quad + \sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\Big(x(t), p_1, p_2, \cdots, q_k, \cdots, p_m\Big). \qquad (18)
\end{aligned}$$

Remark 10. From the above design procedure, we can see that there is an extra transition rate related term in the infinitesimal generator $\mathcal{L}V$. Moreover, this term cannot be incorporated into other terms simply, which will make the stability analysis much more difficult.

5. STABILITY ANALYSIS

Considering the extra transition rate related term, firstly by adding the positive term $c_h \frac{1}{h_{\mathbf{p}}}\hat{h}_{\mathbf{p}}^T \Gamma_h^{-1} \hat{h}_{\mathbf{p}}$ to the right hand side of (18), we obtain that

$$\mathcal{L}V\Big(x(t), x(t-\tau), p_1, p_2, \cdots, p_m\Big)$$

$$\leq -\sum_{i=1}^{n} c_i z_{i,\mathbf{p}}^2 + 2c_h \frac{1}{h_{\mathbf{p}}} \tilde{h}_{\mathbf{p}}^T \Gamma_h^{-1} \hat{h}_{\mathbf{p}} - \lambda \int_{t-\tau}^{t} e^{\lambda(s-t)} q(s) ds$$
$$+ c_h \frac{1}{h_{\mathbf{p}}} \hat{h}_{\mathbf{p}}^T \Gamma_h^{-1} \hat{h}_{\mathbf{p}}$$
$$+ \sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\big(x(t), p_1, p_2, \cdots, q_k, \cdots, p_m\big)$$
$$\leq -cV\big(x(t), p_1, p_2, \cdots, p_m\big) + \delta$$
$$+ \sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\big(x(t), p_1, p_2, \cdots, q_k, \cdots, p_m\big) \quad (19)$$

where $c = min\{2c_1, 2c_2, \cdots, 2c_n, 2c_h, \lambda\}$, $\delta = c_h \frac{\|\Gamma_h^{-1}\| \|h_{\mathbf{p}}\|^2}{h_{\mathbf{p}}}$.

Eq. (8), (15)-(17) show that boundedness of z_i implies the boundedness of x_i, and vice versa, which indicates that

$$\lim_{\|x\| \to \infty} \inf_{t \geq 0} V\big(x(t), \mathbf{r}(t)\big) = \infty. \quad (20)$$

Denote $t_l = \eta_l \wedge t$ for any $t \geq 0$, where η_l is the stopping time defined in Lemma 5. Then it can be obtained that $\|x(t)\| < l$ in the interval $[-\tau, t_l]$ a.s., which implies that $V(x(t), p_1, p_2, \cdots, p_m)$ is bounded on $[-\tau, t_l]$ a.s. by (20). From (19), it is obvious that $\mathcal{L}V(x(t), x(t-\tau), p_1, p_2, \cdots, p_m)$ is also bounded on $[-\tau, t_l]$ a.s.. Then according to Property 4 and (19), we obtain

$$E\Big(\mathcal{L}V\big(x(t), x(t-\tau), \mathbf{r}(t)\big)\Big)$$
$$= \sum_{p_1=1}^{N} \cdots \sum_{p_m=1}^{N} E\Big(\mathcal{L}V\big(x(t), x(t-\tau), p_1, \cdots, p_m\big)\Big) \pi_{p_1} \cdots \pi_{p_m}$$
$$\leq -c \sum_{p_1=1}^{N} \cdots \sum_{p_m=1}^{N} EV\big(x(t), p_1, \cdots, p_m\big) \pi_{p_1} \cdots \pi_{p_m} + \delta$$
$$+ \sum_{p_1=1}^{N} \cdots \sum_{p_m=1}^{N} E\Big(\sum_{k=1}^{m} \sum_{q_k=1}^{N} \gamma_{p_k q_k} V\big(x(t), p_1, \cdots, q_k, \cdots,$$
$$p_m\big)\Big) \pi_{p_1} \cdots \pi_{p_m}$$
$$\leq -cEV\big(x(t), \mathbf{r}(t)\big) + \delta + N\Big(\sum_{k=1}^{m} \max_{p_k, q_k=1}^{N} \Big\{\gamma_{p_k q_k} \frac{\pi_{p_k}}{\pi_{q_k}}\Big\}\Big)$$
$$\times EV\big(x(t), \mathbf{r}(t)\big)$$
$$= -\big(c - d^{'}\big) EV\big(x(t), \mathbf{r}(t)\big) + \delta,$$

where $d^{'} = N\Big(\sum_{k=1}^{m} \max_{p_k, q_k=1}^{N} \big\{\gamma_{p_k q_k} \frac{\pi_{p_k}}{\pi_{q_k}}\big\}\Big)$, and $c_1, c_2, \cdots, c_n$, c_h, λ, are appropriately chosen such that $c > d^{'}$.

According to Property 3, it follows that

$$EV\big(x(t_l), \mathbf{r}(t_l)\big)$$
$$= V\big(x(0), \mathbf{r}(0)\big) + E\int_0^{t_l} \mathcal{L}V\big(x(s), x(s-\tau), \mathbf{r}(s)\big) ds$$
$$\leq V\big(x(0), \mathbf{r}(0)\big) + \delta \cdot t_l - (c - d^{'}) \int_0^{t_l} EV\big(x(s), \mathbf{r}(s)\big) ds$$
$$\leq V\big(x(0), \mathbf{r}(0)\big) + \delta \cdot t_l. \quad (21)$$

From (16), for $i = 1, 2, \cdots, n$, we can verify that $\frac{\partial u_j}{\partial x_i}$ is continuous, which implies that u_j is C^1. Meanwhile f_i and φ_i are locally Lipschitz. Therefore, the local Lipschitz condition of the closed-loop system consisting of (7a), (7b), (15)-(17) is guaranteed. Then according to (20), (21) and Lemma 5, we can obtain that for any initial conditions $\{x(t) : -\tau \leq t \leq 0\} = \xi \in \mathcal{C}_{\mathcal{F}_0}^b([-\tau, 0]; \mathbb{R}^n)$ and $\mathbf{r}(0) \in S^m$, there exists a unique solution $x(t; \xi, \mathbf{r}(0))$ to the closed-loop system.

Furthermore, choosing $c^{'} = c - d^{'}$ and according to Property 3, it yields

$$E\Big(e^{c^{'} t_l} V\big(x(t_l), \mathbf{r}(t_l)\big)\Big) - E\Big(V\big(x(0), \mathbf{r}(0)\big)\Big)$$
$$= E\int_0^{t_l} e^{c^{'} s} \mathcal{L}V\big(x(s), x(s-\tau), \mathbf{r}(s)\big) ds$$
$$+ c^{'} E\int_0^{t_l} e^{c^{'} s} V\big(x(s), \mathbf{r}(s)\big) ds$$
$$\leq -c^{'} E\int_0^{t_l} e^{c^{'} s} V\big(x(s), \mathbf{r}(s)\big) ds + \int_0^{t_l} \delta e^{c^{'} s} ds$$
$$+ c^{'} E\int_0^{t_l} e^{c^{'} s} V\big(x(s), \mathbf{r}(s)\big) ds.$$

That is,

$$E\Big(e^{c^{'} t_l} V\big(x(t_l), \mathbf{r}(t_l)\big)\Big) \leq V\big(x(0), \mathbf{r}(0)\big) + \delta \int_0^{t_l} e^{c^{'} s} ds$$
$$= V\big(x(0), \mathbf{r}(0)\big) + \frac{\delta}{c^{'}}\big(e^{c^{'} t_l} - 1\big),$$

which implies that

$$EV\big(x(t_l), \mathbf{r}(t_l)\big) \leq e^{-c^{'} t_l} V\big(x(0), \mathbf{r}(0)\big) + \frac{\delta}{c^{'}}\big(1 - e^{-c^{'} t_l}\big).$$

Lemma 5 shows that $\eta_l \to \infty$ a.s. when $l \to \infty$. Then letting $l \to \infty$ gives

$$EV\big(x(t), \mathbf{r}(t)\big) \leq e^{-c^{'} t} V\big(x(0), \mathbf{r}(0)\big) + \frac{\delta}{c^{'}}\big(1 - e^{-c^{'} t}\big) \leq d_c, \quad (22)$$

where $d_c = V\big(x(0), \mathbf{r}(0)\big) + \frac{\delta}{c^{'}}$.

From (22) and Theorem 7, we can conclude that all the signals in the closed-loop system are bounded in probability. That is, the stability of the closed-loop nonlinear system in the presence of possible Markovian jumping actuator failures and state-delay can be ensured by the designed controller (16).

6. AN ILLUSTRATIVE EXAMPLE

In this section, we consider the following second-order system with double actuators to demonstrate the effectiveness of our proposed scheme.

$$\dot{x}_1(t) = x_2(t) + x_1(t)\sin(x_1(t)) + \cos(x_1(t-\tau))x_1(t-\tau)$$
$$\dot{x}_2(t) = x_1(t)x_2(t) + \sin(x_2(t-\tau))x_1(t-\tau) + r_1(t)u_1 + r_2(t)u_2$$

where $g_1(r_1) = r_1(t)$, $g_2(r_2) = r_2(t)$. $r_i(t)$, $i = 1, 2$, is a homogeneous irreducible Markovian process, with $S = \{0, 1\}$. For $r_1(t)$, generator $\Gamma = (\gamma_{pq})_{2\times 2} = \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}$. For $r_2(t)$, generator $\Gamma = (\gamma_{pq})_{2\times 2} = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}$.

The initial states and parameter estimates are set as $x_1(0) = 0.1$, $x_2(0) = -0.1$, $\hat{h}(0) = 0.1$, the delay is chosen as $\tau = 0.1$, the rest parameters are chosen as $c_1 = 8.5$, $c_2 = 8.9$, $\Gamma_h = 0.01$, $c_h = 9.1$, and all the ε are equal to 1.

By the proposed adaptive fault tolerant control scheme, we present the states of the system in Fig. 1, from which we can see that both states of the system are bounded in probability. The corresponding control signals are presented in Fig. 2. The results demonstrate the effectiveness of the proposed adaptive compensation scheme.

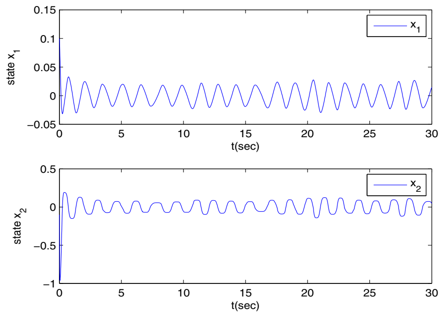

Fig. 1. System states

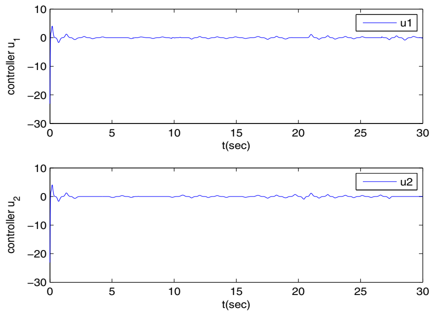

Fig. 2. Control input

7. CONCLUSION

In this paper, adaptive fault-tolerant control problem has been addressed for a class of time-delay nonlinear systems with Markovian jumping actuator failures. Basic properties on the stabilization of systems involving Markovian jumping actuator failures and time-delay have been established and the existence and uniqueness of the solution has been discussed, based on which, an adaptive state-feedback backstepping controller has been proposed, which ensures all closed-loop signals boundedness in probability.

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