## Active Queue Management in Wireless Networks by Using Nonlinear Extended Network Disturbance

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Abstract: This paper studies congestion control in local wireless networks supporting transmission control protocol (TCP) under random early detection (RED). In general, congestion delays may cause instability of the routers running active queue management (AQM) protocols such as RED. Moreover, its management efficiency could be reduced by serious queue oscillation. To improve these two drawbacks, we propose a robust congestion controller focusing on the queue oscillation suppression and time-delay compensation based on an idea of nonlinear extended network disturbance (NEND). The time-delay compensation is further performed via the NEND rejection. Furthermore, the proposed strategy has been validated on NS2 showing their applicability.

#### 1. INTRODUCTION

In recent years, active queue management (AOM) protocols are proposed based on control techniques under a fluid-flow model (Misra et al., 2000), capturing the dynamics of local wired networks with transmission control protocol (TCP) under random early detection (RED). For example, a PID congestion controller (Hollot et al., 2002) was constructed for the local wired network, a feedback congestion controller (Wang et al., 2002) was developed for the network running TCP combined with UDP under RED, an AQM methodology (Ariba et al., 2012) using a time-delay observer was established, and a self-tuning AQM mechanism (Hong et al., 2007) focusing on the change of TCP traffic was proposed. However, the fluid-flow system (Hollot et al., 2002) is only suitable for the wired case. Its extension to the wireless network running TCP with RED was addressed in the work (Zheng et al., 2007), in which a probability for failing data transmission was included in the model development. A robust congestion controller (Zheng et al., 2007) was constructed under the extended model.

Similar to the work (Zheng *et al.*, 2007), some papers (Quet *et al.*, 2004; Quet *et al.*, 2004; Zhang *et al.*, 2007; Yin *et al.*, 2006) designed the robust congestion controllers corresponding to the uncertain network parameters or capacities. Furthermore, bifurcation for the congestion control strategy was studied (Raina *et al.*, 2005). In the research (Zhang *et al.*, 2008), locally and globally asymptotical stabilities have been discussed for a number of congestion controllers. Several kinds of system stability ensured by the congestion control have been analyzed (Peet *et al.*, 2007). Globally asymptotical stability in the congestion control were studied (Deb *et al.*, 2003).

Most published works (Misra *et al.*, 2000; Zheng *et al.*, 2007) treated the time-delay effect as the control design constraints. Instead of this treatment, control strategies (Hsu *et al.*, 2012a,

b) focused on time-delay compensation by implementing either extended network disturbance (END) or nonlinear extended network disturbance (NEND). However, the controllers—consisting of the averaging queue length and window size—in previous mentioned works were all realized by taking the real parameter values as the expected ones. This inevitably causes the queue oscillation.

Motivated by the above drawback, we propose the congestion controller in terms of the queue length and window size with the objective of the time-delay compensation by performing the NEND rejection. Robust asymptotic stability of the network system adopting the proposed mechanism is discussed from the viewpoint of the real dynamics instead of its averaging behavior considered in most works (Hollot *et al.*, 2002; Zheng *et al.*, 2007). This gives rise to the lunching point of this paper: first network stability analysis form its real behaviour.

# 2. SYSTEM DEVELOPMENT AND PROBLEM DEFINITION

2.1 System Development



Fig. 1. Wireless local network system

The local wireless network in Fig. 1 is considered here; it includes the fixed sender sending data packets to the fixed

router and fixed receiver sequentially. Acknowledgement packets are transmitted back to the sender after the receiver receives the data packets. Nevertheless, it might happen that some packets lost under a fixed losing probability during the data transmission within the channel. Moreover, there is only one connection between senders, routers, and receivers successes at the same instant. Given multiple senders/receivers, above characteristics are further formulated by the fluid-flow system (Zheng et al., 2007) except for the characteristic of the single connection. Setting  $P_{ul} = P_{dl} = \gamma \ll 1$  in this system yields

$$\begin{aligned} \dot{\overline{w}}(t) &= 1/\overline{R}(t) - (1 - \gamma) \overline{w}(t) \overline{w}(t - \overline{R}) p(t - \overline{R}) / 2\overline{R}(t - \overline{R}) \\ \dot{\overline{q}}(t) &= n_f \overline{w}(t) (1 - \gamma) / \overline{R}(t) - C_p \end{aligned} \tag{1}$$

where the averaging round-trip time  $\overline{R}(t) = a + \overline{q}/C_p$ , *a* is the propagation delay, the dropping probability  $0 \le p \le 1$ ,  $\overline{q} / \overline{w}$  denotes the averaging queue length/window size,  $n_f$  is the number of TCP flows, and  $C_p$  is the capacity. Given  $(d_{yq}, d_{yw}) \equiv (q - \overline{q}, w - \overline{w})$  with q / w being the real queue length/window size, (1) is rewritten as

$$\dot{w}(t) = \frac{1}{\overline{R}(t)} - \frac{(1-\gamma)\overline{w}(t)\overline{w}(t-\overline{R})p(t-\overline{R})}{2\overline{R}(t-\overline{R})} + \dot{d}_{yw}(t)$$

$$\dot{q}(t) = n_f \overline{w}(t)(1-\gamma)/\overline{R}(t) - C_p + \dot{d}_{yq}(t)$$
(2)

where  $(d_{yw}, d_{yq})$  is bounded due to the TCP definition and hardware restriction. Invariant  $\overline{R}(t)$  may destabilize (2); therefore, (2) desires the time-delay compensation scheme.

## 2.2 Problem Statement

Despite (1) has been asymptotically stabilized by using  $\Delta p \equiv K \begin{bmatrix} \Delta \overline{w} & \Delta \overline{q} \end{bmatrix}^{T}$ (Zheng et al., 2007) with  $(\Delta p, \Delta w, \Delta q) = (p - p_0, \overline{w} - w_0, \overline{q} - q_0)$ and  $(\overline{w}, \overline{q}, p) = (w_0, q_0, p_0)$  being an equilibrium point of (1), this does not mean that (2) adopting  $\Delta p = K [\Delta \overline{w} \quad \Delta \overline{q}]^{T}$ simultaneously holds asymptotic stability. To illustrate, consider Case (i) (Zheng et al., 2007), in which  $p = 0.0032(\overline{w} - w_0) + 0.000015982(\overline{q} - q_0) + p_0$ and  $(w_0, q_0, p_0) = (9.2593 \text{ packets}, 500 \text{ packets}, 0.0216)$ . Fig. 3 (Zheng et al., 2007) illustrates that the network system implementing this controller still suffers queue oscillation. To improve this disadvantage, we must allow  $\Delta p(t)$  to be more sensitive to the queue variation as  $t \rightarrow \infty$ . This brings a possibility to solve the considered problem that how to ensure robust asymptotic stability of (2) with respect to  $(d_{yw}, d_{ya})$  by redesigning  $\Delta p(t)$  on the basis of NEND.

### 3. MAIN RESULTS

Under an assumption that the ergodic hypothesis holds, the expected value of a random matrix  $M(t) = [v_{mij}(t)]_{m_k \times m_k}$  is

defined as 
$$M_m(t) = E[M(t)] \triangleq \left[\int_{t_0}^t v_{mij}(\tau) d\tau / (t-t_0)\right]_{m_1 \times m_2}$$

Moreover,  $\|\cdot\|_2$  represents the induced matrix 2-norm or the 2-norm of a vector and O/0 denotes a zero matrix/vector.

## 3.1 Idea of NEND

The definition of NEND is framed by regarding the timedelay nature of (1) as induced by injecting NEND into a model without any delay. This re-describes (1) as  $\dot{\overline{w}} = 1/\overline{R}(t) - (1-\gamma)\overline{w}^2(t)(p-d_{-1})/2\overline{R}(t)$ .

$$\dot{\overline{q}} = n_f \overline{w}(t) (1 - \gamma) / \overline{R}(t) - C_p, \qquad (3)$$

$$y = [w \quad q]^T$$

where y is the system output and  $d_{nd}$  denotes NEND. Equalizing  $\dot{\overline{w}}$  in (3) and  $\dot{\overline{w}}$  in (1) yields

$$d_{nd} = p - \overline{w} \left( t - \overline{R} \right) p \left( t - \overline{R} \right) \overline{R}(t) / \left[ \overline{R} \left( t - \overline{R} \right) \overline{w}(t) \right]$$
(4)

The value  $|d_{nd}(t)|$  represents the seriousness of time-delay effect of (3). Since the TCP window size in (3) changes upon the ACK receiving rate—delayed by the data congestion,  $d_{nd}$  physically represents the effects of the data congestion on this rate. Furthermore, substituting  $(\overline{q}, \overline{w}) = (q - d_{yq}, w - d_{yw})$  into (3) gives

$$\dot{w} = f_{11}\left(\overline{R}_{0}\right) + f_{12}\left(\overline{w}_{0}, \overline{R}_{0}\right)p + \tilde{d}_{nd} + \dot{d}_{yw},$$

$$\dot{q} = f_{a}\left(\overline{R}_{i0}\right)w_{i} - C_{p} - f_{a}\left(\overline{R}_{0}\right)d_{yw} + d_{est} + \dot{d}_{yq},$$

$$y = \begin{bmatrix} w & q \end{bmatrix}^{T}$$
where
$$(5)$$

$$d_{est} = \left[ f_a\left(\overline{R}\right) - f_a\left(\overline{R}_0\right) \right] \overline{w}(t)$$
(6)

$$\tilde{d}_{nd} = f_{11}\left(\overline{R}\right) - f_{11}\left(\overline{R}_{0}\right) + \left[f_{12}\left(\overline{w},\overline{R}\right) - f_{12}\left(w_{0},\overline{R}_{0}\right)\right]p - \breve{d}_{nd}$$
(7)

$$\overline{R}_{0} = a + q_{0}/C_{p} , \qquad f_{11}(t) = 1/\overline{R}(t) ,$$

$$f_{12}(t) = -(1-\gamma)\overline{w}(t)\overline{w}(t-\overline{R}) / \lfloor 2\overline{R}(t-\overline{R}) \rfloor \qquad ,$$

$$f_a(\overline{R}) = n_f(1-\gamma)/\overline{R}$$
, and  $d_{nd} = f_{12}(\overline{w}(t), \overline{R}(t))d_{nd}$ .

From which one has

$$\begin{aligned} \ddot{q} &= f_a \left( \overline{R}_0 \right) f_{11} \left( \overline{R}_0 \right) + f_a \left( \overline{R}_0 \right) f_{12} \left( \overline{w}_0, \overline{R}_0 \right) p \\ &+ f_a \left( \overline{R}_0 \right) \tilde{d}_{nd} + \dot{d}_{est} + \ddot{d}_{yq} \end{aligned} \tag{8}$$

which is restated as

$$\begin{aligned} x_1 &= x_2, \\ \dot{x}_2 &= f_a \left( \overline{R}_0 \right) f_{11} \left( \overline{R}_0 \right) + f_a \left( \overline{R}_0 \right) f_{12} \left( \overline{w}_0, \overline{R}_0 \right) p \\ &+ f_a \left( \overline{R}_0 \right) \tilde{d}_{nd} + \dot{d}_{est} + \ddot{d}_{yq} \end{aligned} \tag{9}$$

with  $x_1 = q$  and  $x_2 = \dot{q}$ .

Remark 1: In the network applications,  $1 \le \overline{w}(t) \le w_{\max}$  and  $1 \le \overline{q}(t) \le q_{\max}$ ; therefore,  $d_{est}(\overline{w},\overline{q}), \tilde{d}_{nd}(\overline{w},\overline{q}), d_{nd}(\overline{w},\overline{q}) \in U_2(\infty)$  based on (4) and (6) -(7) with  $0 \le p \le 1$  where the continuous functions  $f_{12}(\cdot)$ ,

 $f_{a}(\cdot)$  and  $f_{11}(\cdot)$ have finite values and  $U_{2}(T) := \left\{ M(t) : \left\| M(t) \right\|_{2} \le \varepsilon_{0} \text{ over } [t_{0}, T] \right\} .$ Moreover,  $(d_{yq}, d_{yw})$  must be bounded by the TCP and hardware restriction.

 $M(t) \in U_{2}(T)$ Remark 2: Given  $\dot{M}_{m}(t) = \left[ M(t) - M_{m}(t) \right] / (t - t_{0})$ such that  $\lim_{t \to \infty} \left\| \dot{M}_m(t) \right\|_2 \le \lim_{t \to \infty} 2\varepsilon_0 / (t - t_0) = 0, \text{ showing that } \lim_{t \to \infty} M_m(t)$ remains invariant.

Given  $M(t), d(t) \in U_2(\infty)$  with the random vector  $d(t) \in \mathbb{R}^{m_2}$ ,  $\lim E[M(t)d(t)]$  is constant from Remark 2  $M(t)d(t) \in U_{\gamma}(\infty) \qquad .$ since This implies  $\lim_{t \to \infty} E\left[M(t)d(t)\right] \triangleq \lim_{t \to \infty} M_m(t)d_m(t) + \beta_0$ where  $d_m(t) = E[d(t)]$  and  $\beta_0$  is the constant vector. In other words,  $E[M(t)d(t)] - M_m(t)d_m(t) - \beta_0$  is proportional to  $\dot{d}_{m}(t)$  $\dot{M}_{m}(t)$ ; and consequently,  $E\left[M(t)d(t)\right] \triangleq M_m(t)d_m(t) + \dot{M}_m(t)\Sigma_1(t) + \Sigma_2(t)\dot{d}_m(t) + \beta_0$ for any (M,d) where  $\Sigma_{1,2}(t) \in U_2(\infty)$  are both differentiable at  $\forall t \in [t_0, \infty)$  with appropriate dimensions. Taking equation in  $dE\left[M(t)d(t)\right]/dt = \left[M(t)d(t) - E\left[M(t)d(t)\right]\right]/(t-t_0)$ yields  $\lim dE \left[ M(t) d(t) \right] / dt$ 

$$= \lim_{t \to \infty} \left[ M\left(t\right) - \frac{\Sigma_{2}\left(t\right)}{t - t_{0}} \right] \dot{d}_{m}\left(t\right) + \lim_{t \to \infty} \dot{M}_{m}\left(t\right) \left[ d_{m}\left(t\right) - \frac{\Sigma_{1}\left(t\right)}{t - t_{0}} \right]$$
(10)  
$$= M\left(\infty\right) \dot{d}_{m}\left(\infty\right) + \dot{M}_{m}\left(\infty\right) d_{m}\left(\infty\right)$$

where

 $d(t) = \dot{d}_m(t)(t-t_0) + d_m(t)$ and  $M(t) = \dot{M}_m(t)(t-t_0) + M_m(t).$ 

*Remark 3:* Given the constant  $\lim_{t \to \infty} M_m(t) = \left[\overline{v}_{mij}(\infty)\right]_{m_1 \times m_2}$ norm-bounded with  $M(t_{o})$ 

 $\dot{M}_{m}(t) = \left[ \left( v_{mij}(t) - \overline{v}_{mij}(t) \right) / (t - t_{0}) \right]_{m \times n} = O$  indicates that  $|v_{mij}(\infty) - \overline{v}_{mij}(\infty)| < \infty$  or  $\dot{v}_{mij}(\infty) = \dot{\overline{v}}_{mij}(\infty) = 0$  where  $\overline{v}_{mij}\left(t
ight)$  is the averaging  $v_{mij}\left(t
ight)$ . This implies  $\left|v_{mij}\left(\infty
ight)\right| < \infty$ such that  $M(t) = [v_{mij}(t)]_{m \times n} \in U_2(\infty)$  under  $||M(t_0)||_2 < \infty$ . Remark 4: Given  $M^{(i)}(t) = \left[ d^i v_{mij}(t) / dt^i \right]_{mun} \in U_2(\infty)$ ,  $\lim_{t \to \infty} E \left[ M^{(i+1)}(t) \right] = \lim_{t \to \infty} \left[ M^{(i)}(t) - M^{(i)}(t_0) \right] / (t - t_0) = 0$ which shows  $M^{(i+1)}(t) \in U_2(\infty)$  from Remark 3. After we

repeats this derivation from i = 0 to i = j - 1 with  $j \in \mathbb{N}$ , it

can be concluded that  $M^{(j)}(t) \in U_2(\infty)$  is available for any  $j \in \mathbb{N}$  under  $M(t) \in U_2(\infty)$ .

## 3.2 Observer Design

Owing to that  $x_2$  cannot be measured from y, we need to develop a robust observer of (9) with respect to the unknown  $(d_{yy}, d_{yq})$ . To simplify the design task, the observer is constructed under (5), which is rewritten as

$$\dot{x}_{T} = A_{T} x_{T} + B_{T} u_{T} + d_{T}, y_{T} = C_{T} x_{T}$$
(11)

$$\vec{x}_{T} = \begin{bmatrix} w & q & \tilde{d}_{nd} & d_{ast} \end{bmatrix}^{T}$$

$$u_{T} = \begin{bmatrix} f_{11}(\overline{R}_{0}) + f_{12}(\overline{w}_{0}, \overline{R}_{0})p & -C_{p} \end{bmatrix}^{T} ,$$
  

$$d_{T} = \begin{bmatrix} \dot{d}_{yw} & -f_{a}(\overline{R}_{0})d_{yw} + \dot{d}_{yq} & \dot{\tilde{d}}_{nd} & \dot{d}_{est} \end{bmatrix}^{T} , A_{T} = \begin{bmatrix} M_{0} & I_{2} \\ Q & Q \end{bmatrix} ,$$

$$M_0 = \begin{bmatrix} 0 & 0 \\ f_a(\overline{R}_0) & 0 \end{bmatrix}, \quad B_T = \begin{bmatrix} I_2 & O \end{bmatrix}^T, \text{ and } C_T = \begin{bmatrix} I_2 & O \end{bmatrix}.$$

 $d_T(t) \in U_2(\infty)$  since  $\dot{d}_{yw}(t), \dot{d}_{yq}(t), \tilde{d}_{nd}(t), \dot{d}_{est}(t) \in U_2(\infty)$ based on Remarks 1 and 4. The corresponding observer is proposed as

$$\dot{\hat{x}}_{T} = A_{T}\hat{x}_{T} + B_{T}u_{T} + L(t)(C_{T}x_{T} - C_{T}\hat{x}_{T})$$
(12)

where  $\hat{x}_T$  represents the estimation of  $x_T$  and L(t) is the observer gain. Given  $e_x = x_T - \hat{x}_T$ , it follows

$$\dot{e}_x = A_e(t)e_x + d_T \tag{13}$$

where  $A_e(t) = A_T - L(t)C_T = \left[v_{aij}(t)\right]_{4\times 4}$ . Without loss of generality, the solution of (13) can be denoted by  $e_x = e_m + r_x$ where  $e_m(t) = E[e_x(t)]$  and  $E[r_x(t)] = 0$  for all t. Robust asymptotic stability of (13) is guaranteed by letting  $\lim_{t \to \infty} e_m(t) = 0$  and  $\lim_{t \to \infty} r_x(t) = 0$ . The former can be ensured by the result below.

Theorem 1: Consider the system (13). Suppose  $\alpha_1 > 0$ ,  $( ) [ \mathbf{f}^{t} ( ) ]$ 

$$A_{em}(t) = \left[ \int_{t_0} v_{aij}(\tau) d\tau / (t - t_0) \right]_{4 \times 4} ,$$
  
$$\tilde{z}_{-}(\tau) = \begin{bmatrix} O & I \end{bmatrix}$$

$$\tilde{A}_{T}(t) = \begin{bmatrix} 0 & 1 \\ \dot{A}_{em}(t) & A_{e}(t) \end{bmatrix} , \quad \text{and} \quad$$

 $Q_1(t) = Q_1^T(t) \ge Q_2(t) = Q_2^T(t)$  for all  $t \in [t_0, \infty)$ .  $\lim e_m(t) = 0$  if there are  $L(\cdot)$ ,  $\tilde{A}_a(\cdot)$ ,  $\tilde{A}_b(\cdot)$ , and  $P_1(\cdot) = P_1^T(\cdot) > 0$  guaranteeing

$$\tilde{A}_{T}\left(L(t)\right) = \tilde{A}_{a}\left(t\right) \cdot \tilde{A}_{b}\left(t\right)$$
(14)

$$\tilde{A}_{a}^{T}(t)P_{1}(t)+P_{1}(t)\tilde{A}_{a}(t)=-Q_{1}(t)$$
(15)

$$\dot{P}_{1}(t) = \tilde{A}_{b}^{T}(t)P_{1}(t) + P_{1}(t)\tilde{A}_{b}(t) - \alpha_{1}P_{1}^{2}(t) + Q_{2}(t)$$
(16)

for all  $t \in [t_0, \infty)$  and  $\tilde{A}_{\tau}(\infty)$  is full rank. **Proof:** Expanding

$$E\left[\dot{e}_{x}\left(t\right)\right] = E\left[A_{e}\left(t\right)e_{x}\left(t\right) + d_{T}\left(t\right)\right]$$

$$at \ e_{x}\left(t_{0}\right) \text{ gives}$$

$$(17)$$

$$\dot{e}_{m}(t) = E\left[A_{e}(t)e_{x}(t)\right] + E\left[d_{T}(t)\right] - \frac{e_{m}(t)}{t-t_{0}} + \frac{e_{x}(t_{0})}{t-t_{0}}$$
(18)

 $E\left[\dot{e}_{x}(t)\right] = \dot{e}_{m}(t) + \left[e_{m}(t) - e_{x}(t_{0})\right]/(t-t_{0})$ where and  $E[d_{\tau}(t)]$  becomes constant as  $t \to \infty$  from Remark 2 because of  $d_T(t) \in U_2(\infty)$ . Performing differentiation of (18) corresponding to time derives

$$\lim_{t \to \infty} \ddot{e}_m(t) = \lim_{t \to \infty} \left\{ dE \left[ A_e(t) e_x(t) \right] / dt - \frac{\dot{e}_m(t)}{t - t_0} + \frac{e_m(t)}{\left(t - t_0\right)^2} \right\}$$
(19)

where  $dE \left[ A_e(t) e_x(t) \right] / dt$  in (19) is expended as (10) where  $(M(t), d(t)) \triangleq (A_e(t), e_x(t))$ . Substituting this expansion (19) yields  $\lim_{t \to \infty} \dot{\tilde{x}}_m(t) = \lim_{t \to \infty} \hat{A}_T(t) \tilde{x}_m(t)$  where into

$$\hat{A}_{T}(t) = \begin{bmatrix} O & I \\ \dot{A}_{em}(t) + I/(t-t_0)^2 & A_{e}(t) - I/(t-t_0) \end{bmatrix} \rightarrow \tilde{A}_{T}(t) \text{ as}$$

 $t \to \infty$ , and  $\tilde{x}_m(t) = \left| e_m^T(t) \quad \dot{e}_m^T(t) \right|^2$ . This shows that (17) approaches a limit system (Lee et al., 2001)  $\dot{\tilde{x}}_{m}(t) = \tilde{A}_{T}(t)\tilde{x}_{m}(t)$ (20)

which possesses an equilibrium point  $\tilde{x}_m = 0$  because  $\tilde{A}_{T}(\infty)$  has full rank.

Given 
$$V_m(t) = \tilde{x}_m^T(t)P_1(t)\tilde{x}_m(t),$$
  
 $\dot{V}_m(t) = \tilde{x}_m^T(t) \Big[ \tilde{A}_T^T(t)P_1(t) + P_1(t)\tilde{A}_T(t) + \dot{P}_1(t) \Big] \tilde{x}_m(t)$  (21)

From (14)-(16) one has  $\dot{V}_{m}(t) = -\tilde{x}_{m}^{T}(t) \left[ Q_{1}(t) - Q_{2}(t) \right] \tilde{x}_{m}(t)$  $-\alpha_1 \tilde{x}_m^T(t) P_1^2(t) \tilde{x}_m(t) < 0$ 

at  $\forall t \in [t_0, \infty)$  where  $Q_1(t) \ge Q_2(t)$  over  $[t_0, \infty)$ . Consequently, (17) converges to (20), which is robustly asymptotically stable. This accomplishes the proof. Next, let us identify  $\lim_{t \to \infty} r_x(t)$  by using the following theorem. Theorem 2: Consider (13) featuring  $\lim_{t\to\infty} e_m(t) = 0$  with the determined Suppose  $\alpha_2 > 0$  $\tilde{A}_{r}(t) = \begin{bmatrix} O & I \\ \dot{A}_{e}(t) + \dot{A}_{em}(t) & A_{e}(t) \end{bmatrix} \text{ with } \tilde{A}_{r}(\infty) \text{ being full}$  $\dot{A}_{e}(t) \in U_{2}(\infty)$  ,  $\dot{A}_{em}(t) \in U_{2}(\infty)$  , rank, and  $Q_{r1}(t) = Q_{r1}^T(t) \ge Q_{r2} = Q_{r2}^T(t)$  for all  $t \in [t_0, \infty)$ . This system

is robustly asymptotically stable if there are  $\tilde{A}_{r1}(\cdot)$  and  $P_2(\cdot) = P_2^T(\cdot) > 0$  guaranteeing

$$\left[\tilde{A}_{r}\left(t\right)+\tilde{A}_{r1}\left(t\right)\right]^{T}P_{2}\left(t\right)+P_{2}\left(t\right)\left[\tilde{A}_{r}\left(t\right)+\tilde{A}_{r1}\left(t\right)\right]=-Q_{r1}\left(t\right)$$
(23)

$$\dot{P}_{2}(t) = \tilde{A}_{r_{1}}^{T}(t)P_{2}(t) + P_{2}(t)\tilde{A}_{r_{1}}(t) - \alpha_{2}P_{2}^{2}(t) + Q_{r_{2}}(t)$$
(24)

for all  $t \in [t_0, \infty)$ .

Proof: Substituting 
$$e_x = e_m + r_x$$
 into (13) gives  
 $\dot{r}_x(t) = A_e(t)r_x(t) + A_e(t)e_m(t) + \dot{e}_x(t) - A_ee_x(t) - \dot{e}_m(t)$   
 $= A_e(t)r_x(t) - [A_e(t)\dot{e}_m(t) - \ddot{e}_m(t)](t - t_0) + \dot{e}_m(t)$ 
(25)

and

where

where 
$$e_{m}(t) = e_{x}(t) - \dot{e}_{m}(t)(t-t_{0})$$
 and  
 $\dot{e}_{x}(t) = 2\dot{e}_{m}(t) + \ddot{e}_{m}(t)(t-t_{0})$ . That is,  
 $\lim_{t \to \infty} \dot{r}_{x}(t) = \lim_{t \to \infty} \left[ A_{e}(t)r_{x}(t) - A_{e}(t)\dot{e}_{m}(t)(t-t_{0}) + \ddot{e}_{m}(t)(t-t_{0}) + \dot{e}_{m}(t) \right]$ 

$$= \lim_{t \to \infty} \left\{ A_{e}(t)r_{x}(t) + \left[ A_{e}(t) - A_{em}(t) \right] e_{m}(t) \right\}$$
(26)

where  $\ddot{e}_m(\infty) = \dot{A}_{em}(\infty) e_m(\infty) + A_e(\infty) \dot{e}_m(\infty)$  based on (20),  $\dot{e}_m(\infty) = 0$ , and  $\dot{A}_{em}(t) = \left[A_e(t) - A_{em}(t)\right]/(t - t_0)$ . Furthermore,

$$\lim_{d \to \infty} \ddot{r}_{x}(t) = \lim_{t \to \infty} \left\{ \dot{A}_{e}(t) r_{x}(t) + A_{e}(t) \dot{r}_{x}(t) + \left[ \dot{A}_{e}(t) - \dot{A}_{em}(t) \right] e_{m}(t) + \left[ A_{e}(t) - A_{em}(t) \right] \dot{e}_{m}(t) \right\}$$

$$= \lim_{t \to \infty} \left[ \dot{A}_{e}(t) r_{x}(t) + A_{e}(t) \dot{r}_{x}(t) + \dot{A}_{em}(t) r_{x}(t) \right]$$
(27)

with  $\dot{e}_m(t) = r_x(t)/(t-t_0)$  and  $\left[\dot{A}_e(\infty) - \dot{A}_{em}(\infty)\right] e_m(\infty) = 0$ since  $e_m(\infty) = 0$  and  $\dot{A}_e(t), \dot{A}_{em}(t) \in U_2(\infty)$ . This gives rise to the limit system  $\dot{x}_r(t) = \tilde{A}_r(t) x_r(t)$ , having an equilibrium point  $x_r = 0$  where  $x_r(t) = \left[ r_x^T(t) \quad \dot{r}_x^T(t) \right]^T$ since  $\tilde{A}_r(\infty)$  has full rank.

(1) Given  $V_r(t) = x_r^T(t) P_2(t) x_r(t)$  where  $P_2(t) > 0$ at  $\forall t \in [t_0, \infty),$ 

$$\dot{V}_{r}(t) = -x_{r}^{T}(t) \left[ \mathcal{Q}_{r1}(t) - \mathcal{Q}_{r2}(t) \right] x_{r}(t) -\alpha_{2} x_{r}^{T}(t) P_{2}^{2}(t) x_{r}(t) < 0$$
(28)

from (23)-(24) and  $Q_{r1}(t) \ge Q_{r2}$  at  $\forall t \in [t_0, \infty)$ . The dynamics (13) therefore admits  $e_m(\infty) = 0$  and  $r_x(\infty) = 0$ , revealing robust asymptotic stability.

The observer design is completed by determining L(t) in (13) to hold Theorems 1-2.

## 3.3 Control Design

The controller is aimed at ensuring  $\lim(x_1(t), x_2(t)) = (q_0, 0)$  in (9); it is defined as

$$p = \left[\frac{-k_1(t)}{f_a\left(\overline{R}_0\right)f_{12}\left(w_0,\overline{R}_0\right)} \quad \frac{-k_2(t)}{f_a\left(\overline{R}_0\right)f_{12}\left(w_0,\overline{R}_0\right)}\right] \begin{bmatrix} (q-q_0)\\ \dot{q} \end{bmatrix}$$
(29)

where  $k_{1,2}(t) > 0$  for  $\forall t \in [t_0, \infty)$ . The development of (29) in terms of  $(q, \dot{q})$  releases the assumption  $(w, q) = (\overline{w}, \overline{q})$ (Hollot et al., 2002; Zheng et al., 2007) in the control realization. Substituting (29) into (9) gives

(22)

$$\dot{e} = A(t)e + d_e + \begin{bmatrix} 0 & k_2(t)F_0\dot{e}_x \end{bmatrix}^T$$
(30)

where 
$$e = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T$$
,  $e_1 = q - q_0$ ,  $e_2 = \dot{q}$ ,  
 $d_e = \begin{bmatrix} 0 & f_a(\overline{R}_0) f_{11}(\overline{R}_0) + f_a(\overline{R}_0) \tilde{d}_{nd} + \dot{d}_{est} + \ddot{d}_{yq} \end{bmatrix}^T$ ,  
 $A(t) = \begin{bmatrix} 0 & 1 \\ -k_1(t) & -k_2(t) \end{bmatrix}$ , and  $F_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$ .  
 $d_e(t) \in U_2(\infty)$  since  $f_a(\overline{R}_{i0}) \tilde{d}_{nd}(t) \in U_2(\infty)$  and  
 $\dot{d}_{est}(t), \ddot{d}_{yq}(t) \in U_2(\infty)$  from Remarks 1 and 4. Combining  
(30), (13), and  $\ddot{e}_x = \dot{A}_e(t) e_x + A_e(t) \dot{e}_x + \dot{d}_{T_1}$  (derived from

(13)) gives

$$\tilde{e} = A(t)\tilde{e} + d_e$$

where

$$\tilde{d}_{e}(t) = \begin{bmatrix} d_{e}^{T}(t) & d_{T}^{T}(t) & \dot{d}_{T}^{T}(t) \end{bmatrix}^{T}$$

(31)

and

$$\tilde{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1(t) & -k_2(t) & 0 & k_2(t)F_0 \\ 0 & 0 & A_e(t) & O \\ 0 & 0 & \dot{A}_e(t) & A_e(t) \end{bmatrix} \cdot \tilde{d}_e(t) \in U_2(\infty)$$

 $\tilde{e} = \begin{bmatrix} e^T(t) & e_x^T(t) & \dot{e}_x^T(t) \end{bmatrix}^T$ 

because of  $d_e(t), d_T(t), \dot{d}_T(t) \in U_2(\infty)$  based on Remark 4 and  $d_T(t) \in U_2(\infty)$ . Robust asymptotic stability of (31) is ensured by designing  $(k_1(t), k_2(t))$  according to Theorems 1-2 with  $A_e$  substituted by  $\tilde{A}$ .

To satisfy  $0 \le p \le 1$ , we redefine

$$p \triangleq \overline{p} + u_p \tag{32}$$

where  $\overline{p}(t)$  is in the form of (29) and

$$u_{pi} = \begin{cases} 1 - \overline{p}, \text{if } \overline{p} > 1\\ 0, & \text{if } 0 \le p \le 1\\ -\overline{p}, & \text{if } \overline{p} < 0 \end{cases}$$

Without loss of generality, we assume  $\overline{p}(t) \in U_2(\infty)$  so that

$$u_{p}(t) = p(t) - \overline{p}(t) \in U_{2}(\infty). \text{ Taking (32) in (9) generates}$$
$$\dot{\tilde{e}} = \tilde{A}(t)\tilde{e} + \tilde{d}_{eT}$$
(33)

where

where 
$$d_{eT} = d_e + \begin{bmatrix} d_a^T & 0 & 0 \end{bmatrix} \in U_2(\infty)$$
 with  
 $d_a = \begin{bmatrix} 0 & f_a(\overline{R}_0) f_{12}(w_0, \overline{R}_0) u_p \end{bmatrix}^T$  due to

 $\tilde{d}_{e}(t), f_{a}(\overline{R}_{0})f_{12}(w_{0},\overline{R}_{0})u_{a}(t) \in U_{2}(\infty)$ . The system (33) features robust asymptotic stability since it has guaranteed Theorems 1-2 with  $A_{e} \triangleq \tilde{A}$  under the designed  $(k_{1}(t), k_{2}(t))$ .

## 4. SIMULATION

This section states a verification on NS2 with the topology in Fig. 2, in which it has a base-station BS, a wireless router  $n_w$ , FTP source routers  $\overline{Src_1} - \overline{Src_{40}}$ ,  $n_f = 40$ , a destination router Dst, a wired router  $Rr_w$ , the propagation delay 0.01 s,

 $C_p = 6$  Mbps with the packet size 500 bytes,  $\gamma = 0.01$ ,  $(w_0, q_0, p_0) = (150 \text{ packets}, 4.1667 \text{ packets}, 0.1164)$ , and the activating time 0 s with the terminating time 200 s for all FTP sources. The base station supported RED while others worked under DropTail. Given the environment settings, the was evaluated observer (12)with  $L(t) = \begin{bmatrix} (4t+4)I_2 & (4t^2+8t+4)I_2 \end{bmatrix}^T$ . Moreover, based on Theorems 1-2, the controller (32) implemented on BS was designed with  $(k_1(t), k_2(t)) = (0.1t^2, 0.1t)$ .

Fig. 3 illustrates the verification results of our control strategies. In this figure, the data transmission starts after 20 s since the wireless routers could not access BS until 20 s. The queue length converges to 150 packets in Fig. 3(a), in which the queue oscillation was induced by that the fluid-flow system construction (Zheng et al., 2007) did not consider that there is only one connection between senders, routers, and receivers successes at the same instant. However, this does destroy the feasibility of our method since the oscillation amplitude was less than 25 packets, which is only 25/150 = 17% of  $q_0$ . Consequently, the network system was stabilized via the proposed strategy. Moreover, the error oscillations in Figs. 3(c)- 3(d) were induced by the same cause as that of the queue oscillation. This does not destroy the feasibility of the proposed manner because the estimation errors possess zero means in Fig. 3.



Fig. 2. Wireless network topology.



Fig. 3. Verification results, (a) queue length, (b) dropping probability, (c) queue length estimation error, and (d) window size estimation error.

### 5. CONCLUSION

This paper has dealt with a problem of enhancing AQM efficiency for local wireless networks with the goal of suppressing its time-delay effect based on NEND, characterizing the cause of the time-delay property. A

NEND-based solution, consisting of real the queue length and window size, has been developed for the oscillation reduction and time-delay effect suppression. The former reduction is achieved by identifying the convergence of the state's mean value while the latter is performed by the NEND rejection. Furthermore, robust asymptotic stability of the network system using the proposed controller is discussed in view of its real behavior instead of the averaging dynamics. Finally, a case study has been presented to address superiority of our method.

Not only the network dynamics but also other industrial systems feature the time-delay control problem. Applying NEND in solving their control problem gives rise to a possibility that such issues can be solved via disturbance rejection techniques. This reduces the complicity of the considered problem and further makes the control realization much easier.

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### REFERENCES

- Misra, V., W.B. Gong and D. Towsley (2000). Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED. *Proceedings of SIGCOMM*, pp. 151-160.
- Lee, T.C., D.C. Liaw and B.S. Chen (2001). A general invariance principle for nonlinear time-varying systems and its applications. *IEEE Transactions on Automatic Control*, vol. 46, no. 12, pp. 1989-1993.
- Hollot, C.V., V. Misra and D. Towsley (2002). Analysis and design of controllers for AQM routers supporting TCP flows. *IEEE Transactions on Automatic Control*, vol. 47, no. 6, pp. 945-959.
- Deb, S. and R. Srikant (2003). Global stability of congestion controllers for the internet. *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1055-1060.
- Quet, P.F. and H. Ozbay (2004). On the design of AQM supporting TCP flows using robust control theory. *IEEE Transactions on Automatic Control*, vol. 49, no. 6, pp. 1031-1036.
- Wang, L., Z.Z. Li, Y.P. Chen and K. Xue (2005). Fluid-based stability analysis of mixed TCP and UDP traffic under RED. *IEEE International Conference on Engineering of Complex Computer Systems*, pp. 341-348.
- Raina, G. (2005). Local bifurcation analysis of some dual congestion control algorithms. *IEEE Transactions on Automatic Control*, vol. 50, no. 8, pp. 1135-1146.
- Yin, F.J., G.M. Dimirovski and Y.W. Jing (2006). Robust stabilization of uncertain input delay for internet congestion control. *American Control Conference*, pp. 5576-5580.
- Hong, Y. and O.W.W. Yang (2007). Self-turing TCP traffic controller using gain margin specification. *IET Communications*, vol. 1, no. 1, pp. 27-33.
- Peet, M. and S. Lall (2007). Global stability analysis of a nonlinear model of internet congestion control with

delay. *IEEE Transactions on Automatic Control*, vol. 52, no. 3, pp. 553-559.

- Zheng, F. and J. Nelson (2007). An  $H_{\infty}$  approach to congestion control design for AQM routers supporting TCP flows in wireless access networks. *Computer Networks*, vol. 51, pp. 1684-1704.
- Zhang, G.H., Y.Y. Wu and Y.H. Liu (2007). Stability and sensitivity for congestion control in wireless mesh networks with time varying link capacities. *Ad Hoc Networks*, vol. 5, pp. 769-785.
- Zhang, Y.P. and D. Loguinov (2008). Local and global stability of delayed congestion control systems. *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2356-2360.
- Ariba, Y., F. Gouaisbaut, S. Rahme and Y. Labit (2012). Traffic monitoring in transmission control protocol/active queue management networks through a time-delay observer. *IET Control Theory & Applications*, vol. 6, no. 4, pp. 506-517.
- Hsu, P.M. and C.L. Lin (2012). Time-delay compensation for time-varying delayed systems using extended network disturbance. 7<sup>th</sup> IEEE Conference on Industrial Electronics and Applications, pp. 388-393.
- Hsu, P.M. and C.L. Lin (2012). Time-delay compensation using nonlinear extended network disturbance. *IEEE International Conference on Control Applications*, pp. 1400-1404.