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Control of an Electromechanical Control Actuation System Using a Fractional Order Proportional, Integral, and Derivative-Type Controller

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Abstract: In this study, the usefulness of a fractional order PID (proportional plus integral plus derivative)-type controller is investigated on the control of an electromechanical control actuation system. In this extent, the mathematical model of an electromechanically-actuated control actuation system considered is obtained in the continuous time domain first and then a control system with an integer PID-type controller is constructed by regarding this model. Having performed the computer simulations using the fractional order controllers which are designated by defining the integral and derivative actions in a fractional manner, the transient response characteristics of the proposed fractional order control systems are compared to those of the integer order control systems. Eventually, it is observed that satisfactory results are attained such that they can safely be implemented on a physical control actuation system mechanism developed.

Keywords: Fractional order control, control actuation system, electromechanical actuation, PID action, computer simulation.

1. INTRODUCTION

One of the most important subsystems of the aerial systems such as unmanned aerial vehicles and guided munitions is control actuation systems (CASs) which are utilized to travel those systems along specified trajectories. The CASs which are designed in an electromechanical manner for short- and medium-range aerial systems are developed so as to realize the command signals to the deflections of the control surfaces under the effect of aerodynamic loads acting on these surfaces. The mentioned control problem becomes more complicated especially within the transonic flight regime in which both the amplitude and direction of the aerodynamic loads might alter and in such cases; it may be not possible to keep the stability of the CAS considered by means of the classical control approaches. To resolve this problem, robust control methods including sliding mode control and H_∞ control have been introduced as alternatives to classical control with PID action in recent studies (Özkan, 2005).

The algorithms involving fractional order PID-type controllers constitute one of the design choices for CASs. Specifically, the ability of the fractional order integral action on describing and modeling real objects more accurately than the classical integral action leads the fractional order control approach to gain more popularity (Kumar, 2013). On the other hand, the fractional order control algorithms which were not implemented by a near-past due to the insufficiency of the mathematical tools used in the solution could have been realized thanks to the new mathematical methods supported by the recent improvements in the computer technology (Chen *et al.*, 2009 and Khalil *et al.*, 2009). As symbols λ and μ denote the orders of the integral and derivative gains, respectively, the fractional order PID-type controllers which are often expressed as $PI^\lambda D^\mu$ ($PI^\lambda D^\delta$ in some of the studies by replacing μ with δ) provide designers with optimizing the performance parameters for the CAS specified by adjusting coefficients λ and μ as well as the proportional, integral, and derivative gains (Bhaskaran, 2007, Chen *et al.*, 2009, Petráš, 2009, Kumar, 2013, and Zhao *et al.*, 2005). In some applications including direct current (DC) electrical motor control problems, the fractional order controller having either PI^λ rule only by setting μ=0 or PD^μ rule only by assigning λ=0 are encountered along with certain parameter tuning methods (Copot *et al.*, 2013 and Melício *et al.*, 2010). The optimization task mentioned above can be carried out using the cost functions designated with regard to the requirements and relevant constraints in addition to utilizing several optimization algorithms defined in the frequency domain (Zhao *et al.*, 2005 and Bettou and Charef, 2006). One of the most widely-used parameter determination studies done in the time domain is the approach in which the λ and μ values are obtained by minimizing the sum of the squares of the real and imaginary parts of the poles of the control system and the phase angle of the system response so as to satisfy the specified bandwidth and damping ratio quantities (Biswas *et al.*, 2009). Besides, the works handling the determination of parameters λ and μ according to the Hall-Sartorius Method which regards minimizing the sum of the squares of the error terms are encountered in the literature (Bettou and Charef, 2006). While the controller gains computed by considering the models constructed in the continuous time domain are usually used in the discrete time domain, PID-type controllers which are directly designed in the discrete time domain are also available (Khalil *et al.*, 2009 and Petráš, 2009). In transferring the controller gains obtained in the continuous time domain to the discrete time domain, some convenient approximation techniques such as the expansion to power series and modified estimation methods are utilized (Chen *et al.*, 2009 and Xue *et al.*, 2006).

Unlike the optimization approach explained above, the applications in which the gains of a classical integer PID-type controller where $\lambda=\mu=1$ are found by means of an appropriate method such as the pole placement technique and then parameters λ and μ are treated as additional design variables and they are usually reached by trial and error within selected ranges appear as well (Bettou and Charef, 2006 and Podlubny, 1999). In this extent, studies handling the trial and error iteration according to a phase margin specification are also faced with (Shekher *et al.*, 2012).

In this study, an electromechanically-actuated CAS which is developed to be utilized in a guided munition is considered. In this extent, the aim is at designing a control system upon the CAS such that it maintains the stability of the CAS under the aerodynamic loads acting on the end effectors, i.e. fins, connected to the rods of the CAS while catching the specified performance requirements. Once the most appropriate control system is decided at the end of the relevant computer simulations, the forthcoming task is to implement it on a manufactured physical CAS. That means the present work has an importance because its results will be directly applied on a physical CAS mechanism. For this purpose, after evaluating all the possible fractional order control system design approaches briefly explained above, first the gains of a classical integer PID-type controller are obtained for an electromechanically-actuated CAS. Afterwards, it is tried to be decided on the values of parameters λ and μ considering the numerical values given in the related works. While making this decision, a number of different λ and μ values are evaluated. In the end of the computer simulations performed using modules SIMULINK® and NINTEGER® of the software MATLAB® in accordance with the parameter ranges that are set by respecting the numerical values presented in the relevant studies, the performance characteristics of the designed fractional order control systems are compared to those of their integer order counterparts (Valério and Costa, 2004). Using the data acquired from the simulations, the effect of the variations in parameters λ and μ is investigated on the transient system response under the electrical current restriction coming from the driving card utilized in the control of the CAS.

2. DYNAMIC MODELING OF THE CONTROL ACTUATION SYSTEM

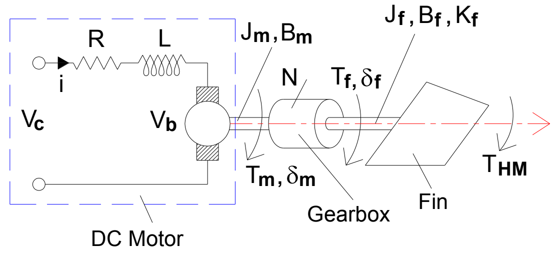

Fig. 1. Schematic view of the electromechanically-actuated control actuation system.

The schematic representation of the considered electromechanical CAS which is essentially composed of a DC electrical motor, gearbox, and fin is given in Fig. 1.

The definitions given in Fig. 1. are listed as follows:

- V_c : Supply voltage of the DC motor
- R : Internal resistance of the DC motor
- L : Inductance of the DC motor
- V_b : Armature voltage of the DC motor.
- J_m : Moment of inertia of the rotor of the DC motor which is reduced to the output shaft of the motor
- B_m : Viscous friction coefficient between the output shaft of the DC motor and support bearing
- T_m : Torque on the output shaft of the DC motor
- δ_m : Angular displacement of the output shaft of the DC motor
- N : Reduction ratio of the gearbox
- T_f : Torque on the fin connecting rod
- δ_f : Angular displacement of the fin connecting rod
- J_f : Moment of inertia of the fin which is reduced to the fin connecting rod
- B_f : Viscous friction coefficient between the fin connecting rod and support bearing
- K_f : Torsional stiffness of the fin
- T_{HM}: Hinge moment resulted from the aerodynamic effects on the fin

The equation of motion of the CAS under consideration is obtained in the following manner as J_e, B_e, and K_e stand for the equivalent moment of inertia, equivalent viscous friction coefficient, and equivalent stiffness on the fin connecting rod, respectively (Özkan, 2005):

$$T_f = J_e \ddot{\delta}_f + B_e \dot{\delta}_f + K_e \delta_f + T_{HM} \tag{1}$$

where $J_e = J_f + N^2 J_m$, $B_e = B_f + N^2 B_m$, and $K_e = K_f$.

After making intermediate calculations, the dynamic model of the CAS is found in the following form as V_c constitutes the system input (Özkan, 2005):

$$V_c = R\left(J_e \ddot{\delta}_f + B_e \dot{\delta}_f + K_e \delta_f + T_{HM}\right)/(K_t N) \tag{2}$$

where K_t denotes the DC motor constant.

Although the derived mathematical model of the CAS given in equation (2) seems to be quite simple, it is sufficient enough to represent the basic dynamic behaviour of an electromechanical actuation system which involves a DC electrical motor especially in its physical implementation. Moreover, K_t parameter can be kept almost constant within the operation region of the CAS and the gearbox has a constant N value. Evaluating all these properties together as well as the accurately-calculated J_e, B_e, and K_e parameters, it can be claimed that this linear model suffices in describing the dynamic of the considered CAS.

3. DESIGN OF THE CONTROL SYSTEM

The block diagram of the control system which is constructed for the considered electromechanically-actuated CAS to bring the fin into desired angular positions is shown in Fig. 2. Here, symbols δ_{fd}, $G_c(s)$, E, and I correspond to the desired fin angle, controller transfer function, angular position error, and controller output current, respectively.

In the present study, the control of a CAS whose dynamic model is established as given in equation (2) by means of a fractional order PID-type controller is dealt with. As K_p, K_i, and K_d represent the proportional, integral, and derivative gains of the controller, respectively, the transfer function of the fractional order PID-type controller [G_c(s)] can be written in its most general form as follows (Chen *et al.*, 2009 and Biswas *et al.*, 2009):

$$G_c(s) = K_p + \left(K_i / s^{\lambda}\right) + K_d\, s^{\mu} \tag{3}$$

One of the possible ways in the determination of the gains of the fractional order controller is to design the controller as an integer PID and then to assign the fraction rates of the integral and derivative actions in a manner suitable with the specified ranges (Bettou and Charef, 2006). Obeying this procedure for $\lambda=\mu=1$, the forthcoming expression is reached for the fin angle in the Laplace domain by regarding the block diagram in Fig. 2.:

$$\delta_f(s) = G_{\delta_d\delta}(s)\delta_{fd}(s) - G_{T\delta}(s)T_{HM}(s) \tag{4}$$

Here, transfer functions $G_{\delta_d\delta}(s)$ and $G_{T\delta}(s)$ are defined in the following fashion:

$$G_{\delta_d\delta}(s) = \frac{n_{\delta 2}\, s^2 + n_{\delta 1}\, s + 1}{d_3\, s^3 + d_2\, s^2 + d_1\, s + 1} \tag{5}$$

$$G_{T\delta}(s) = \frac{n_{T1}\, s}{d_3\, s^3 + d_2\, s^2 + d_1\, s + 1} \tag{6}$$

In equation (6), as $\mu = N K_t K_i$, definitions $n_{\delta 1} = K_p / K_i$, $n_{\delta 2} = K_d / K_i$, $n_{T1} = R/\mu$, $d_1 = (R K_e + N K_t K_p)/\mu$, $d_2 = (R B_e + N K_t K_d)/\mu$, and $d_3 = R J_e / \mu$ are introduced.

The controller gains can be determined by placing the poles of the closed loop control system on the complex plane such that they satisfy the specified bandwidth and damping ratio values (Özkan, 2005). For this purpose, as ω_c and ζ_c indicate the desired bandwidth in rad/s and damping ratio, respectively, the characteristic polynomial seen in the denominators of equations (5) and (6) as given in equation (7) as well is equated to the characteristic polynomial of a standard third-order system as in equation (8):

$$D(s) = d_3\, s^3 + d_2\, s^2 + d_1\, s + 1 \tag{7}$$

$$D_3(s) = \frac{s^3}{\omega_c^3} + \left(\frac{2\zeta_c + 1}{\omega_c^2}\right)s^2 + \left(\frac{2\zeta_c + 1}{\omega_c}\right)s + 1 \tag{8}$$

By equating the coefficients of the terms at the same order of variable "s" for equations (7) and (8), K_p, K_i, and K_d are found as follows:

$$K_p = (2\zeta_c + 1)J_e\, \omega_c^2 / (N K_t) \tag{9}$$

$$K_i = \left(J_e\, \omega_c^3\right)/(N K_t) \tag{10}$$

$$K_d = \left[(2\zeta_c + 1)J_e\, \omega_c - B_e\right]/(N K_t) \tag{11}$$

Although it is not a strict rule, the fractional order control system which is established by the use of the controller gains calculated above along with λ and μ that are clarified in accordance with a certain performance criterion guarantees the stability of the system provided that magnitudes of λ and μ do not exceed 1 and also contributes the robustness of the system to be improved thanks to the advantage of capability for adjusting these two extra parameters as well (Khalil *et al.*, 2009 and Xue *et al.*, 2006).

4. COMPUTER SIMULATIONS

The numerical values considered for the parameters of the electromechanical control actuation system are submitted in Table 1.

Table 1. Numerical values used in the simulations

Parameter	Numerical Value	Parameter	Numerical Value
J_e	0.25 kg·m^2	N	45
B_e	0.01 N·m·s/rad	f_c	10 Hz
K_t	0.35 N·m/A	ζ_c	0.7

The relevant computer simulations are carried out in the MATLAB® SIMULINK® environment and using the NINTEGER® tool developed for the design of the fractional order and nonlinear control systems. Since the primary goal is to observe the effects of the different values of λ and μ parameters of a PID type controllers, the integer order ($\lambda=\mu=1$) PID controller is selected for performance comparison. Also, the Crone Method is utilized in the design of the fractional order controller. In the simulations performed in the discrete time domain with sampling frequency of 2 kHz, the duration of each simulation is set to be 0.5 s and the ODE5 Dormand-Price solver is selected with a constant step. The gain of the electronic driving card used in the control of the DC motor of the electromechanical system is assumed to be unity because its operating frequency

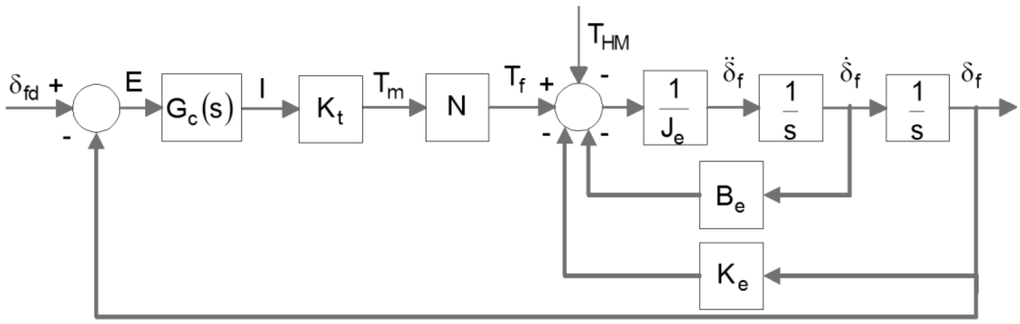

Fig. 2. Block diagram of the control actuation system.

is at the order of 100 times the bandwidth of the closed loop control system and the current limit is added to the model within the range of ±10 A. Moreover, the reference fin angle is supplied with the amplitude of 5°. Lastly, it is supposed that the aerodynamic hinge moment acting on the fins varies randomly within the interval of ±100 N·m. The ranges considered for the numerical values of parameters λ and μ for the designed fractional order PID-type controllers are assigned by accounting the optimized quantities given in the related studies as in Table 2 (Khalil *et al.*, 2009, Petráš, 2009, Zhao *et al.*, 2005, Bettou and Charef, 2006, Biswas *et al.*, 2009, and Xue *et al.*, 2006).

In the end of the computer simulations performed under the conditions explained above, the maximum current, maximum overshoot, settling time, and steady state error results are presented in Table 2 for the integer order PID (IPID) and fractional order PID (FPID) controllers. The plots showing the system response and control current requirement (current command) are given in Fig. 3 through Fig. 12 for the chosen sample control systems proposed.

Tablo 1: Results obtained for the considered controllers with the current limit of 10 A

Configuration	Order of the Controller		Maximum Current (A)	Maximum Overshoot (°)	Settling Time (ms)	Steady State Error (°)
	λ	μ				
IPID	1	1	10	6.88	98	0
FPID-1	0.1	0.5	> 10	8.30	100	0
FPID-2	0.5			4.67	110	0.33
FPID-3	0.8			3.35	200	1.65
FPID-4		0.9		3.28	200	1.72

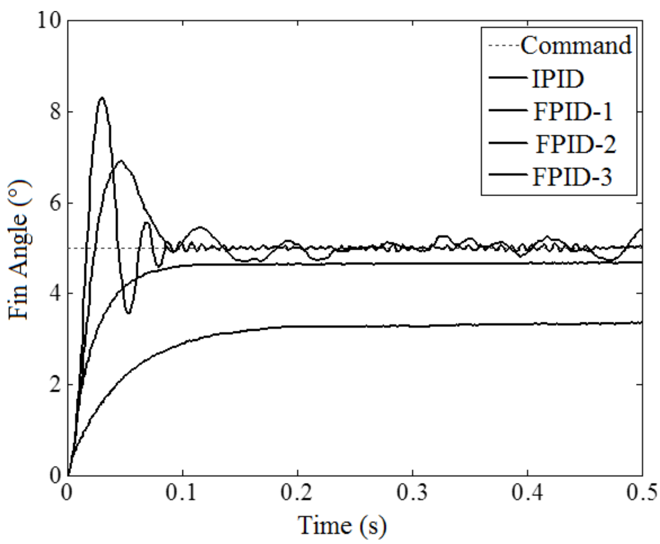

Fig. 3. Responses of the designed control systems with the current limit of ±10 A.

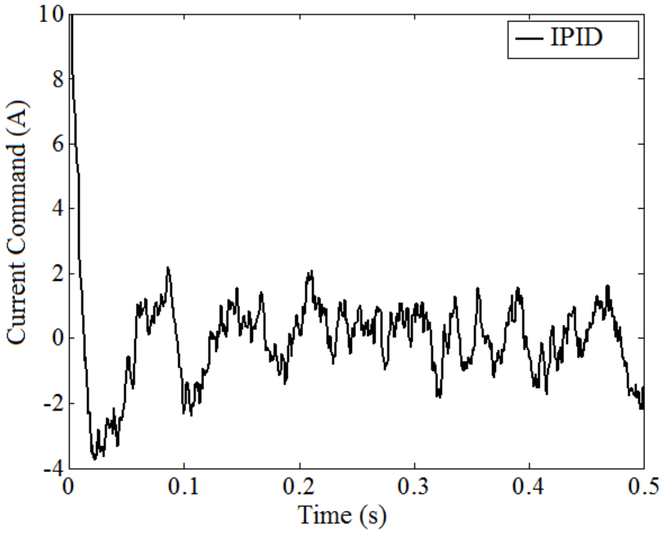

Fig. 4. Current requirement of the IPID control system with the current limit of ±10 A.

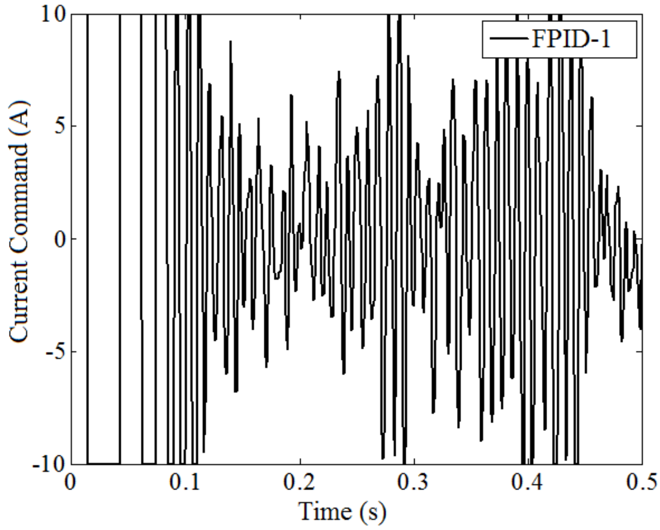

Fig. 5. Current requirement of the FPID-1 control system with the current limit of ±10 A.

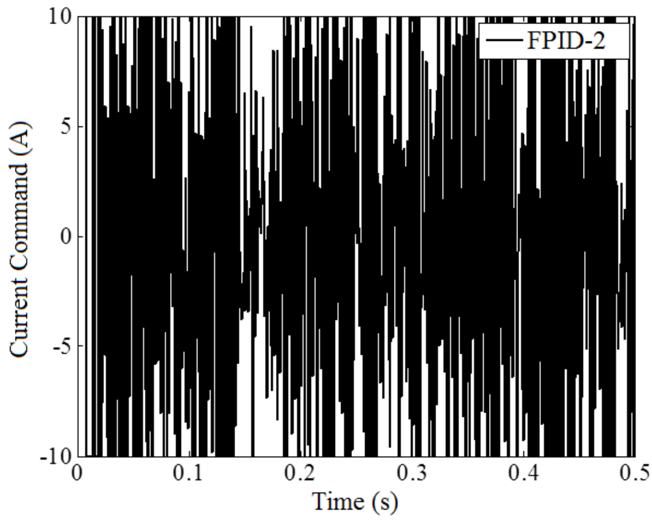

Fig. 6. Current requirement of the FPID-2 control system with the current limit of ±10 A.

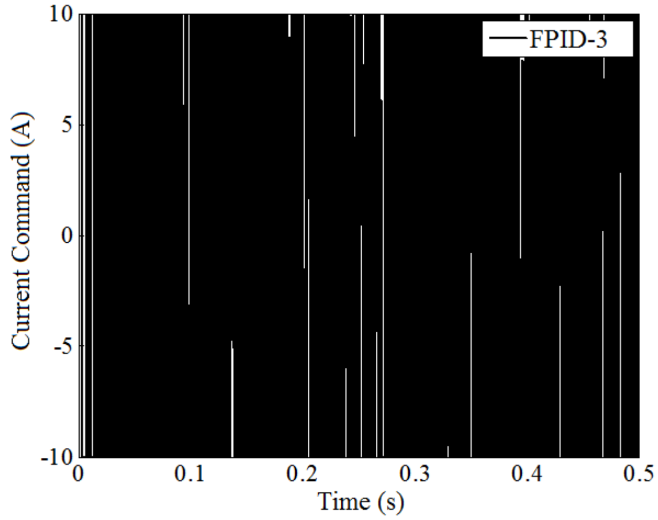

Fig. 7. Current requirement of the FPID-3 control system with the current limit of ±10 A.

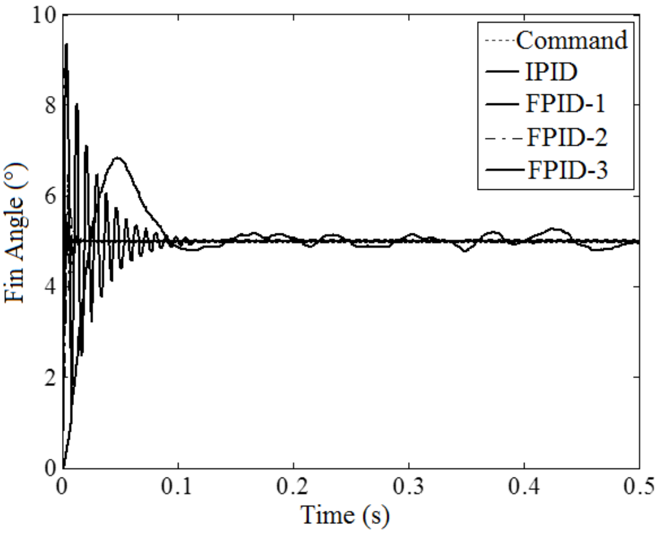

Fig. 8. Responses of the designed control systems without any current limit.

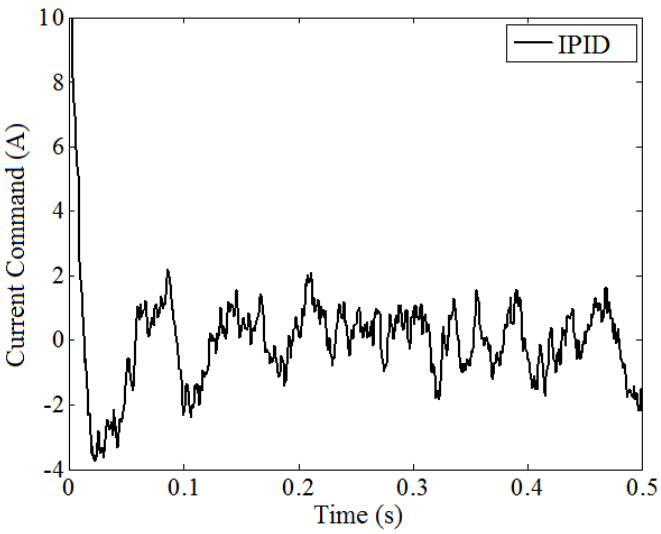

Fig. 9. Current requirement of the IPID control system without any current limit.

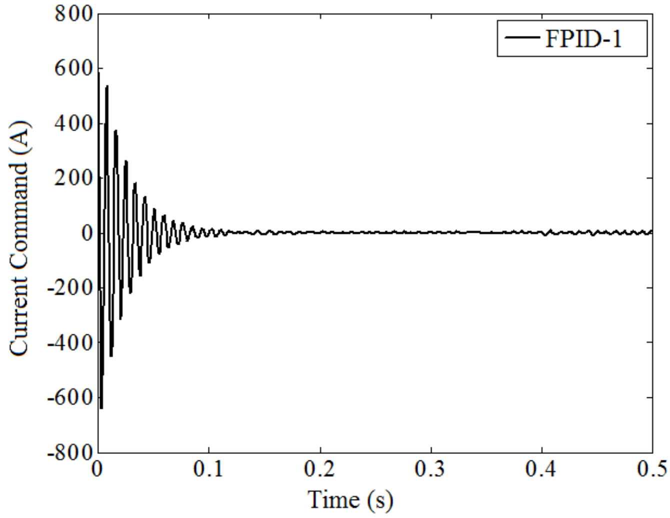

Fig. 10. Current requirement of the FPID-1 control system without any current limit.

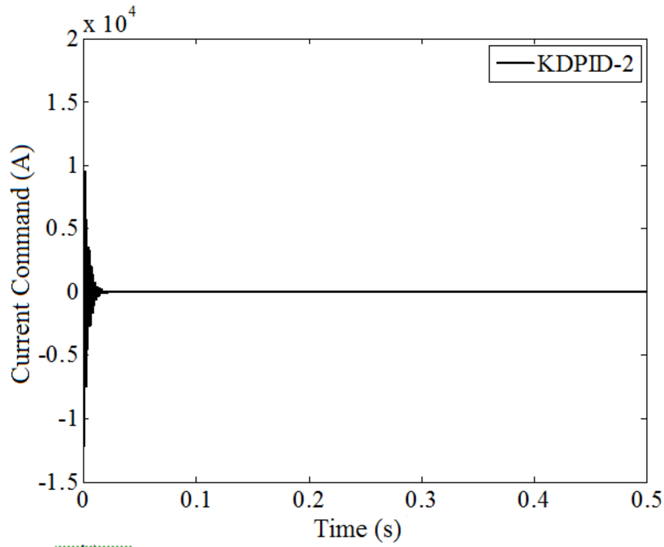

Fig. 11. Current requirement of the FPID-2 control system without any current limit.

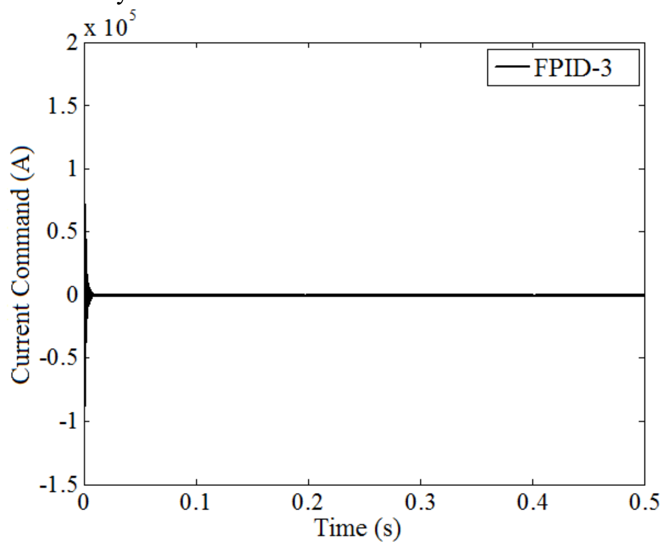

Fig. 12. Current requirement of the FPID-3 control system without any current limit.

5. DISCUSSION AND CONCLUSION

As per the computer simulations for the electromechanically-actuated CAS considered, it can be concluded that the proposed fractional order control systems yield satisfactory results especially in the sense of the system stability and robustness against the disturbance which is regarded as the hinge moment influencing the CAS with a randomly-varying characteristic within a specified interval. When the data acquired from the computer simulations are evaluated, it is obvious that the maximum overshoot diminishes as the values of parameters λ and μ approach unity whereas the settling time and steady state error grow. Regarding the current capacity of the driving card of the DC motor, the steady state error is nullified for all of the simulation models constructed with regard of the different values of λ and μ. Yet, the current requirement of the control system reaches huge levels that cannot be satisfied physically accordingly with the increments in λ and μ. On the other hand, smaller steady state errors are encountered in more realistic simulations accounting the limitations of the driving card of the DC motor involving λ and μ parameters with lower values but more maximum overshoot and more oscillations in the system response. This situation is originated from the fact that the current demand required to nullify the steady state error cannot be satisfied under the current limit regarded. Due to the unsatisfied current demand, it seen from the graphs that the control current become quite oscillatory. In fact, this behaviour can be eliminated or at least minimized by elaborating λ and μ parameters a little more. Also, increasing λ and μ leads the oscillations in the system response to decrease considerably but to enlarge the steady state error.

Another significant output of the present study is that the increase in parameter μ which corresponds to the fractional order of the derivative action makes the maximum overshoot decrease. This growth does not cause any change in the settling time while it results a higher steady state error.

Actually, the fractional order PID controller has two more parameters, namely λ and μ, to be adjusted for optimization the performance of the considered system than the classical integer PID-type controller. Therefore, the present study does not reflect the results of an exact and fair comparison of fractional and integer type controllers but it can be used to gain a basic insight on the advantages of the implementation of a fractional order controller.

In the adaptation of the fractional order PID-type controllers to the real-time systems via test setups developed in a specific manner, it is recorded in the relevant literature that certain problems come into the picture in the sense of catching the control system requirements including speed (Chen *et al.*, 2009, Petráš, 2009, and Bettou and Charef, 2006). Thus, the computer simulations performed should be repeated using a test setup designed on purpose so that such difficulties can be observed and recommendations on the solution can be made. Furthermore, it is advised that the performance characteristics of the proposed fractional order PID-type control systems be investigated against parameter uncertainties and control strategies other than the integer order PID-type controller be utilized for comparison purpose.

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