# Modeling, Analysis, and Improvement of a Multi-product Furniture Assembly Line * 

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#### Abstract

In this paper, a case study of modeling, analysis, and continuous improvement of a multi-product assembly line at a furniture manufacturing plant is introduced. In such system, cabinets with multiple design options are assembled. Analytical models have been developed and recursive procedures have been derived to evaluate line production rate. It is shown that such a method results in high accuracy in performance evaluation. Continuous improvement efforts, such as lot size adjustment and bottleneck analysis, have been carried out to improve the system throughput. Such methods provide quantitative tools for plant engineers and managers to operate and improve the assembly line to achieve high productivity.


Keywords: Multiple products, bernoulli reliability model, production rate, bottleneck, lot size, furniture assembly line.

## 1. INTRODUCTION

Furniture products are essential and important elements in general business and domestic life. Due to fierce competition in this industry, being flexible to market, reducing labor cost and operating large volume on high productivity are the key factors linked to the profitability of the industry (Furniture Industry Profile (2013); Pirc and Vlosky (2010)). Most of the literatures on furniture production focus on planning, quality control, and supply chain research (e.g., representative papers by Alem et al. (2012); Radharamanan et al. (1996); Vickery et al. (1997)). To our best knowledge, no research has been conducted yet from a production system point of view. However, performance analysis and continuous improvement of furniture manufacturing systems are critical to ensure productivity, flexibility and quality. As in furniture manufacturing, assembly lines are the last and one of the most important production sections, analyzing and improving the productivity of them have significant importance.

Manufacturing systems have attracted substantial research efforts during the last 50 years (see, for example, monographs by Buzacott and Shantikumar (1993); Gershwin (1994); Li et al. (2009)). Both simulation analysis and analytical investigation have been carried out extensively. In particular, analytical methods can provide a quick analysis of system behavior, and enable us to investigate the nature of the system. Although flexible manufacturing systems have been studied for almost 30 years (see reviews by Buzacott and Yao (1986); Sethi and Sethi (1990); Beach et al. (2000); Li et al. (2009)), the

[^0]case of multi-product line with unreliable machines and finite non-dedicated buffers is less studied, except serial line studies by Zhao and Li (2013a); Zhao and Li (2013b).
In this paper, we introduce a case study of modeling, analysis, and improvement of an assembly line at a furniture plant, which is a leading provider of healthcare related cabinetry, millwork and built-in workspace solutions. Business office and storage application units, special work surface tops, landscape panel units, home office and storage units, etc., are the main products, and all of them will go through the final assembly line. To analyze highly flexible furniture assembly lines, a new method is introduced to address the assembly operations in the system. Specifically, structural modeling is first introduced to simplify the complicated system layout to an assembly system without losing the fidelity. Next, an iterative procedure is presented to approximate the production rate of the system. The method is validated by comparing with the actual throughput observed on the factory floor. It is shown that the model has achieved a high accuracy. Then, using the validated model, lot size study is carried out to reduce the impact of setup times, and bottleneck analysis is introduced to identify the machines that impede line production rate in the strongest manner. By adjusting lot size and mitigating the bottleneck machines, the system productivity can be improved significantly.

The remainder of the paper is structured as follows: Section 2 describes the system layout. Structural modeling is introduced in Section 3. Section 4 outlines the performance evaluation method and validates the model. The continuous improvement study is presented in Section 5. Finally, conclusions are provided in Section 6.


Fig. 1. Assembly line layout

## 2. SYSTEM DESCRIPTION

In the furniture manufacturing plant under study, when a customer order is received, it gets confirmed within two days, and the shipment date is determined. It takes an average of two weeks to build a cabinet and ship it to the customer. Prior to assembly, raw blanks will be pressed, machined, and then installed at the kitting area, preparing for assembly. The layout of the assembly line is shown in Figure 1.
There are two serial processes that join at the assembly station. The drawers are assembled in one of the serial lines and the outer casing is put together in the other. The drawers and the outer casing are then assembled and sent for quality check. On successful completion of the quality check the furniture is sent for packing and shipment. However, in the presence of defects, the cabinet will be sent to the "hospital" area where the rework is carried out. Most of the operations are manual, except the case clamp station provides an automatic pressing of blanks and counting of units. All raw material are stored in the kitting area, and small components (such as screws, hinges, etc.) are prepared on the desks. The blanks, drawers, cases, and units are transported on the conveyors.
The operation performed in each work station are described below.

- Drawer Build: Wooden blanks are gathered based on the required specification and put together. These wooden blanks are picked by the operator who assembles.
- Drawer Hardware: The sliders and other hardware are attached to the drawers with the help of a stapling machine.
- Drawer Fronts and Doors: Cleaned wooden blanks are attached to the front of the drawer box which are then passed on for assembly
- Case Preparation: The required wooden blanks for the outer casing are picked by the worker who will carry out the operation. Hinges and slides are attached to the blank of the outer casing at this station.
- Build Case: Wooden blanks are put together and the outer casing is made for the drawer.
- Case Clamp: The outer casing once prepared is passed on to the case clamp station, which is an automatic
machine that presses the blanks together. The sensor in the station regulates the number of units that enter the station
- Assembly: The drawers, doors and outer casing are assembled together at this station. The furniture is then tipped and the bottom blank and the rollers are fixed.
In summary, a production flow of the furniture assembly line can be described as follows (Figure 2): build drawer, drawer hardware, and drawer fronts consist of the drawer line, while care preparation, build case, and case clamp comprise the case line. These two lines merge at the assembly station, and then followed by quality check.


Fig. 2. Production flow in assembly line

## 3. SYSTEM MODELING

Given the production system layout, structural modeling is introduced to develop a simplified model, which is tractable for analysis but without losing the fidelity. Such a simplification procedure is introduced below.

### 3.1 Structural Modeling

In this study, simplification procedure is carried out step by step based on the observations in the production line. First, a flow chart can be drawn to describe the process flow, shown in Figure 2. Then, each module (or station) in the Figure 2 have the following characteristics:
(1) On each station described above, the multiple tasks carried out by the workers to finish the assigned jobs are considered one operation. For example, on the assembly station, the job is not done until hinges are put on, doors are placed and drawer sliders are installed. Thus, a station is considered as an aggregated virtual machine.

Table 1. Machine cycle times

| $m_{i}$ | Average processing time (sec) |  | Average setup time (sec) |  | Average cycle time (sec) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Simple | Complex | Simple | Complex | Simple | Complex |
| 1 | 72 | 72 | - | - | 72 | 72 |
| 2 | 89 | 89 | - | - | 89 | 89 |
| 3 | 100 | 180 | 9.164 | 13.070 | 109.164 | 193.070 |
| 4 | 110 | 190 | 10.309 | 14.216 | 120.309 | 204.216 |
| 5 | 80 | 130 | 3.436 | 7.343 | 83.436 | 137.343 |
| 6 | 112 | 112 | - | - | 112 | 112 |
| 7 | 70 | 70 | - | - | 70 | 70 |

(2) All stations are mutually independent, due to its unique functionality in the process. They are always separated by buffers (conveyors), intended to decouple or reduce the impact of any random overtime or downtime in production to other stations.
(3) According to the data collected, the assembly line is seldom blocked by the shipping section; the beginning processes of the assembly lines are never starved since the kitting area is large enough and coordination of machining orders is efficient enough to sustain assembly production without frequent waiting.
(4) Buffers between each pair of consecutive stations are identified; their capacities can be found by observing the number of raw parts hold in between. In addition, "block before service" regime is applied in the assembly line to regulate the material flow.
(5) By observing the processing times distribution of 51 different products as shown in Figure 3, it's a bimodal instead of unimodal distribution. Therefore, they are characterized by two product types, simple (or standard) and complex. Then, a two-product assembly line analysis is developed to calculate the performance of the system.
(6) There is a setup time when product type is switched. It has been observed that the operators take a significant amount of time to read the configurations and set up tooling before carrying out the assigned task. To incorporate the setup time in the model, the cycle time for a product is adjusted by adding a portion of setup time in the lot to the net processing time of the product.
(7) The quality inspection is carried out by sampling only and it never blocks the assembly station. Therefore, such a station is ignored in the simplified model.


Fig. 3. A histogram of 51 different products at the drawer front station

After simplifications, the final model is shown in Figure 4, where the circles represent the machines (or stations),
denoted as $m_{1}, m_{2}, \ldots, m_{7}$; and the rectangles are the buffers, denoted as $b_{1}, b_{2}, b_{3}, b_{5}, b_{6}, b_{7}$; with buffer capacity marked in the boxes. The identification of the parameters of the machines and buffers will be introduced in next subsection.


Fig. 4. Assembly line model

### 3.2 Parameter Identification

In this study, 28 days of active production data is collected through the factory information system. The products are grouped into two categories, simple and complex, for machines $m_{3}, m_{4}$, and $m_{5}$. The cycle time on these machines include net processing time of the product, and a percentage of setup times, proportional to lot size. Note that the setups not only exist between two groups (simple and complex), but also exist between different product types within each group. Specifically, by summing up all the setup times within the simple product group, and dividing by the lot size for the simple group, we obtain the additional time to be added into cycle time. Adding this time to the net processing time of the simple product, the final cycle time for simple product group is obtained. Similar procedure can be applied to complex product group. By analyzing the production lot data, the average lot sizes for both complex and simple products is 8.73. The percentages of complex and simple product counts are $50.82 \%$ and $49.17 \%$, respectively. Therefore, the cycle times for all machines can be calculated, and are summarized in Table 1.
Denote the shortest cycle time of all machines (i.e., $c_{7,1}=$ $c_{7,2}=70 \mathrm{sec}$ ) as the system cycle time $\tau$, the Bernoulli success probability $p_{i, j}$ for machine $m_{i}, i=1, \ldots, 7$, and product type $j, j=1,2$, can be calculated as follows:

$$
p_{i, j}=\min \left\{0.999, \frac{\tau}{c_{i, j}}\right\}
$$

where $c_{i, j}$ is the cycle time for the $j$ th type product on machine $m_{i}$. To avoid $p_{i, j}=1,0.999$ is set to $p_{i, j}$ when $\tau=c_{i, j}$. Thus, the model parameters are obtained and shown in Table 2.

Table 2. Machine and buffer parameters

| $m_{i}$ | $p_{i, 1}$ | $p_{i, 2}$ | $N_{i}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.970 | 0.970 | 12 |
| 2 | 0.784 | 0.784 | 100 |
| 3 | 0.641 | 0.363 | 21 |
| 4 | 0.582 | 0.343 |  |
| 5 | 0.839 | 0.510 | 6 |
| 6 | 0.625 | 0.625 | 1 |
| 7 | 0.999 | 0.999 | 2 |

Remark 1. Note that due to confidentiality, the data has been modified and is for illustration purpose only. However, the basic property of the data does not change.

## 4. PERFORMANCE ANALYSIS AND MODEL VALIDATION

Due to the complexity of the multi-product system, directly analysis by explicitly numbering the states of multiple products system is impossible. Therefore, an approximation approach is pursued. The idea is to aggregate the multiple products into a virtual "single" product. Then, overlapping decomposition method is used to analyze the "single" product assembly line.

### 4.1 Product Aggregation

To start with, consider machine $m_{i}, i=3,4,5$. Two product types can be made, simple and complex, denoted as 1 and 2, respectively. Four possible states exist in this scenario: $\left\{m_{i}\right.$ is up, processing type $1 ; m_{i}$ is up, processing type $2 ; m_{3}$ is down, should process type $1 ; m_{i}$ is down, should process type 2$\}$. These four states are denoted as $s_{1}, s_{2}, s_{3}$, and $s_{4}$, respectively. The transitions between the states are illustrated in Figure 5.


Fig. 5. Transition diagram of a machine producing two part types

Note that $m_{i}$ can go to any other state if it is up in current cycle. However, if it is in down state currently, it must return to its own up state first and cannot go to an up state of another part type directly. Based on these transitions, let $p_{i 1}^{\prime}$ and $p_{i 2}^{\prime}$ denote the probability that the machine is up and processing part types 1 and 2 , respectively. We obtain

$$
\begin{align*}
& p_{i 1}^{\prime}=\frac{\alpha_{1}}{\frac{\alpha_{1}}{p_{i 1}}+\frac{\alpha_{2}}{p_{i 2}}}, \quad i=3,4,5,  \tag{1}\\
& p_{i 2}^{\prime}=\frac{\alpha_{2}}{\frac{\alpha_{1}}{p_{i 1}}+\frac{\alpha_{2}}{p_{i 2}}}, \quad i=3,4,5, \tag{2}
\end{align*}
$$

where $\alpha_{j}, j=1,2$, is the percentage of counts of part type $j$. Then the probability that the machine is up and processing any type of product is equal to

$$
\begin{equation*}
p_{i}=p_{i 1}^{\prime}+p_{i 2}^{\prime}=\frac{1}{\frac{\alpha_{1}}{p_{i 1}}+\frac{\alpha_{2}}{p_{i 2}}}, \quad i=3,4,5 \tag{3}
\end{equation*}
$$

and probability $p_{i}$ can be viewed as the production rate of an aggregated part for a single machine system.
Remark 2. In general, when there are more than two product types, we have

$$
\begin{equation*}
p_{i j}^{\prime}=\frac{\alpha_{j}}{\sum_{l=1}^{K} \frac{\alpha_{l}}{p_{i l}}}, \tag{4}
\end{equation*}
$$

and the production rate for the aggregated part will be

$$
\begin{equation*}
p_{i}=\sum_{j=1}^{K} p_{i j}^{\prime}=\frac{1}{\sum_{l=1}^{K} \frac{\alpha_{l}}{p_{i l}}} . \tag{5}
\end{equation*}
$$

In addition, such an aggregation is not a simple average, since $p_{i} \neq \sum_{j} p_{i j} \alpha_{j}$.

By aggregating the two product types, simple and complex, into one at machines $m_{3}, m_{4}$ and $m_{5}$, we obtain the aggregated parameters $p_{3}, p_{4}$, and $p_{5}$, respectively, using equation (3). Since machines $m_{1}, m_{2}, m_{6}, m_{7}$ have identical speeds for different product types, we have

$$
p_{i}=p_{i 1}=p_{i 2}, \quad i=1,2,6,7
$$

### 4.2 Overlapping Decomposition

The assembly system can be analyzed using an approximation method based on overlapping decomposition. Specifically, the assembly system described in Figure 4 is decomposed into two serial lines, $L_{1}$ and $L_{2}$, with the merge machine $m_{4}$ as the overlapped machine, i.e.,

> Line 1: $m_{1}, m_{2}, m_{3}, m_{4}$
> Line 2: $m_{5}, m_{6}, m_{7}, m_{4}$

First, consider Line 1. The overlapped assembly machine $m_{4}$ will be modified to $m_{4}^{\prime}$ to accommodate the effects from Line 2. Let $s t_{4,2}$ denote the probability that buffer $b_{7}$ in Line 2 is empty. Such starvation time can be viewed as $m_{4}$ 's downtime in Line 1 so that its reliability will be decreased. Therefore, the modified machine reliability $p_{4}^{\prime}$ of $m_{4}^{\prime}$ can be defined as:
$p_{4}^{\prime}=p_{4}\left(1-\operatorname{Pr}\left\{b_{7}\right.\right.$ in Line 2 is empty $\left.\}\right)=p_{4}\left(1-s t_{4,2}\right)$.
Thus, a serial line with a modified machine $m_{4}$ can be obtained, $m_{1}, m_{2}, m_{3}, m_{4}^{\prime}$. Using the serial line analysis method, the empty probability of buffer $b_{3}$ at machine can be calculated, denoted as $s t_{4,1}$.

$$
s t_{4,1}=\Psi\left(p_{1}, p_{2}, p_{3}, p_{4}^{\prime}, N_{1}, N_{2}, N_{3}\right)
$$

where operator $\Psi(\cdot)$ is introduced to denote the calculation of empty probability of the last buffer in a serial line.
Next, using probability $s t_{4,1}$, we consider Line 2 . The overlapped assembly machine $m_{4}$ will be modified to $m_{4}^{\prime \prime}$ by taking into account of $s t_{4,1}$, i.e.,
$p_{4}^{\prime \prime}=p_{4}\left(1-\operatorname{Pr}\left\{b_{3}\right.\right.$ in Line 1 is empty $\left.\}\right)=p_{4}\left(1-s t_{4,1}\right)$.
Again, the serial line analysis method is applied to calculate empty buffer probability $s t_{4,2}$.

$$
s t_{4,2}=\Psi\left(p_{5}, p_{6}, p_{7}, p_{4}^{\prime \prime}, N_{5}, N_{6}, N_{7}\right)
$$

Since probabilities $s t_{4,1}$ in Line 1 and $s t_{4,2}$ in Line 2 are unknown, we introduce iterations. In the first iteration, assume $s t_{4,2}$ is known (e.g., equal to 0.5 ), we calculate $s t_{4,1}$ and then obtain the new values for $s t_{4,2}$. In the second iteration, this probability are replaced into Line 1 again
to generate $s t_{4,1}$ and then update $s t_{4,2}$. The process is repeated anew until it is convergent. Mathematically, such a process can be described as follows:
Procedure 1.

$$
\begin{align*}
p_{4}^{\prime}(s) & =p_{4}\left(1-s t_{4,2}(s-1)\right),  \tag{6}\\
s t_{4,1} & =\Psi\left(p_{1}, p_{2}, p_{3}, p_{4}^{\prime}(s), N_{1}, N_{2}, N_{3}\right),  \tag{7}\\
p_{4}^{\prime \prime}(s) & =p_{4}\left(1-s t_{4,1}(s-1)\right),  \tag{8}\\
s t_{4,2} & =\Psi\left(p_{5}, p_{6}, p_{7}, p_{4}^{\prime \prime}(s), N_{5}, N_{6}, N_{7}\right) . \tag{9}
\end{align*}
$$

with initial conditions

$$
s t_{4,2}(0)=0.5,
$$

and $s$ is iteration number,

$$
s=1,2, \ldots
$$

The operator $\Psi(\cdot)$ is obtained through serial line aggregation method introduced in Li and Meerkov (2009). To make this paper self-contained, the serial line aggregation procedure and operator $\Psi(\cdot)$ are described in the Appendix.
It can be shown that Procedure 1 is convergent and it leads to an estimate of system production rate.
Theorem 1. Procedure 1 is convergent, i.e.,

$$
\begin{aligned}
& \lim _{s \rightarrow \infty} p_{4}^{\prime}(s)=p_{4}^{\prime}, \quad \lim _{s \rightarrow \infty} p_{4}^{\prime \prime}(s)=p_{4}^{\prime \prime} \\
& \lim _{s \rightarrow \infty} s t_{4,1}(s)=s t_{4,1}, \quad \lim _{s \rightarrow \infty} s t_{4,2}(s)=s t_{4,2}
\end{aligned}
$$

The proof of the theorem is omitted due to space limitation, and can be found in Zhao and Li (2013c). Then the production rate of the assembly system will be

$$
\begin{equation*}
P R=p_{4}\left(1-s t_{4,1}\right)\left(1-s t_{4,2}\right) \tag{10}
\end{equation*}
$$

The production rate for each product type, $P R_{j}, j=1,2$, can be obtained as well.

$$
\begin{equation*}
P R_{j}=\alpha_{j} P R, \quad j=1,2 \tag{11}
\end{equation*}
$$

### 4.3 Model Validation

Using the method introduced above and the data shown in Table 2, we calculate the total throughput of the assembly line as 205.86 parts (including both simple and complex units) per day. Comparing with the actual throughput of 204.12 parts per day, the difference is less than $1 \%$. Therefore, the model is validated and can be used for subsequent analysis.

## 5. IMPROVEMENT ANALYSIS

### 5.1 Lot Sizing Improvement

To improve the throughput of the furniture assembly system under consideration, a direct way is to increase the lot size so that changeovers will be reduced. Using the method introduced above, we investigate the impact of increasing lot size by $100 \%, 200 \%$ and $300 \%$. The results are shown in Table 3.
It can be observed that there will be a considerable increase of throughput if lot size is doubled. However,

Table 3. Lot size improvement

| Lot size | Current | $100 \%$ <br> increase | $200 \%$ <br> increase | $300 \%$ <br> increase |
| :---: | :---: | :---: | :---: | :---: |
| Throughput | 205.07 | 212.20 | 214.66 | 215.74 |
| Improvement | - | $3.48 \%$ | $4.36 \%$ | $4.88 \%$ |

continuous increase will lead to a diminishing return. In practice, the lot size could not be arbitrary big since the plant has to satisfy due day order of all product types. Therefore, the plant management has accepted recommendation and planed to double the lot size.

### 5.2 Bottleneck Analysis

Bottleneck analysis has been shown as the most effective way to improve production system performance ( Li and Meerkov (2009)). To improve the production rate of the assembly line, we need to identify the machine that impedes the line performance in the strongest manner. Here we define the joint bottleneck machine ( $\mathrm{BN}-\mathrm{m}$ ) as:
Definition 1. Machine $m_{i}$ is the joint bottleneck machine if

$$
\sum_{l} \frac{\partial P R}{\partial p_{i, l}}>\sum_{l} \frac{\partial P R}{\partial p_{j, l}}, \quad \forall j \neq i
$$

Such a bottleneck indeed represents the machine that the improvement of machine speed on both product types, for instance, reducing the cycle times of both simple and complex part types, will lead to the largest improvement in system production rate comparing with improving all other machines. Such a machine is referred to as a joint bottleneck machine (Zhao and Li (2013a)). To identify the bottleneck machine, an arrow assignment rule based on probabilities of blockage and starvation has been proposed for serial production lines. Here we extend the method to assembly systems. Specifically, the arrows are assigned from the upstream machine to the downstream if $B L_{i}>$ $S T_{i+1}$ (i.e., machine $m_{i}$ 's blockage probability is greater than $m_{i+1}$ 's starvation probability), otherwise, the arrow should be in opposite direction. Then, the bottleneck machine ( $\mathrm{BN}-\mathrm{m}$ ) is the one with no emanating arrows. For assembly machine $m_{4}$, the starvation due to $b_{3}$ and $b_{7}$ are used to compare with blockages of $m_{3}$ and $m_{7}$, respectively. An illustration of such identification is shown in Figure 6. Since machine $m_{4}$ has no emanating arrows, it is the bottleneck machine. Such results match with floor observations. By adding one more worker at the assembly station, processing time can be reduced by $1 / 3$, i.e., to 73 and 127 seconds for simple and complex products, respectively. The throughput is then increased to 225 parts per day, which is a $9.7 \%$ improvement.

Therefore, the following suggestion has been recommended to plant management: increasing lot size by $100 \%$ and adding a labor to improve the assembly station. Such an effort can lead to almost $15 \%$ improvement, i.e., throughput is increased to 236 parts per day. The resulting benefits through improved line throughput would be much larger than investment cost (adding labor).

## 6. CONCLUSIONS

This paper introduces a case study of multi-product furniture assembly line. Through structural modeling of the


Fig. 6. Bottleneck identification by arrow assignment rule line, an assembly system model has been developed and an iterative procedure has been introduced to evaluate line production rate. Using the data collected on the factory floor, the model has been validated with high accuracy. Lot size analysis is proposed and bottleneck identification has been carried out to improve system throughput. Such methods provide quantitative tools for analysis and improvement of multi-product assembly systems, which are not only useful for furniture production, but also for other manufacturing industries.

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## APPENDIX

Consider a serial production line with $M$ machines and $M-1$ buffers. The machines have parameters $p_{k}, k=$ $1, \ldots, M$, and buffers have capacities $N_{k}, k=1, \ldots, M-1$. The following aggregation procedure is introduced:

## Procedure 2.

$$
\begin{gathered}
p_{k}^{b}(n+1)=p_{k}\left[1-Q\left(p_{k+1}^{b}(n+1), p_{k}^{f}(n), N_{k}\right)\right] \\
k=1, \ldots, M-1 \\
p_{k}^{f}(n+1)=p_{k}\left[1-Q\left(p_{k-1}^{f}(n+1), p_{k}^{b}(n+1), N_{k-1}\right)\right] \\
k=2, \ldots, M \\
n=0,1,2, \ldots
\end{gathered}
$$

with initial conditions

$$
p_{k}^{f}(0)=p_{k}, \quad k=1, \ldots, M
$$

and boundary conditions

$$
p_{1}^{f}(n)=p_{1}, \quad p_{M}^{b}(n)=p_{M}, \quad n=0,1,2, \ldots
$$

where,

$$
\begin{aligned}
Q\left(p_{1}, p_{2}, N_{1}\right) & = \begin{cases}\frac{\left(1-p_{1}\right)(1-\phi)}{1-\frac{p_{1}}{p_{2}} \phi^{N_{1}}}, & \text { if } p_{1} \neq p_{2} \\
\frac{1-p}{N_{1}+1-p}, & \text { if } p_{1}=p_{2}=p\end{cases} \\
\phi & =\frac{p_{1}\left(1-p_{2}\right)}{p_{2}\left(1-p_{1}\right)}
\end{aligned}
$$

The convergence of the procedure has been proved in Li and Meerkov (2009). Therefore,

Theorem 2. Procedure 2 is convergent and the following limits exist:

$$
p_{k}^{f}=\lim _{n \rightarrow \infty} p_{k}^{f}(n), \quad p_{k}^{b}=\lim _{n \rightarrow \infty} p_{k}^{b}(n), \quad k=1,2, \ldots, M
$$

Using the converged limits, we obtain

$$
\Psi=Q\left(p_{M-1}^{f}, p_{M}^{b}, N_{M-1}\right)
$$

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