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Supply/demand correlation as an auxiliary variable for smart grid control design ⋆

René Boel * **Pravin Varaiya** **

* *SYSTeMS Research Group, Faculty of Engineering, University of Ghent, B-9052 Zwijnaarde, Belgium, and NICTA, Melbourne University, Melbourne, Australia,* Rene.Boel@UGent.be
** *EECS, University of California, Berkeley,* varaiya@eecs.ucb.edu

Abstract: Stable and efficient operation of a smart grid requires coordination of the actions of many independent decision makers or agents that must cooperate in globally balancing the demand for power to the partly unpredictable and uncontrollable supply. This paper proposes the use of the correlation coefficient between the renewable supply and the (controllable) demand, measured in a given region over a specified time scale, as a performance measure that is imposed on lower level control agents by a higher level controller in order to achieve coordination between the actions of these individual agents. Using elementary calculations for simple distribution networks this paper illustrates that the expensive peak value of power imported from the grid, and the distribution losses, depend linearly on this correlation coefficient. Operational behavior is improved way by achieving a high correlation coefficient. The correlation coefficient they achieve can be used as a measure for determining the rewards control agents should get for their contribution to the ancillary services. Achieving a high correlation coefficient clearly will contribute to the stability of the smart grid. In this paper we briefly discuss some requirements for implementing a coordination strategy based on the correlation coefficients. Broadcasting of signals from a top level in the control hierarchy to the lower layer controllers only is needed.

Keywords: smart grid, coordinated control, demand side management, correlation coefficient, intermittent power supply.

1. INTRODUCTION

The liberalization of the electrical power system, combined with the need for future decarbonization, has led to a rising interest in novel control problems for what has become known as the smart grid. This paper grew out of discussions on possible future architectures for the coordination control for this smart grid, and from some theoretical considerations on coordination control. It does not intend to present a fully detailed view of the future smart grid control architecture. Rather it attempts to sketch a possible new avenue for coordinating the actions of local control agents (e.g. aggregators, cluster managers, or balance responsible parties) operating in the grid.

Renewable supply of power often is unpredictable and uncontrollable. This intermittency must be compensated by controlling demand or by importing power from other sources [1], in order to guarantee the required quality of service. Novel developments in ICT that enable control of many loads, via demand shifting and demand flattening [2], make it possible for many agents to control the local supply and demand thereby contributing to this goal. These many different decision makers, acting in different locations over different time scales, try to achieve local, often conflicting, goals. Only if the actions by these competing agents are properly coordinated will the stability and the efficient operation of the smart grid be guaranteed [2]. Robustness requirements for reliable and efficient operation of the smart grid imply that as little communication as possible should be used for the coordination of these decision makers. Therefore we look in this paper for a control architecture that only requires broadcasting signals from a top layer controller to lower level local demand shaping controllers, without any need for local control agents to transmit information to the central controller nor to their neighbors.

The approach proposed in this paper could be implemented at different levels in the control hierarchy of the smart grid, but in order to simplify the presentation we only consider the interaction between two layers, taking as an example a distribution net, operated by a "smart" distribution grid operator (this higher level supervisory controller is called a GO from now on) owning some of the renewable generation or buying renewable power. This GO has many customers, some large customers with their own local demand shaping controller, others like intermediate aggregating providers that influence the demand of smaller indirect customers. The legal and technical structure of the interactions between the different players in the smart grid operation, esp. for demand shaping controllers, is not the topic of this paper, but in order to simplify the presentation we will call these local controllers aggregators from now on (as in [3], while in [6] a very similar decision

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maker is called the CM, the cluster manager for a resource cluster).

Power available at time t to the smart grid consists of two components, some controllable power $g(t)$ supplied by the main grid or obtained locally, e.g. from storage devices or from small CHP installations, and some unpredictable and uncontrollable renewable power $w(t)$ from wind and PV installations (the methods of this paper could also be applied to a microgrid in islanded mode, with some minor modifications). The renewable power $w(t)$ is assumed to be a stochastic process with known probabilistic characteristics. Power demand $\ell(t)$ at time t consists of an uncontrollable time-varying base load, and some controllable loads. Controllable loads like batteries or thermostatic loads can be shifted in time subject to some constraints [3-7]. These control actions u of shifting load implemented by the different aggregators transform the uncontrolled demand $\ell(t)$ into a controlled load $\ell(t,u)$. The control action $u(t)$ at time t depends on the history (and if possible predictions) of demand an supply, as far as this is known to the local aggregator at time t.

The uncontrolled load $\ell(t)$ and the renewable supply $w(t)$ are stochastic processes, whose second order properties [9] are characterized by their mean $E\ell(t)$, resp. $Ew(t)$, their variance $\sigma_\ell^2 = E[\ell(t) - E\ell(t)]^2$, resp. $\sigma_w^2 = E[w(t) - Ew(t)]^2$, and by their correlation coefficient $\rho_{\ell,w}(t) = E[\ell(t) - E\ell(t)] \times [w(t) - Ew(t)]$. The variance σ_ℓ^2 expresses how likely large excursions away from the mean are for the load. The correlation coefficient $\rho_{\ell,w}(t)$ expresses how similar $\ell(t)$ and $w(t)$ look as random processes, i.e. whether the load is likely to be high when the supply is high - as would be very desirable in order to optimally use renewable resources - or whether the opposite is true.

Aggregators can control the load $\ell(t)$ in many ways: by shifting battery charging times subject to meeting deadlines for full battery availability, or by switching on or off thermal loads loads like heaters or freezers as long as their temperature remains inside the bounds given by the thermostat (this basically amounts to using thermal storage), by switching off non-essential devices, and so on. These control actions transform the load $\ell(t)$ in the controlled load $\ell(t,u)$ which in general has the same mean value. The control actions executed by the aggregator at time t depend on the information which the aggregator has available at time t concerning the past local load, the past renewable supply, the value $\rho_{min}(t)$ of the desirable correlation coefficient sent by the GO, and perhaps also on some predictions of both the local load $\ell(\tau)$ and of the GO-supply $w(\tau), \tau > t$ (these predictions will of course have a variance that increases very quickly the farther in the future they go). These control actions can flatten the demand, reducing $\sigma_{\ell(t,u}^2 < \sigma_\ell^2$, reducing one source of uncertainty in the operations of the smart grid. As shown in [8] this flattening of the load actually uses the negative correlation among the prosumers that belong to one single aggregator (in [8] the members of an aggregator are called producers, but in fact negative production, that is consumption, is allowed). Reducing the variance reduces the peak power demand, which typically is the most expensive power demand.

This flattening should also take into account that demand should be matched as closely as possible with the randomly and unpredictably varying renewable supply $w(t)$, in order to maximize the desired greening effect, and in order to reduce the cost due to expensive unscheduled import of non-renewable supply (or the equally expensive drawing of power from storage). A high correlation coefficient, meaning that peaks and valleys in demand more or less coincide in time with peaks and valleys in the renewable supply, implies that little expensive non-renewable power will be needed. Moreover it will be easy to stabilize the system in that case. Ancillary services for frequency and voltage stabilization will not be required as often.

This paper explains how imposing minimal values ρ_{min} on the correlation coefficients $\rho_{\ell,w}(t)$ that is achieved by an aggregator can help in improving the performance of the smart grid. The value ρ_{min} can be broadcast by the GO to all its aggregators. The specified minimal correlation coefficients broadcast to aggregators are easy to understand and to implement by each aggregator control agent (and their designers), and require little bandwidth for communication. A minimal correlation coefficient between supply and demand is indeed a good candidate as a signaling variable used for coordinating control actions of different aggregators and GO, requiring limited communication only. It does not require the GO to know any on-line details about the load at aggregators.Moreover aggregators could get a financial reward for accepting to achieve higher correlation coefficients, reducing the specifications for other aggregators (that of course will face a higher price). Note that if such a scheme with price dependent specifications were implemented it would require two-way communication, not only broadcasting information from the GO to the aggregators, but also transmitting information from aggregators to GO about their willingness to pay or their ability to achieve a certain specification.

The next section of this paper presents a general discussion of the proposed architecture, with some intuitive explanations. Section 3 presents the rather trivial calculations explaining how $\rho_{\ell,w}(t,u)$ influences cost of grid import and distribution losses for a simple DC model of a distribution system consisting of one single line. Section 4 explains some extensions to more realistic cases, with an AC model, for a tree structured distribution net. These two sections also show how the correlation coefficients achieved by different aggregators, using the same renewable supply $w(t)$, influence the imported power from the grid and the distribution losses. This is important because it shows that this performance measure $\rho_{\ell,w}(t,u)$ can indeed be used in order to allocate to different aggregators a reward for providing ancillary services. The final section of this paper briefly describes in general terms how the proposed approach using $\rho_{\ell,w}(t,u)$ as a coordinating signal can be applied for some examples of load management.

2. PROPOSED COORDINATION ARCHITECTURE

The basic idea of this paper is that a high correlation coefficient between controllable load and renewable supply, achieved by achieved globally and by each aggregator, will improve the efficiency and the reliability of the smart grid by reducing the peak demand for non-renewable grid power, and by reducing the losses in the distribution network. Below we show that the expensive peak import of grid power as well as the losses in the distribution network depend on the expected square value

$$Eg^2(t) = E(\ell(t,u) - w(t))^2 = \quad [E\ell(t,u)]^2 + [Ew(t)]^2 \\ +\sigma_\ell^2(t,u) + \sigma_w^2(t) - 2.\rho_{\ell,w}(t,u).\sigma_\ell(t,u).\sigma_w(t)$$

The difference $\ell(t,u) - w(t) = g(t)$ represents the import of grid power needed to balance supply and demand. A large value of $Eg^2(t)$ implies that there is a high probability of large peaks in this import $g(t)$. In order to minimize this expensive peak import, and as will be shown below in order to minimize distribution losses, aggregators should control the load by implementing feedback control actions $u(t)$ so that the variance $\sigma_\ell^2(t,u)$ is reduced and the so that the correlation coefficient $\rho_{\ell,w}(t,u)$ is as large as possible. The control actions that influence $\sigma_\ell^2(t,u)$ correspond to flattening the load, and must take into account mainly the local limitations and specifications of each local load. An aggregator will e.g. try to flatten its total load by generating negative correlation between the different tasks it must serve [8], and by shifting demand taking into account deadlines and storage capabilities [4]. Different aggregators compete for the same (renewable and grid) power, and for the same flow capacity along the distribution lines. This interaction between aggregators depends on, as well as influences, the expected evolution of the market price for electricity.

Besides flattening demand it is also useful to match demand $\ell(t,u)$ and renewable supply $w(t)$ by increasing $\rho_{\ell,w}(t,u)$ in order to reduce $Eg^2(t)$. This by the way also can be seen as an ancillary service limiting voltage and frequency perturbations. Designing properly coordinated control laws for the different decision makers each attempting to match supply and demand requires the availability of some simple measures for quantifying how well each aggregator succeeds in matching is local load $\ell(t,u)$ to the unpredictable evolution of the renewable supply $w(t)$. Setting specifications on the minimal correlation coefficient $\rho_{min}(t)$ that the aggregator must achieve is one way to guarantee that each aggregator gets a fair share of the renewable supply $w(t)$, and of the flow capacity, and at the same time to ensure that they contribute their fair share to the ancillary services. Setting specifications on the correlation coefficient $\rho_{\ell,w}(t,u)$ is thus a reasonable way of achieving coordination between aggregators, since $\rho_{\ell,w}(t,u)$ expresses the common goal of all the users of the distribution net. This specification on the correlation coefficient can be set by the distribution net operator, the GO, acting as a hierarchically higher level supervisor. The GO, as a hierarchically higher control layer, broadcasts the specification $\rho_{min}(t)$ for the correlation coefficient $\rho_{\ell,w}(t,u)$ to all aggregators. Each aggregator must implement a demand shifting feedback control $u(t)$ (for one large user of power, or for a group of customers including EV charging with deadlines, or for aggregators with a lot of storage capacity by using storage capacity) thus flattening its raw uncontrolled demand $\ell(t)$ to a controlled load $\ell(t,u)$.

Each aggregator must calculate a local estimate $\hat{\rho}_{\ell,w}(t,u)$ of the correlation coefficient that it achieves in order to check whether it is complying with its specifications, by using $\hat{\rho}_{\ell,w}(t,u) = \frac{1}{N}\sum_{n=0\ldots N-1}(\ell(t-n,u) - \hat{\ell}(t,u)).(\widehat{w}(t-n) - \widehat{w}(t))$ over a window of recent values (where the mean values $\hat{\ell}(t,u)$ and $Ew(t)$ are assumed to vary sufficiently slowly so that they can be estimated online) . This requires that the aggregator has access to some approximation $\widehat{w}(t)$ of the global variable $w(t)$. Since the coordination among local agents does require broadcasting signals from the GO to the different aggregators anyway it is a reasonable assumption that the GO also broadcasts some information on $w(t)$. Note that this does not require the aggregator to know a detailed model of the probability distributions of its own load (this probabilistic model might be useful for the demand shaping control decisions though), nor a model of the evolution of the renewable generation $w(t)$. The method does not seem to require more probabilistic information than other proposals for similar goals, like the use of stochastic network calculus [5].

Of course for different applications of this approach the size N of the window for measuring the correlation, and the time step between successive measurements $\ell(t,u)$ and $\ell(t-1,u)$ will be different. This will be related with the time constant of the autocorrelation function of the supply process $w(t)$. This will differ for PV and for wind power, and also will differ depending on the location, and particularly also on the size of the area over which the supplies are aggregated. Both temporal and spatial statistics must be considered.

The presentation of flattening in this section is actually incomplete. Reducing the variance of the load will in fact only reduce the uncertainty at a given time around the mean value of this load. This mean value is also varying in time. Flattening should in fact deal with the autocorrelation function of the load, which means both with the unpredictability at a given time, and with how fast the stochastic process varies. A correct solution would have to deal with all the different time scales at which ancillary services must be provided to the grid, and how this relates to the design of demand shaping controllers. This is a different research topic from the results reported here, and for the sake of a simple presentation we assume as a first approximation that the aggregator simply tries to reduce the variance $\sigma_\ell^2(t,u)$ and increase the correlation coefficient $\rho_{\ell,w}(t,u)$.

3. CORRELATION COEFFICIENTS AND PERFORMANCE FOR SIMPLE DC MODEL

Consider first a very simple model of a microgrid with one single user, as shown in figure 1. Only active power is considered, so all currents and voltages are taken as DC values. The controlled power demand $\ell(t,u)$, further on also called the load, is connected via a short line with resistance R_1 to an uncontrollable, renewable source generating a random amount $w(t)$. The imbalance $\ell(t,u) - w(t) + losses(t)$, supplied by the main grid or from storage devices, consists of a predictable part $g_{scheduled}(t)$ delivered from the main grid according to day-ahead contracts, and an unpredictable part ($losses(t)$ denotes the Ohmic losses in the distribution line at time t that were so far ignored). The unscheduled imported grid power is more expensive since it is essentially provided by ancillary services. Of course the instantaneous price at the time of the peak could by chance be low, or even negative, but this will happen only in the very unlikely case that locally there is a shortage of renewables, while elsewhere there is a significant oversupply of renewables. Moreover the imported grid power passes via a long line with resistance R_0 (typically $R_0 > R_1$) causing more losses. Reducing unscheduled power import and avoiding storage also avoids higher cost for installing extra transmission capacity, transformer capacity, and (if storage is also used for demand balancing) extra costs for batteries. It is thus obvious that especially the peaks of $\ell(t,u) - w(t) + losses(t)$ must be reduced. All these arguments together show the importance of making the variance $\sigma_{\ell(t,u)}$ small while ensuring that the correlation coefficient $\rho_{\ell,w}(t)$ is large enough.

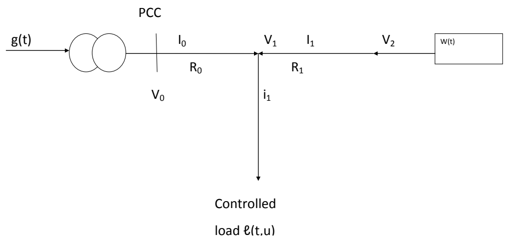

Fig. 1. a simple DC microgrid model

There exists a huge literature on demand side management (for some references see e.g. [4,8]). Given some requests $\ell(t)$ from users for power or energy, the local controller, either the user itself or an aggregator or a balance responsible party (BRP), can shift some of this demand in time to achieve a controlled load $\ell(t,u)$, as explained above.

As can be seen from figure 1

$$g(t) = \ell(t,u) - w(t) + losses(t) \tag{1}$$

where

$$\begin{aligned} losses(t) &= R_0.I_0(t)^2 + R_1.I_1(t)^2 \\ &= R_0.(i_1(t) - I_1(t))^2 + R_1.I_1(t)^2 \\ &= R_0.(\frac{\ell(t,u) - w(t) - losses(t)}{V_1})^2 + R_1.(\frac{w(t)}{V_2})^2 \\ &\approx R_0.(\frac{\ell(t,u) - w(t)}{V_0})^2 + R_1.(\frac{w(t)}{V_0})^2 \end{aligned}$$

where the last approximation follows from the assumption that even in this distribution net the $losses(t)$ are small, and that voltage drops along the lines are small. Of course control actions must still minimize the losses, and keep voltages within bounds, but for the purpose of understanding the effect of correlation on the systems performance through these formulas this approximation is sufficient. It can be shown that the expressions obtained here are first order Taylor series expansions. In the above formulas V_0 is assumed to be kept constant by an appropriate higher level controller, while $w(t)$ is a stochastic process observed by the GO.

All other variables characterizing the behavior of the smart grid are also stochastic processes, parameterized by V_0, and depending on the basic stochastic processes $w(t)$, and on $\ell(t,u)$, thus also depending on the control actions u. The currents and voltages are obtained from: $i_1(t) = \frac{\ell(t,u)}{V_1} \approx \frac{\ell(t,u)}{V_0}$, $I_1(t) = \frac{w(t)}{V_2(t)} \approx \frac{w(t)}{V_0(t)}$, $I_0(t) = i_1(t) - I_1(t)$, $V_1(t) = V_0 - R_0.I_0(t)$, and $V_2(t) = V_1(t) + R_1.I_1(t)$.

The expected import of power from the grid is

$$Eg(t) = E\ell(t,u) - Ew(t) + E(losses(t))$$

where

$$\begin{aligned} E(losses(t)) = \quad & R_0.E(\frac{\ell(t,u) - w(t)}{V_1})^2 + R_1.E(\frac{w(t)}{V_2})^2 \\ \approx \quad & \frac{R_0}{V_0^2}.E(\ell(t,u) - w(t))^2 + \frac{R_1}{V_0^2}.(E(w(t)^2) \end{aligned} \tag{2}$$

which, assuming that $R_1 \ll R_0$ depends mainly on

$$\begin{aligned} E(\ell(t,u) - w(t))^2 = [E(\ell(t,u) - w(t))]^2 + \sigma_{\ell(t,u)}(t)^2 \\ + \sigma_w(t)^2 - 2.\rho_{\ell,w}(t).\sigma_{\ell(t,u)}(t).\sigma_w(t) \end{aligned} \tag{3}$$

where

$$\sigma_{\ell(t,u)}(t)^2 = E[\ell(t,u) - E\ell(t,u)]^2$$

is the variance of the controlled load (measuring the performance of demand flattening), while

$$\sigma_w(t)^2 = E^[w(t) - Ew(t)]^2$$

is the variance of the random renewable supply. The correlation coefficient

$$\rho_{\ell,w}(t) = E[(\ell(t,u) - E\ell(t,u)).(w(t) - Ew(t))]$$

describes how closely the demand shaping control u succeeds in matching demand to renewable (and cheap) supply.

The expected losses depend on

- how well the long term control actions - day-ahead or hour-ahead contracts - match the average $E(\ell(t,u))$ to the expected $Ew(t)$,
- how well the local control actions smoothen out the randomness in the controlled demand by reducing its variation $\sigma_{\ell(t,u)}(t)^2$, which is partly achieved by flattening demand,
- how large the correlation coefficient $\rho_{\ell,w}$ can be made.

Selecting a large value of the correlation coefficient reduces the distribution losses. Indirectly this reduces the average import of power from the grid. More importantly a large value of the correlation coefficient reduces the variance of the imported load (to simplify the calculation we ignore below the correlation between the losses, the load $\ell(t,u)$, and the renewable power $w(t)$) :

$$\begin{aligned} \sigma_g(t)^2 = & E[g(t) - Eg(t)] \approx [E(\ell(t,u) - w(t))]^2 \\ & + \sigma_{\ell(t,u)}^2 + \sigma_w^2 - 2.\rho_{\ell,w}.\sigma_{\ell(t,u)}.\sigma_w + Elosses(t)^2 \end{aligned} \tag{4}$$

The smaller $\sigma_g(t)$ is the smaller the risk is that there is a high peak in the imported power $g(t)$. Indeed assume for simplicity that $g(t)$ has a Gaussian distribution, then the probability

$$\wp(g(t) \geq Eg(t) + k.\sigma_g(t))$$

(which can be found by looking up tables of Q-functions or complimentary error functions in statistics books) is a complicated increasing function of $k.\sigma_g(t)/Eg(t)$. Keeping the peaks in $g(t)$ small is important because the smaller the peaks are the lower the risk is that one needs to buy a large amount of power from the grid at the short-term spot price, which may be much higher than the price paid for day-ahead scheduled power import.

4. CORRELATION COEFFICIENTS FOR MORE REALISTIC MODELS

4.1 Several interacting loads

The purpose of using correlation coefficients is to coordinate many different interacting demand shaping local controllers $u_n, n = 1, \ldots, N$ that each determine a local load $\ell_n(t, u_n)$, receiving power from the same renewable sources, with as ultimate goal that

$$\sum_{n=1,\ldots,N} \ell_n(t, u_n) - w(t)$$

have a small variance. Hence it is necessary to investigate how the formulae look like in the case of N loads competing for the same power:

$$E[\sum_{n=1,\ldots,N} \ell_n(t, u_n) - w(t)]^2 = [E(\sum_{n=1,\ldots,N} \ell_n(t, u_n) - Ew(t))]^2 + \sum_n \sigma^2_{n,\ell} + \sigma^2_w - 2.\sum_n \rho_{\ell_n,w}.\sigma_{n,\ell}.\sigma_w \quad (5)$$

where $\rho_{\ell_n,w}$ and $\sigma_{n,\ell}$ represent the correlation coefficient with $w(t)$ and the variance as calculated for the individual loads $\ell_n(t, u)$.

If one would know a priori the relative importance of the loads one could divide up the renewable supply a priori according to the relative loads and try to find a minimal value for

$$\sum_{n=1,\ldots,N} (\ell_n(t, u_n) - \alpha_n.w(t))]^2$$

where $\sum_n \alpha_n = 1$, and where α_n should be selected so as to somehow optimize overall behavior. However in practice the GO does not know this relative importance in advance, since this would require transmitting very detailed information from aggregators to GO. The best possible choice for the different specified minimal correlation coefficients should be determined according to some long term average load information, and should lead to fair and robust specifications for each local aggregator.

Evaluating the losses in this case becomes a lot more complicated. Consider one single branch of the distribution net in figure 2 (DC case, with resistances instead of impedances, to keep the notation simple):

$$losses_n(t) = \sum_{j=1,\ldots,N} R_{N-j}(\frac{\sum_{k=0,\ldots,j-1} \ell_{N-k}(t, u) - w_n(t)}{V_{N-j}})^2 \quad (6)$$

Ignoring voltage drops along the lines, i.e. $V_k \approx V_0$ its average can be written as

$$Elosses_n(t) = \sum_{j=1,\ldots,N} \frac{R_{N-j}}{V_0^2}[N.(E(w(t))^2 + \sigma^2_w) + \sum_{k=0,\ldots,j-1} ((N-k).(E(\ell_{N-k})^2 + \sigma_{\ell_{N-k}})^2 - 2.\rho_{\ell_{N-k},w}).\sigma_{\ell_{N-k}}.\sigma_w)] \quad (7)$$

(assuming somewhat unrealistically that even after control actions u_n, that depend on the same information, have influenced them the different stochastic processes $\ell_{n,u_n}(t, u_n)$, are stochastically independent of each other, and also indepedent of $w_n(t)$). Minimizing expected losses now depends on the current distribution over the different sections of the branch. One can expect that the currents will inevitably be large near the bus and near the renewable source, and will be small in the middle (where the voltage also is lowest). A good compromise must be found by cooperation among the different aggregators, taking into account the long term averages $(E(\ell_n))^2$, the local variances $\sigma^2_{\ell_n}$, and the costs due to the local mismatch between supply and demand as measured by the correlation coefficients $\rho_{\ell_n,w}$. This optimization depends also on satisfying all constraints like maximal line currents in each section, keeping voltages between bounds. The performance depends on the control actions by slow controllers at a hierarchically higher level selecting both the average values $(E(\ell_n))$ and the specifications for the correlation coefficients to be achieved by each local controller $\rho_{\ell_n,w}$. The performance will also depend on the fast local controllers that try to flatten local demand $\ell_n(t, u_n)$ thus reducing $\sigma^2_{\ell_{N-k}}$, while achieving a sufficiently high value of the correlation coefficient $\rho_{\ell_n,w}$ as imposed by the slower higher level controller. Finally the assignment should be fair, meaning that the performance seen by a customer should not depend on its geographical location (unless this difference is reflected in its rates). While the formulae are certainly more complicated in this case it is still clear that imposing a minimal correlation coefficient is a possible approach to force many different aggregators competing for power on the same distribution grid to act in a coordinated way.

4.2 AC models, with active and reactive power balancing, along one single distribution line

The calculations above can be repeated in the realistic case of AC grids, where both active and reactive load and supply must be balanced. All the resistances must be replaced by complex impedances ($Z_n = R_n + j.X_n$), the voltages $V_n(t)$ and currents $I_n(t)$ now become complex phasors, and the power is now expressed by the complex power $\ell(t, u) = V_1(t).i_1(t)^*$, $w(t) = V_2(t).I_1(t)^*$, and $g(t) = V_0.I_0(t)^* = \ell(t, u) - w(t) - losses(t)$. What does become more complicated is the evaluation of the losses: the losses in line 0 are $Z_0.I_0.I_0^* = (R_0 + j.X_0).((ReI_0)^2 + (ImI_0)^2)$ and a similar formula holds for the losses in other lines. The aggregators now must balance both active power and reactive power. Imbalances must now be compensated not just by import of expensive active power, but also by import of reactive power, in current practice from the main grid. Note however that in case the inverter at the renewable source is not required to work at unity power factor, as is currently required, then an extra degree of freedom becomes available for the control since the renewable source can be used as a local source of reactive power, contributing to the voltage stabilization. The correlation coefficients for matching reactive power will in this case be imposed not on the demand shaping of loads, but must be shared by demand shaping of load and control of the power factor of renewables. This problem is more involved, and needs further investigation.

Calculations identical to those in subsection 3.1 show that in the AC case the losses are minimized, and the peak import of expensive grid power is reduced, by the local decision of flattening the local (active and reactive) power demand, and by making the correlation coefficient $\rho_{a,\ell,w}$ between $Re(\ell(t,u))$ and $Re(w(t)$ large, as well as making now also the correlation coefficient $\rho_{i,\ell,w}$ between $Im(\ell(t,u))$ and $Im(w(t))$ large. Again it is reasonable to use this correlation coefficient as a coordinating variable set by the GO, the same controller that influences the reactive power supplied by the renewable source, and by designing the local demand shaping controllers in the aggregators so that they reduce the variance of the active and reactive power demand, subject to achieving sufficiently high correlation coefficients $\rho_{a,\ell,w}$ and $\rho_{i,\ell,w}$.

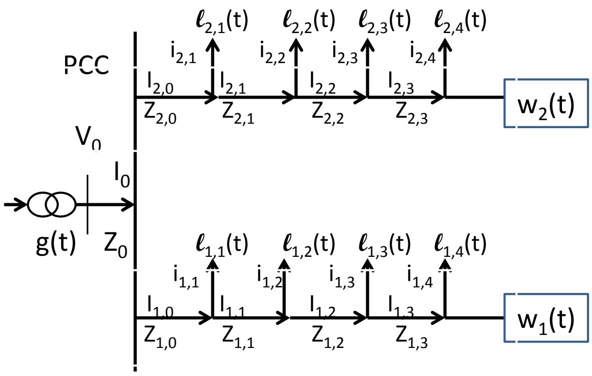

Fig. 2. model of a simple distribution grid with 2 renewable sources

4.3 Microgrids with several renewable sources

The correlation coefficient between load and supply can also be used as a variable for coordinating the actions of several providers, each acting as a GO, in the case where many renewable sources (possibly operated by different suppliers) are connected to a loop-free distribution network, as shown in figure 2. For simplicity we assume that the significant renewable power sources in this distribution net are still concentrated at the end of branches, the nodes farthest removed from the PCC bus. This assumption only has minimal effect on the the calculations for the import of power from the grid, as only the losses (always small even in the distribution network) are influenced by the location of the renewable resources.

The import of grid power is:

$$g(t) = \sum_{n,k}(\ell_{n,k}(t,u_{n,k}) + losses_n(t)) - \sum_n w_n(t) \tag{8}$$

where the index n refers to the n-th branch of the distribution tree, $w_n(t)$ represents the renewable power on the n-th branch (only 2 branches are shown in figure 2), and $\ell_{n,k}(t,u_{n,k})$ the load connected to the k-th node of the n-th branch, controlled by local demand shaping controller $u_{n,k}$. If the losses can be ignored and if the impedances in the branches beyond the bus are so small that the renewable power can be transported from one branch to another without significant losses (insignificant compared to the loss encountered by transporting grid power over the line with impedance Z_0) then the problem reduces to the same problem as before: the correlation coefficients between $\ell_{n,k}(t,u_{n,k})$ and $\sum_n w_n(t)$ determine how well coordinated the different local controllers are in reducing the peak imported power. If however the distribution losses for transporting power between the 2 branches are significant then it will be better to match each local load to the renewable supply at its own branch only in the first place, transferring renewable power from one branch to the other only when this turns out to be cheaper than importing grid power. In other words then the aggregator in node (n,k) should flatten its local demand, taking into account the correlation coefficient between $\ell_{n,k}(t,u_{n,k})$ and $w_n(t)$, again the same problem as treated before.

It is also interesting to note that these formulae remain valid irrespective of whether the different branches of the distribution tree are owned and operated by the same power provider, or whether they are operated by independent providers. In the last case the formulae can be used to evaluate the contribution of each provider to the distribution net performance. The correlation coefficient that each provider imposes on the local controllers under its supervision could be used, besides more obvious variables as generated renewable power $w_n(t)$, as a measure of their contribution to keeping the losses and the peak import from the grid small. This in turn can be used as a tool for calculating the reward each provider should get for the power as well as for the ancillary services it provides.

5. HOW TO IMPLEMENT LOCAL CONTROLLERS USING CORRELATION COEFFICIENT SPECIFICATIONS?

5.1 Computation and communication requirements

It is obvious that in order for aggregator (k,n) to be able to adhere to the minimal correlation coefficient $\rho_{\ell_{k,n},w}(t,u_{k,n})$ imposed on it by the GO, it is necessary that control agent (k,n) receives information on the value of $w_n(t)$. This requires a communication network that can, at times $t_j = j.\delta$, broadcast the value of $w_n(t_j)$ measured at the renewable source of branch n to all the nodes on that branch n of the distribution network (and if available also broadcast predictions on future values of this renewable supply). At time t the agent implementing control law $u_{k,n}$ will thus know a discrete sequence of measurements $w_n(h.\delta), h = 1,\ldots,\lceil t/\delta\rceil$, (and possibly predictions). Observing the local controlled demand by measuring $i_{k,n}(j.\delta), V_n(j.\delta), j = 1,\ldots,\lceil t/\delta\rceil$, the local control agent can then evaluate at time t a finite window estimate $\hat{\rho}_{\ell_{k,n},w}(h.\delta)$ of the correlation coefficient $\rho_{\ell_{k,n},w}(t)$:

$$\hat{\rho}_{\ell_{k,n},w}(h.\delta) = \frac{1}{J}\sum_{j=0,\ldots,J-1}[(\ell_{k,n}((h-j).\delta) - \hat{\ell}_n((h-j).\delta)). \\ .(w_n((h-j).\delta) - \hat{w}_n((h-j).\delta))] \tag{9}$$

where $\hat{\ell}_{k,n}((h-j).\delta) = \frac{1}{M}\sum_{m=0,\dots,L-1}\ell_{k,n}((h-j-m).\delta)$ is a time adaptive estimate of the average local load in the recent past, while $\hat{w}_n((h-j).\delta) = \frac{1}{M}\sum_{m=0,\dots,L-1} w_n((h-j-m).\delta)$ is a window based average of the renewable power generation on that branch (this value is the same for all local controllers and could be calculated by the GO at the renewable source, and broadcast to all aggregators together with the current values $w_n(h.\delta)$).

In order to calculate the best local controller each aggregator also must know an estimate

$$\hat{\sigma}_{\ell_{k,n}}(h.\delta) = \frac{1}{J}\sum_{j=0,\dots,J-1}[\ell_{k,n}((h-j).\delta) - \hat{\ell}_{k,n}(h.\delta)]^2 \quad (10)$$

of the variance of its local load, and an estimate (since it appears in (3) as a multiplier of $\rho_{k,n}(\ell_{k,n},u)$)

$$\hat{\sigma}_{w_n}(h.\delta) = \frac{1}{J}\sum_{j=0,\dots,J-1}[w_n((h-j).\delta) - \hat{w}_n(h.\delta)]^2 \quad (11)$$

of the variance of the renewable supply (which again can be calculated locally or broadcast by the renewable source node).

Notice that the above estimates implicitly assume that the averages of $\ell_{k,n}(t)$ and of $w(t)$ vary slowly, so that they can be treated as constant over an estimation window of length $J.\delta$. All the other estimated parameters also are assumed to vary slowly over time, in order for these estimates to be sensible. Sudden changes in some of the loads or in the uncontrollable supply of renewable power will cause transients that deteriorate the performance of the system. This limitation however is inevitable unless a fault detection algorithm is installed that quickly detects these disturbances, and then broadcasts this information to all the other aggregators and to the GO.

5.2 Demand shaping control using correlation coefficients

Summarizing the computational and communication requirements we see that

- each GO_n must be able to measure its renewable generation $w_n(t)$ at times $t = \ell.\delta$ for an appropriately selected value of δ, and must estimate the windowed average $\hat{w}_n((h-j).\delta)$ and the variance of $w_n(t)$ ($\hat{\sigma}_{w_n}(h.\delta)$ as defined in (11);
- each GO_n must broadcast its measurements $w(k.\delta)$ and its estimates $\hat{w}_n((h-j).\delta)$ and $\hat{\sigma}_w(h.\delta)$;
- each aggregator receives the information $w_n(h.\delta)$, $\hat{w}_n((h-j).\delta)$, and ($\hat{\sigma}_{w_n}(h.\delta)$ for all $h \le t/\delta$, as broadcasted by its GO_n supervisor, and uses this information together with local probabilistic information on its local load in order to calculate the demand shifting controller $u_{k,n}$ that provides the best compromise between meeting demands as timely as possible, minimizing if possible expensive imported grid power, and satisfying the specification $\hat{\rho}_{\ell,w}(t,u) \le \rho_{min}$ (which of course also implies that the aggregator must calculate its value $\hat{\rho}_{\ell,w}(t,u)$).

How the optimization in the last bullet of the above itemization is actually carried out depends on the particular application. The cost to be minimized must be a weighted sum of the cost related to the peak power (as calculated in (3) or in (5)), the losses according to (2), but also the cost for deviating from the raw demand as requested by the customers in $\ell(t)$. If (3) is used, then a fully distributed approach is taken, where each agggregator optimizes its behavior, with only the specified correlation coefficient ρ_{min} as coordinating information. In many applications one also uses common price information in order to find the optimal demand shifting controller. If (5) is used for the peak demand evaluation, together with price information, then more complicated strategies that explicitly take into account the interactions can be obtained.

6. CONCLUSION

This paper proposes the correlation coefficient between the controlled load of each aggregator, and the renewable source that it can be fed from, as a signal to be broadcast by a central controller to each aggregator. This defines the minimal correlation that must be achieved by each aggregator in implementing its demand shaping actions. Adding the correlation coefficient specification to other local optimization considerations makes it possible to achieve a fair division of the renewable supply, such that the smart grid is operating efficiently and reliably.

7. REFERENCES

[1] Vandoorn, T.L.; Zwaenepoel, B.; De Kooning, J. D M; Meersman, B.; Vandevelde, L., "Smart microgrids and virtual power plants in a hierarchical control structure," Innovative Smart Grid Technologies (ISGT Europe), 2011 2nd IEEE PES International Conference and Exhibition on , vol., no., pp.1,7, 5-7 Dec. 2011 doi: 10.1109/ISGTEurope.2011.6162830

[2] D. Bakken et al.: "GRIP - Grids with Intelligent Periphery: Control architectures for Grid2050", IEEE Int. Conference on Smart grid Communication, Oct. 17-20, 2011, Brussels, 7-12

[3] M. Amizadeh, G. Kesidis, A. Scaglione: Clustering consumption in queues: a scalable model for electric vehicle scheduling,

[4] J. de Hoog et al. "Optimal Charging of Electric Vehicles taking Distribution Network Constraints into Account", submitted

[5] K. Wang et al. "How Stochastic Network Calculus Concepts Help green the Power Grid", Architectures and Models for the Smart Grid (IEEE SmartGridComm), http://smart.caltech.edu/papers/stochcalculus.pdf.

[6] A. Nayyar, K. Poola, P. Varaiya: A statistically robust sharing mechanism for an aggregate of renewable energy producers, 2013 European Control Conference, July 17-19 2013, Zürich.

[7] A. Subrmanian et al.: Real-time scheduling of deferrable loads, ACC 2013, pp.3643-3650.

[8] L. Gan et al.: Real-time deferrable load control: handling uncertainties or renewable generatio, ACM eEnergy 2013.

[9] S. Ross: A First Course in Statistics, Pearson New International Edition, 2013.